Homework/Problem Set I

MAS 961 / API-309
Due March 8 (in class)
Instructor: César A. Hidalgo

120 points in total (-10 points for each day after the due date)

I Fun with Random Networks and Graph Theory (35 points)

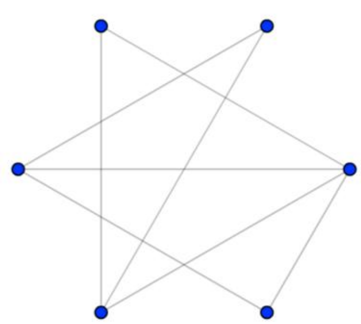
Percolation in Random Networks

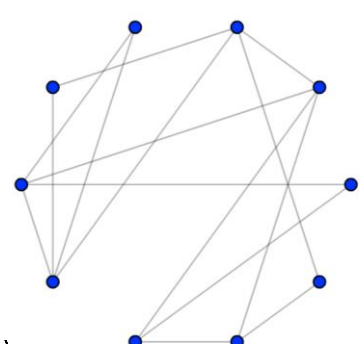
1) In class we saw that properties of random graphs, such as the emergence of a cycle of a given length, or a clique with a certain number of nodes, appear suddenly. The critical point at which these structures appear can be expressed as a function of the network *density* or *fill*, which is simply the ratio between the number of links present in the network, over the total number of possible links.

a) At what density, or fill ,does a random network become connected? **(1 point)**
b) At what density, or fill, is a random network expected to have its first triangle (3-clique)? How does this compare to the number of links required for a network to become connected? (express your answer as a percentage increase) **(4 points)**
c) Social networks are characterized by having many cliques. Yet cliques are infrequent in random networks. What is the average degree of the nodes in a random network in which we expect to find a 5-clique? a 10-clique? **(10 points)**

Eulerian and Planar Graphs

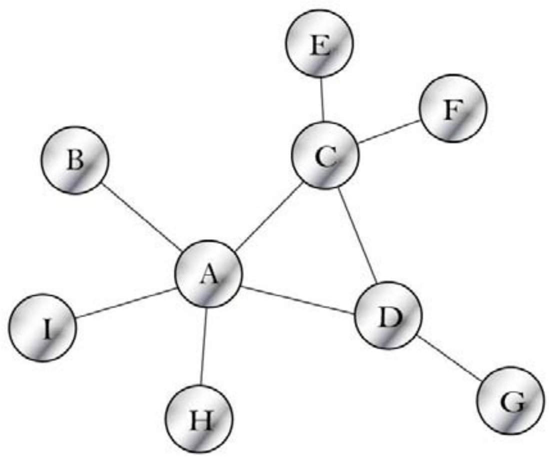
Consider the following two networks and determine if they are planar and/or Eulerian. Explain why (use diagrams and drawings to help illustrate your point) **(20 points)**


a)


b)

II Fun with Scale-Free Networks (40 points):

1) When epidemics spread over scale-free networks they can do so even if they are not very infectious. Explain why this is something on scale-free networks, but not in random networks or lattices. **(16 points)**



2) Assume this network is an incomplete realization of the BA (Barabási-Albert) model: **(10 points)**

a) If a new node with m=2 links arrives, what is the probability that it will connect to node A? to node B? to node C? to either of the three?

3) In class we solved the BA model analytically and derived an expression for the degree of a node as a function of both, the time in the network and its time of incorporation. Using this, and m=1, calculate:

a) The expected number of links, or degree, of the first node to be incorporated in the network after 99 new nodes are added. **(4 points)**

b) The expected number of links, or degree, of the 100th node to be incorporated into the network when the network has 1000 nodes. **(4 points)**

c) Estimate the total amount of time that it would take the 1000th node added to the network to reach the degree that the first node to be added to the network had at the time the 1000th node was incorporated. Comment. **(6 points)**

III Fun with Basic Network Properties **(45 points)**

Consider the undirected network defined by the following set of links:

Alice	**B**ob
Carl	**A**lice
Alice	**D**avid
Alice	**E**rnst
Alice	**F**rank
Bob	**G**ail
Gail	**H**arry
Harry	**J**en
Jen	**G**ail
Harry	**I**rene
Irene	**G**ail
Irene	**J**en
Ernst	**F**rank
David	**C**arl
Carl	**F**rank

1) Draw the network (by hand) **(4 points)**

2) a) How many nodes and links are there? **(1 points)**

b) What is the density of the network? **(1 points)**

3) a) Calculate the degree of each node. Who is/are the most central nodes according to this measure? **(4 points)**

b) Calculate the clustering of each node, and the average clustering of the network. **(3 points)**

c) Calculate the closeness centrality for each node. Who is/are the most central according to this measure? **(4 points)**

d) Calculate the betweenness centrality of each node. Who is/are the most central according to this measure? **(4 points).**

4) a) Calculate the topological overlap of each link (using the geometric version). **(4 points)**

b) Calculate the betweenness of each link. **(4 points)**

5) Plot the degree of a node against its betweenness. Are these variables correlated? (Calculate the correlation coefficient and its p-value.) What are the main outliers of this relationship? Use the figure you made in 1 to interpret the outliers. **(5 points)**

6) Consider a random network with the same number of nodes and links than the network documented above. What would it clustering be? What would its average path length be? How do this compare to this network? Would you say this is a small world-network? **(6 points)**

7) There is a conspiracy against the most central node according to betweenness (the one you identified in 3d). If you were to add only one link in the network to maximally reduce the betweenness centrality of this node. What link would that be? **(5 points)**