2.882 System Design and Analysis

February 14

What we'll do today

- Information content
- Robustness

Review

- Design process / Domain
- Functional Requirements
- Design Parameters

Review

Design Domain

- How do you go about ‘design’?

What do customers want? → ***How*** does our product/system satisfy it?

What do we want to achieve? → ***How*** do we want to achieve it?

What are the solutions to be generated? → ***How*** do we want to generate it?

Review

Functional Requirement (FR)

- Functional requirements (FRs) are a *minimum set* of independent requirements that completely characterize the *functional needs* of the artifact (product, software, organization, system, etc.) in the functional domain.

  – Independently achievable, in principle.

  – “The determination of a good set of FRs …… requires skill, …, and many iterations”

Review

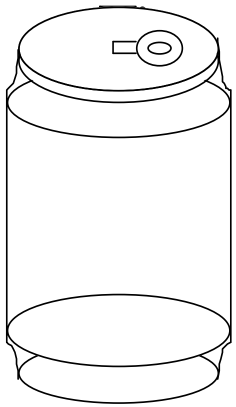
What constitute a GOOD set of FRs?

- MECE: Mutually Exclusive and Collectively Exhaustive
- One FR carries only one requirement
  - Juxtaposing two requirements into one doesn't make them a single requirement
- Solution neutral
  - "Cover the battery contact by a plastic sliding door"
- Clarity/specificity
  - Bad example: missing target range, time factor, etc.
- Logical hierarchical structure

Review

Design Parameter (DP)

- Design parameters (DPs) are the key physical (or other equivalent terms in the case of software design, etc.) variables in the physical domain that characterize the design that satisfies the specified FRs.



DPs of a aluminum beverage belonging to the main body piece
- Body thickness
- Shape of the bottom
- …

Review

Searching for a DP

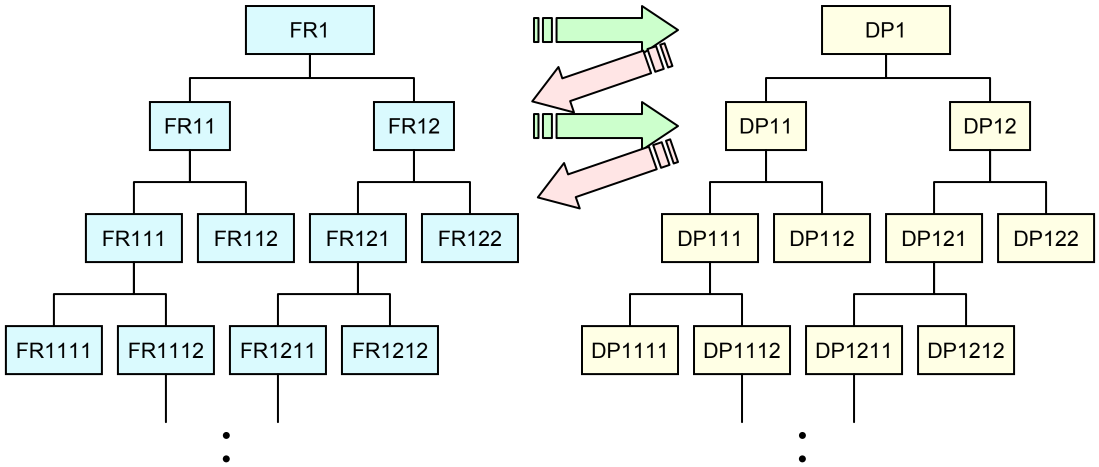
- "Nothing substitutes for Knowledge"
- Be open to wild idea

- Analogies
  - Benchmarking, Patents, Literatures, etc. in OTHER application area
- Visualization
  - Sketch your idea
- Stimuli
  - Circulating ideas
  - Get exposed to foreign situations

Review

Design Hierarchy

- Decomposition by zigzagging
  - Process of developing detailed requirements and concepts by moving between functional and physical domain
  - Yields a hierarchical FR-DP structure



- Decomposition
  - Process of breaking down a large problem into a set of smaller ones so that each of the sub-problems is manageable
- Zigzagging
  - Decomposition process must involve both functional and physical domain by moving between the two domains
  - Upper-level choice of DP drives the FRs at the next level
- Lower-level FRs are a complete description of functional needs of the upper-level FR-DP pair
  - Parent-Child relationship

Information content

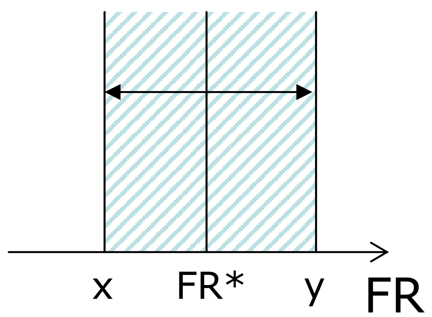
- Design range
- System range
- Probability of success
- (Allowable) Tolerance

Design Range

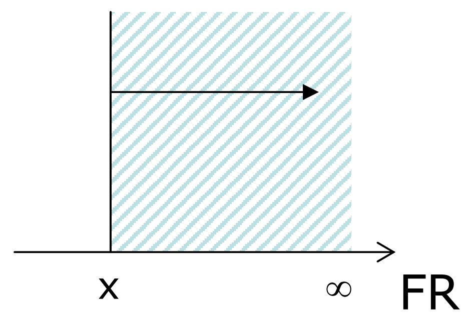
- Examples of "range" in FR statements
  - Maintain the speed of a vehicle at a designated mph *in the range of 0mph - 90mph*
  - Maintain the speed of a vehicle *at a x mph +/- 5mph*
  - Ensure *no* leakage under pressure *up to 100 bar*
  - Ensure *99%* successful ignition at the first attempt in the temperature *range of -30℃ ~ 80℃*
  - Generate nailing forces of *up to 2,000 N*

Design Range

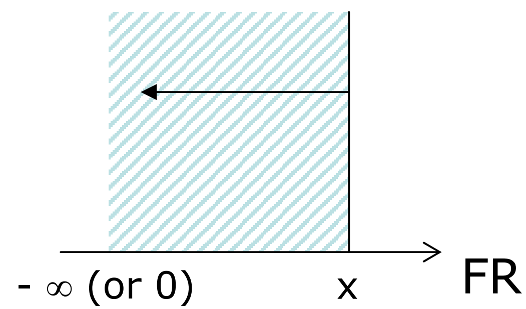
- Specification for FR
- Acceptable range of values of a chosen FR metric; Goal-post
- Different from “tolerance”
- Different from “operating range”
- Target value (nominal), Upper bound, Lower bound



Between x and y



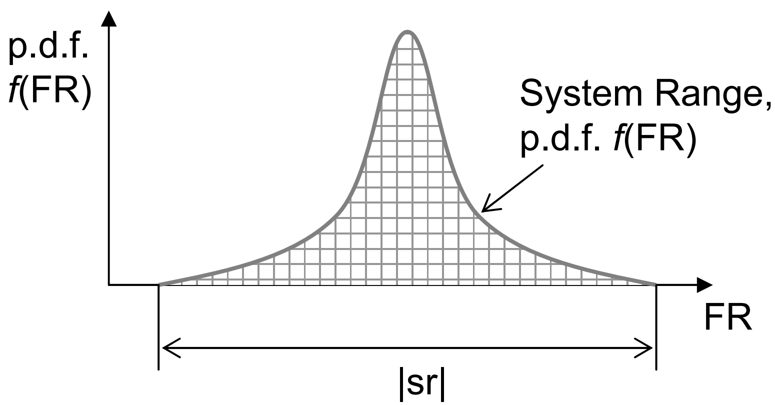
Greater than x
(Larger the better)

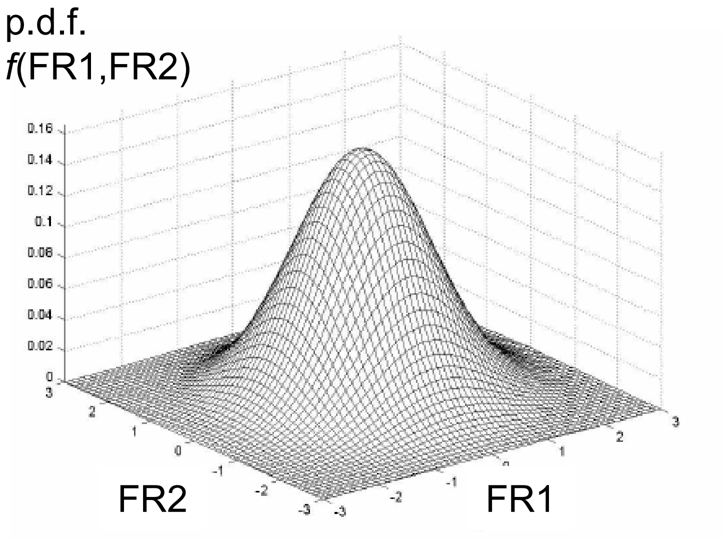


Smaller than x
(Smaller the better)

System Range

- Response/performance in FR domain, resulting from the chosen 'design'
  - Here, 'design' includes both a chosen set of DPs and the way they deliver/affect FRs
- Due to various factors such as the input (DP) variation, internal/external noise, etc., FR takes a range of values, forming a range

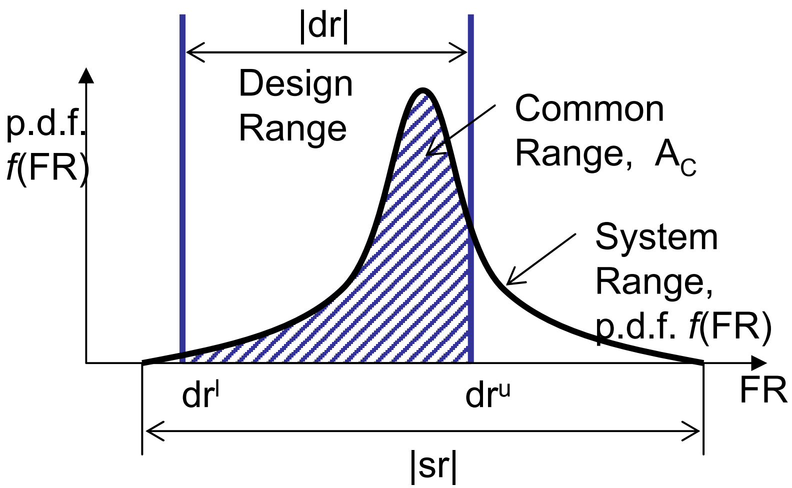


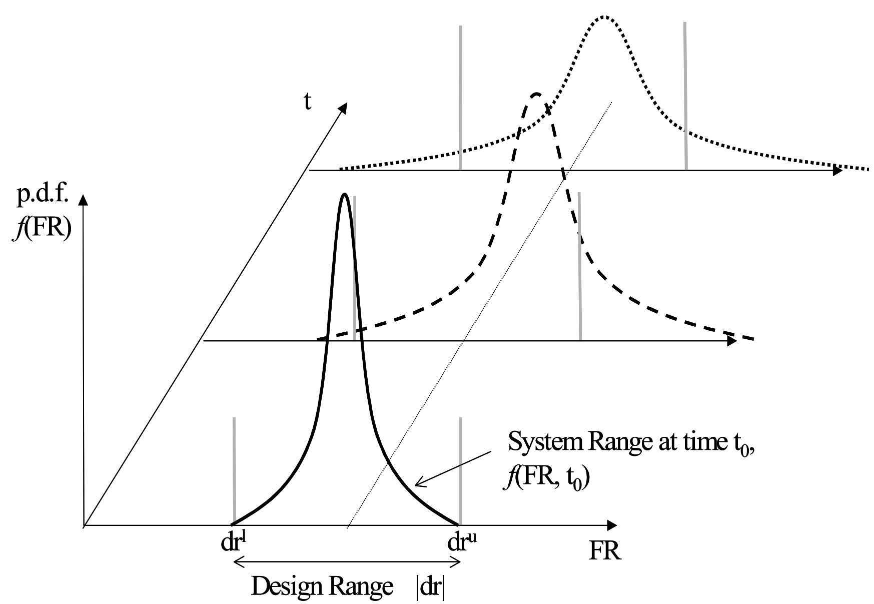


Information content

$$P(FR) = \int_{dr^l}^{dr^u} f(FR)dFR$$

$$I = -\log_2 P = -\log_2 P(FR) = -\log_2 \int_{dr^l}^{dr^u} f(FR)dFR$$





Where do we get $f(FR)$?

Information content for multiple FRs

$$I(FR_1, FR_2, \cdots, FR_n) = -\log_2 p_{1,2,\ldots,n}$$

$$p_{1,2,\ldots,n} = \int_{\text{design hyperspace}} f(FR_1, FR_2, \cdots, FR_n) dFR_1 dFR_2 \cdots dFR_n$$

If Uncoupled,

$$p_{1,2,\ldots,n} = \int_{dr1}\int_{dr2} \cdots \int_{drn} f(FR_1, FR_2, \ldots, FR_n) dFR_1 dFR_2 \ldots dFR_n$$

$$= \int_{dr1} f_1(FR_1) dFR_1 \times \int_{dr2} f_2(FR_2) dFR_2 \times \cdots \times \int_{drn} f_n(FR_{1n}) dFR_n$$

$$= p_1 p_2 \ldots p_n$$

$$I(FR_1, FR_2, \cdots, FR_n) = I(FR_1) + I(FR_2) + \ldots + I(FR_n)$$

If Decoupled,

$$p_{1,2,...,n} = \int_{dr1}\int_{dr2}\cdots\int_{drn} f(FR_1, FR_2, ..., FR_n)\, dFR_1 dFR_2 ... dFR_n$$

$$= \int_{dr1}\int_{dr2}\cdots\int_{drn} f(FR_n|FR_1, FR_2, ..., FR_{n-1}) f(FR_{n-1}|FR_1, FR_2, ..., FR_{n-2}) \cdots$$

$$\cdots f(FR_2|FR_1) f(FR_1) \quad dFR_1 dFR_2 ... dFR_n$$

For example,
$$\begin{Bmatrix} FR_1 \\ FR_2 \end{Bmatrix} = \begin{bmatrix} a & O \\ b & c \end{bmatrix} \begin{Bmatrix} DP_1 \\ DP_2 \end{Bmatrix}$$

$$p_{1,2} = \int_{dr1}\int_{dr2} f(FR_1, FR_2)\, dFR_1 dFR_2$$

$$= \int_{dr1}\int_{dr2} f(FR_2|FR_1) f(FR_1)\, dFR_1 dFR_2$$

$$= \int_{dr1} \left\{ \int_{dr2} f(FR_2|FR_1)\, dFR_2 \right\} f(FR_1)\, dFR_1$$

$$\begin{Bmatrix} FR_1 \\ FR_2 \end{Bmatrix} = \begin{bmatrix} a & O \\ b & c \end{bmatrix} \begin{Bmatrix} DP_1 \\ DP_2 \end{Bmatrix} \Rightarrow \begin{aligned} DP_2 &= \frac{1}{c} FR_2 - \frac{b}{c} \cdot DP_1 \\ &= \frac{1}{c}\left(FR_2 - \frac{b}{a} FR_1 \right) \end{aligned}$$

Let g1 and g2 be the pdf of DP1 and DP2.

From $$f(FR) = \frac{1}{|a|} g\left(\frac{FR}{a}\right)$$

We get f(FR1) and f(FR2|FR1) :

$$f(FR_1) = \frac{1}{|a|} g_1\left(\frac{FR_1}{a}\right) \qquad f(FR_2 \mid FR_1) = \frac{1}{|c|} g_2\left(\frac{FR_2 - b/a \times FR_1}{c}\right)$$

Since $$p_{1,2} = \int_{dr1} \left\{ \int_{dr2} f(FR_2 | FR_1) dFR_2 \right\} f(FR_1) dFR_1$$

$$p_{1,2} = \int_{dr1} \left[\int_{dr2} \frac{1}{|c|} g_2 \left(\frac{FR_2 - b/a \times FR_1}{c} \right) dFR_2 \right] \frac{1}{|a|} g_1 \left(\frac{FR_1}{a} \right) dFR_1$$

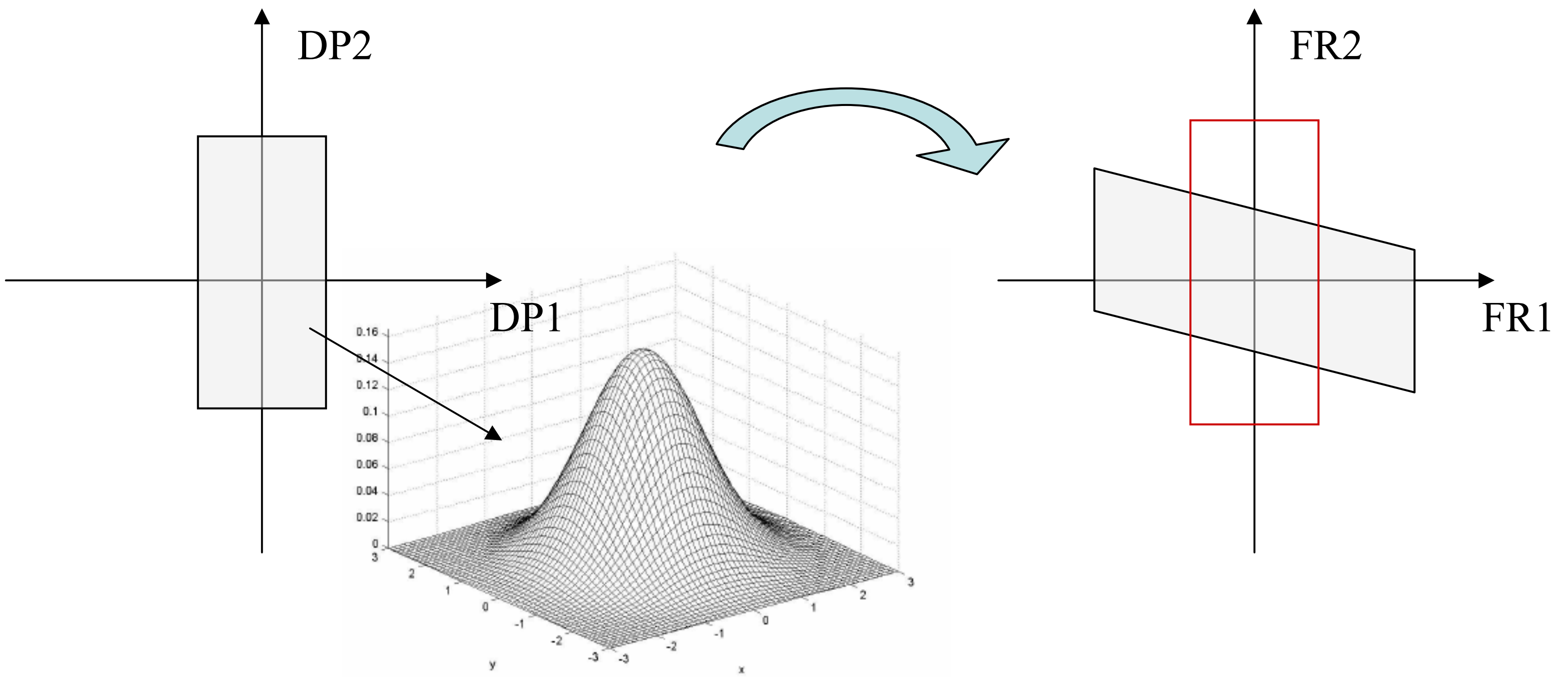
Multiple FR system range

Example

$$\begin{Bmatrix} FR1 \\ FR2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \end{Bmatrix}$$

Design range

FR1: [-0.5 , 0.5]
FR2: [-2.0 , 2.0]



Information Contents

Allowable tolerance

- Defined for DP
- Tolerances that DPs can take while FRs still remaining completely inside design ranges
- Unconditional tolerance
- Conservative tolerancing

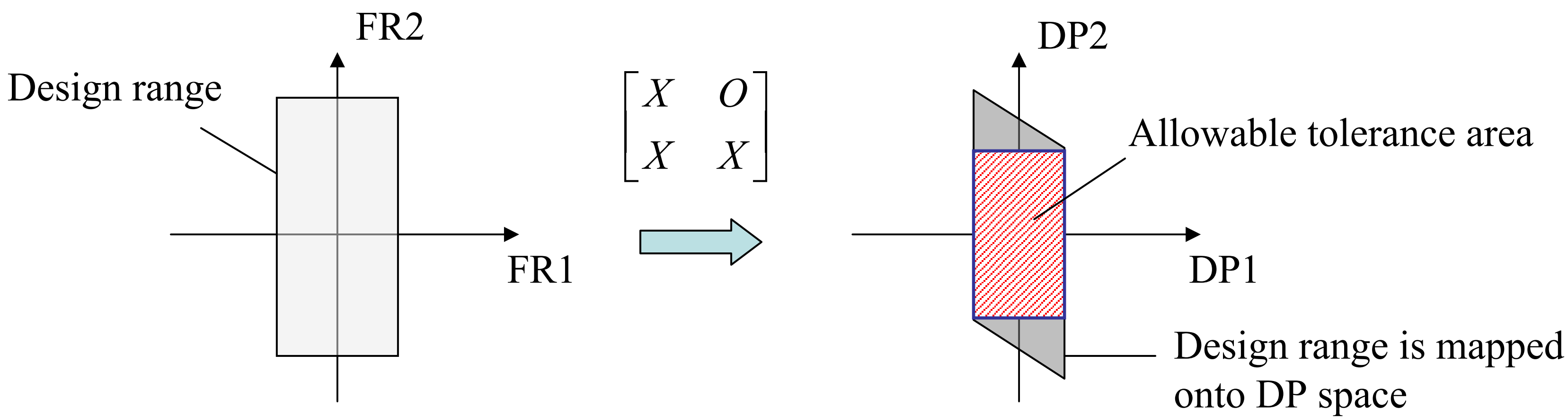
$$\begin{Bmatrix} FR1 \\ FR2 \\ FR3 \end{Bmatrix} = \begin{bmatrix} A11 & 0 & 0 \\ A21 & A22 & 0 \\ A31 & A32 & A33 \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \\ DP3 \end{Bmatrix} \Rightarrow$$

$$\Delta DP1 = \frac{\Delta FR1}{A11}$$

$$\Delta DP2 = \frac{\Delta FR2 - |A21 \cdot \Delta DP1|}{A22}$$

$$\Delta DP3 = \frac{\Delta FR3 - |A31 \cdot \Delta DP1| - |A32 \cdot \Delta DP2|}{A33}$$

Allowable tolerance



$$\Delta DP1 = \frac{\Delta FR1}{A11}$$

$$\Delta DP2 = \frac{\Delta FR2 - |A21 \cdot \Delta DP1|}{A22}$$

Detecting change in system range

"Monitoring marginal probability of each FR is not only inaccurate but potentially misleading"

Example

$$\begin{Bmatrix} FR1 \\ FR2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \end{Bmatrix}$$

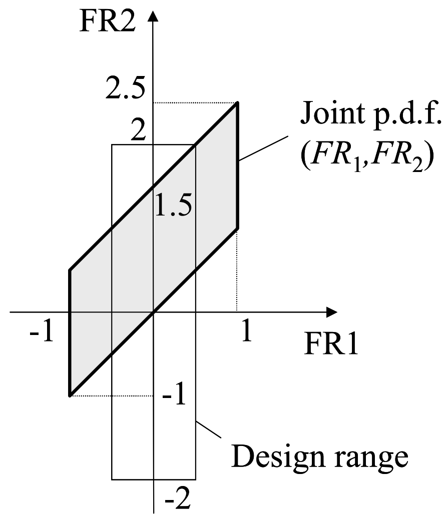
Design range

FR1: [-0.5,0.5]
FR2: [-2,2]

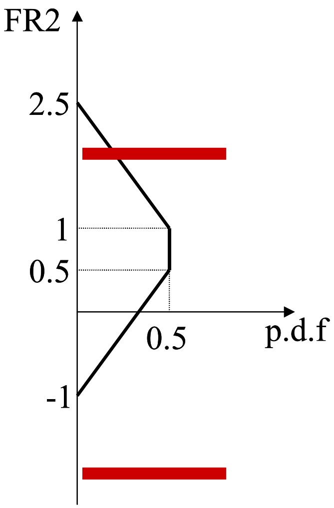
Design parameter variation

Initial
DP1: [-1,1]
DP2: [0,1.5]

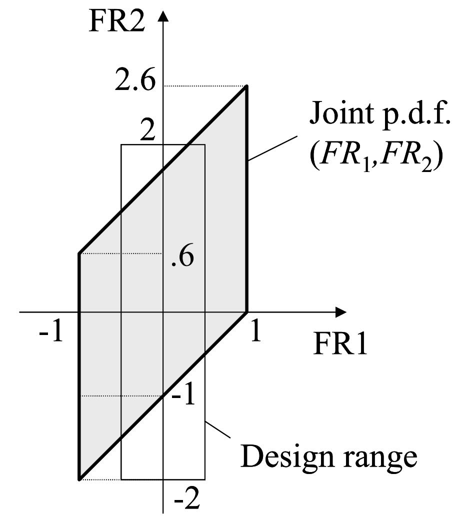
After change
DP1: [-1,1]
DP2: [-1,1.6]



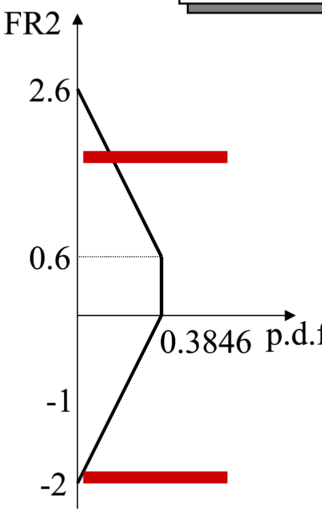
(a)



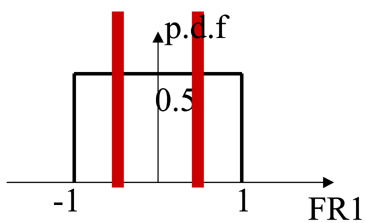
(b)



(a)

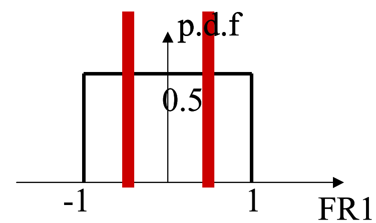


(b)



(c)

DP1: [-1,1]
DP2: [0,1.5]



(c)

DP1: [-1,1]
DP2: [-1,1.6]

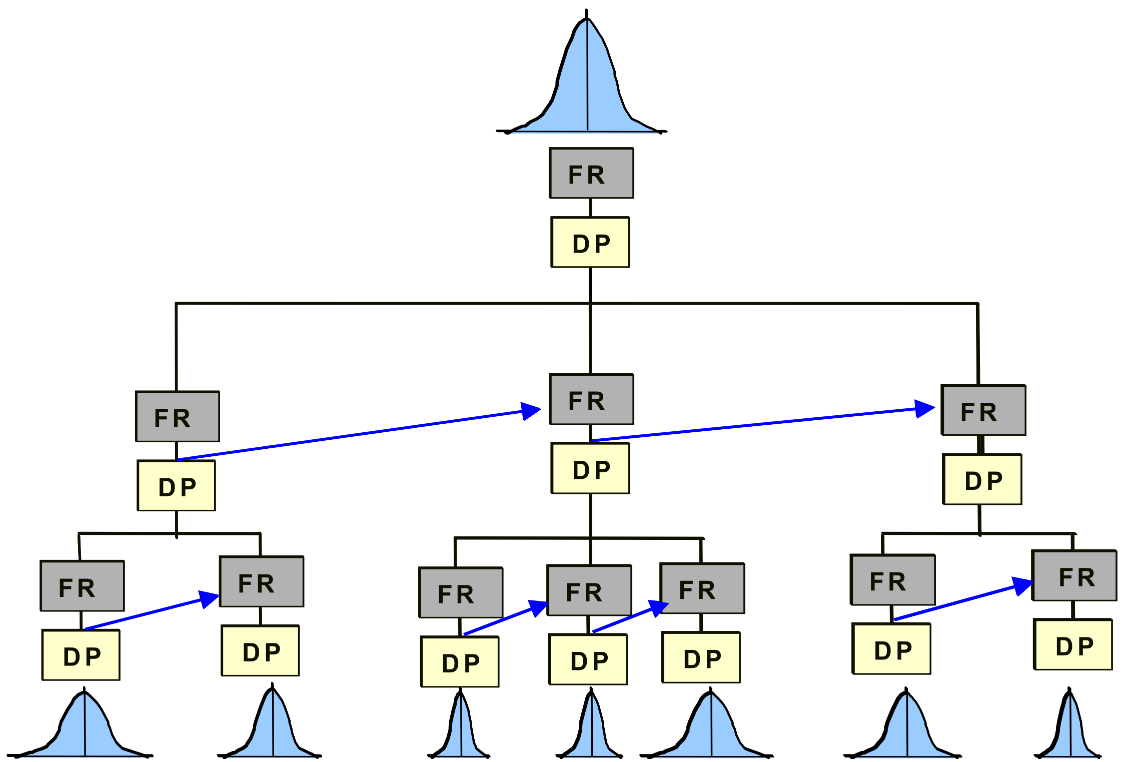
Before DP2 change

After DP2 change

	p_{FR1}	p_{FR2}	$p_{FR1} \times p_{FR2}$	$p_{FR1,FR2}$
Before	0.5	0.9583	**0.4792**	**0.5**
After	0.5	0.9654	**0.4827**	**0.499**

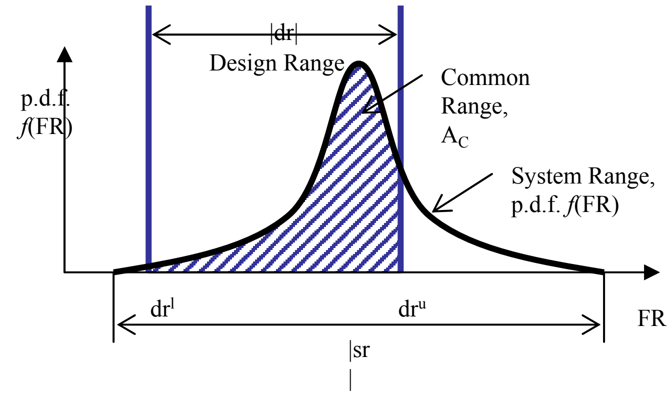
Summary

- Joint probability unless it is uncoupled design
- Assuming DPs are statistically independent, working in DP domain is typically easier.



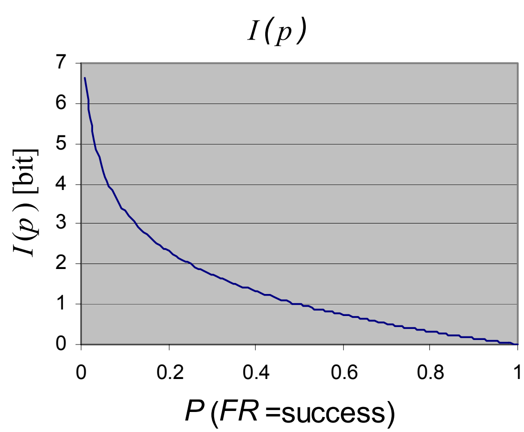
"Information"

- In AD information content, by imposing design range, FR is transformed into a binary random variable.

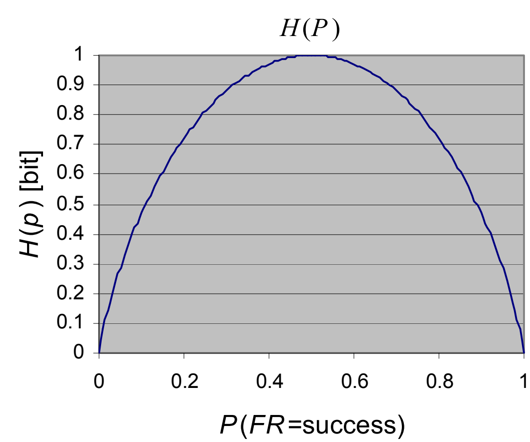


$u_i = \begin{cases} 1 \text{ (success) with} & P(\mathrm{FR}_i = \text{success}) \\ 0 \text{ (failure)} & 1-P(\mathrm{FR}_i = \text{success}) \end{cases}$

$I(u_i = 1) \equiv -\log_2 P(\mathrm{FR}_i = \text{success})$ $H(X) = -\sum pi \log 2\, pi = E[I]$



(a)



(b)

Robustness

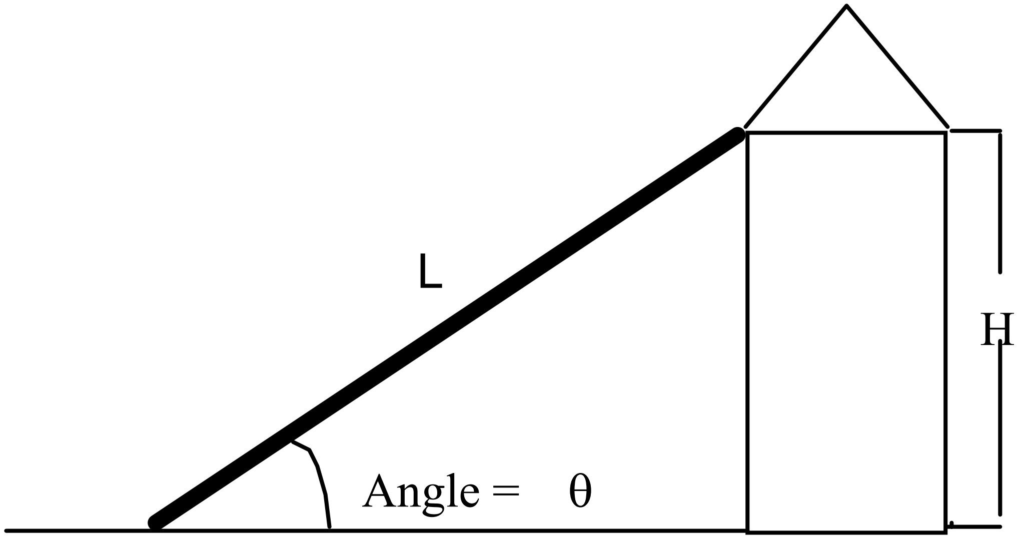
- In axiomatic design, robust design is defined as a design that always satisfies the functional requirements,

  $\Delta FRi > \delta FRi$

  even when there is a large random variation in the design parameter δDPi.

- Two different concepts in robustness
  - Insensitive to ‘noise’
    - Information Axiom
    - Traditional robust design
  - Adaptive to change
    - Independence Axiom
    - Hod Lipson, Jordan Pollack, and Nam P. Suh, "On the Origin of Modular Variation", *Evolution*, *Evolution*, 56(8) pp. 1549-1556, 2002

Example: Measuring the Height of a House with a Ladder



Example: Measuring the Height of a House with a Ladder

Solution:

$$H + \Delta H = \sin(\theta + \delta\theta)L = (\sin\theta\cos\delta\theta + \cos\theta\sin\delta\theta)L$$

For small $\delta\theta$,

$$H + \delta H = \sin\theta L + L\cos\theta\delta\theta$$

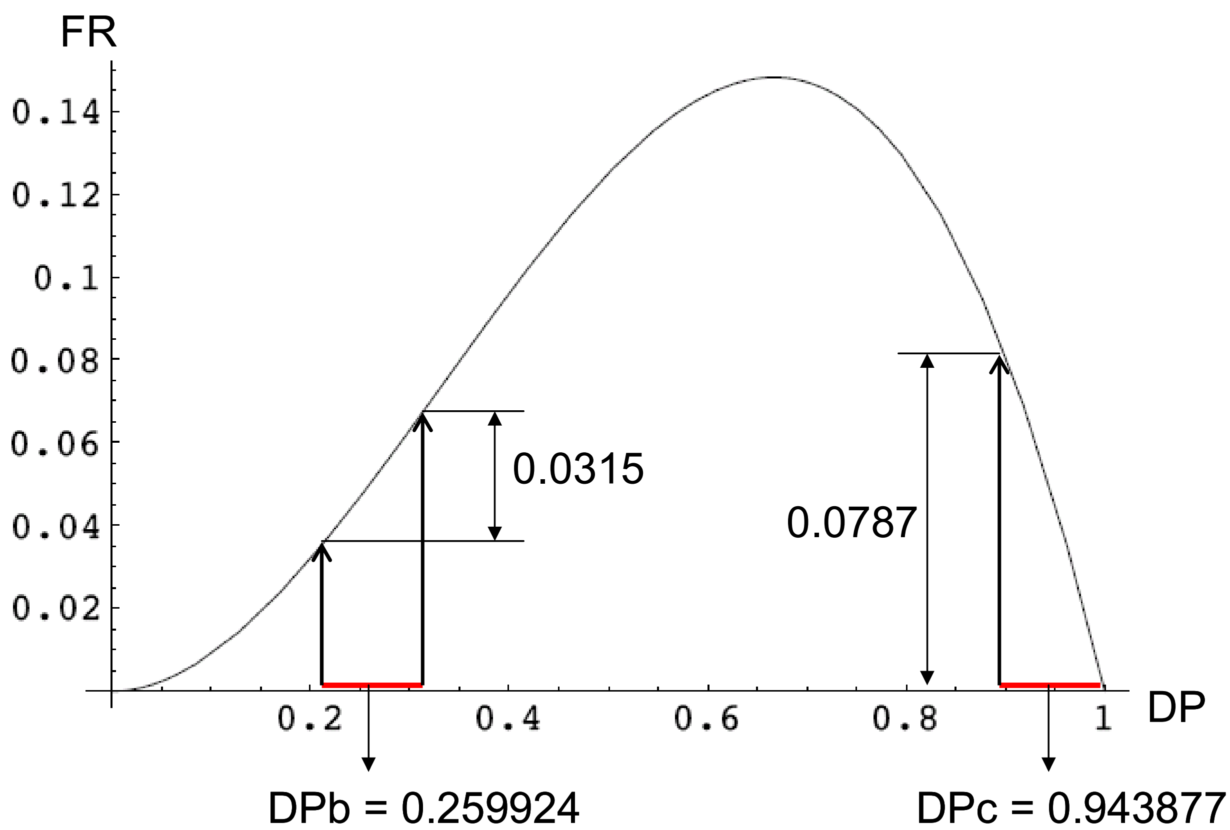
$$\delta H = L\cos\theta\delta\theta$$

where θ is the mean value of the estimated angle, L the length of the ladder, and H the height.

Carefully selecting parameter values can make a system much more robust at almost no additional cost.

Example

- FR = $DP^2 (1 - DP)$



How to make a system robust?

- Where does the variation come from?

$$\overrightarrow{FR} - \overrightarrow{FR}^* = \left.\frac{\partial \overrightarrow{FR}}{\partial \vec{n}}\right|_{\vec{n}=0} \delta\vec{n} + \left.\frac{\partial \overrightarrow{FR}}{\partial \overrightarrow{DP}}\right|_{\overrightarrow{DP}=\overrightarrow{DP}^*} (\overrightarrow{DP} - \overrightarrow{DP}^*) + \left.\frac{\partial \overrightarrow{FR}}{\partial \vec{C}}\right|_{\vec{C}=\vec{C}^*} (\vec{C} - \vec{C}^*)$$

Robustness

$$\overrightarrow{FR} - \overrightarrow{FR}^* = \left.\frac{\partial \overrightarrow{FR}}{\partial \vec{n}}\right|_{\vec{n}=0} \delta\vec{n} + \left.\frac{\partial \overrightarrow{FR}}{\partial \overrightarrow{DP}}\right|_{\overrightarrow{DP}=\overrightarrow{DP}^*} (\overrightarrow{DP} - \overrightarrow{DP}^*) + \left.\frac{\partial \overrightarrow{FR}}{\partial \vec{C}}\right|_{\vec{C}=\vec{C}^*} (\vec{C} - \vec{C}^*)$$

2 1 3

0. Assign the largest possible tolerance
0. Eliminate the bias (E[FR] = FR*)

1. Eliminate the variation: SPC, Poka-Yoke, etc.
2. De-sensitize: Taguchi robust design
3. Compensate

$$\left.\frac{\partial \overrightarrow{FR}}{\partial \vec{C}}\right|_{\vec{C}=\vec{C}^*} (\vec{C} - \vec{C}^*) = -\left(\left.\frac{\partial \overrightarrow{FR}}{\partial \vec{n}}\right|_{\vec{n}=0} \delta\vec{n} + \left.\frac{\partial \overrightarrow{FR}}{\partial \overrightarrow{DP}}\right|_{\overrightarrow{DP}=\overrightarrow{DP}^*} (\overrightarrow{DP} - \overrightarrow{DP}^*) \right)$$

Bring the system range back into design range: Re-initialization

Example: Design of Low Friction Surface

- Dominant friction mechanism: Plowing by wear debris

- System range (particle size) moves out of the desired design range
  $\Rightarrow$ Need to re-initialize

Figure removed for copyright reasons.

Figure removed for copyright reasons.

N. P. Suh and H.-C. Sin, Genesis of Friction, Wear, 1981

S. T. Oktay and N. P. Suh, Wear debris formation and Agglomeration, Journal of Tribology, 1992

Design of Low Friction Surface

- Periodic undulation re-initializes the system range

Figures (6-part diagram and two graphs)
removed for copyright reasons.

S. T. Oktay and N. P. Suh, Wear debris formation and agglomeration, Journal of Tribology, 1992