mas.s62 lecture 3 signatures

2018-02-14

Tadge Dryja

signatures

pset01 was about lamport signatures

There are other signature schemes, some with cool features

Hash-based, RSA, ECDSA, EC schnorr

multiple use hash signatures

Problem with lamport signatures:

Multiple signatures from one key allows forgeries

Solution: use more public keys

multiple use hash signatures

Easy way:

make 2 public keys, concatenate and publish

For signatures, indicate use of key1 or key2

multiple use hash signatures

Signatures: same size

Public keys: 2X size

Private keys: 2X size..?

multiple use hash signatures

Signatures: same size

Public keys: 2X size

Private keys: 2X size..?
make a root private key, and hash(root,1) for key 1, hash(root, 2) for key 2.
In fact, private key can be 32 bytes

multiple use hash signatures

howto 32 byte privkey:

0row hash(seed,0,0), hash(seed,0,1)..

1row hash(seed,1,0), hash(seed,1,1)..

multiple use hash signatures

from 16KB to 32B. That's quite nice!

Can we do that with the pubkey..?

32B pubkey…?

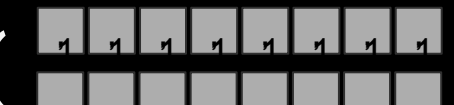
multiple use hash signatures

from 16KB to 32B. That's quite nice!

Can we do that with the pubkey..?

32B pubkey…?

Commit to pubkey with

hash() (hash of all 16KB)

multiple use hash signatures

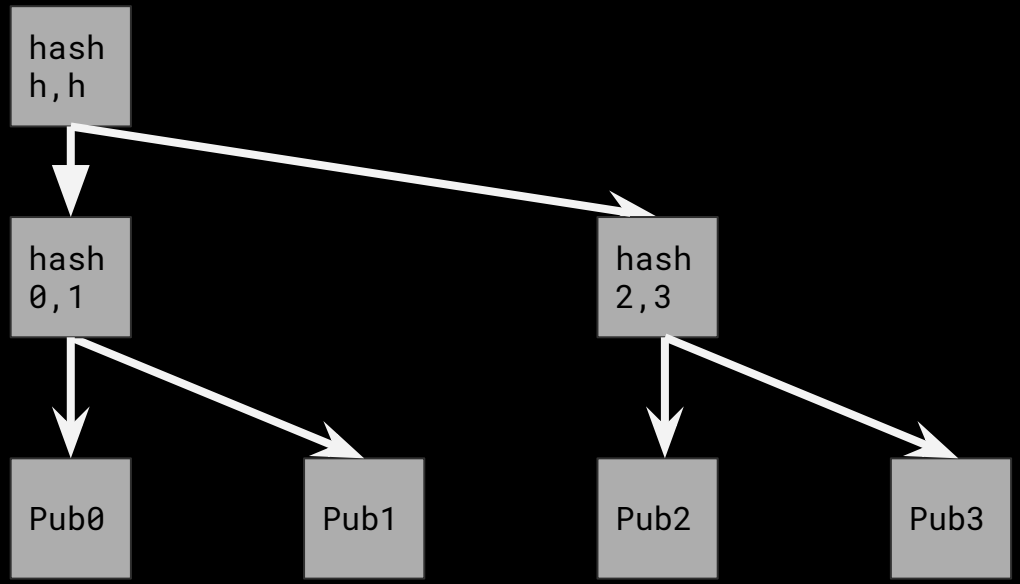
private key -> 32B

public key -> 32B

signatures still big; actually get bigger. Include full pubkey in signature.

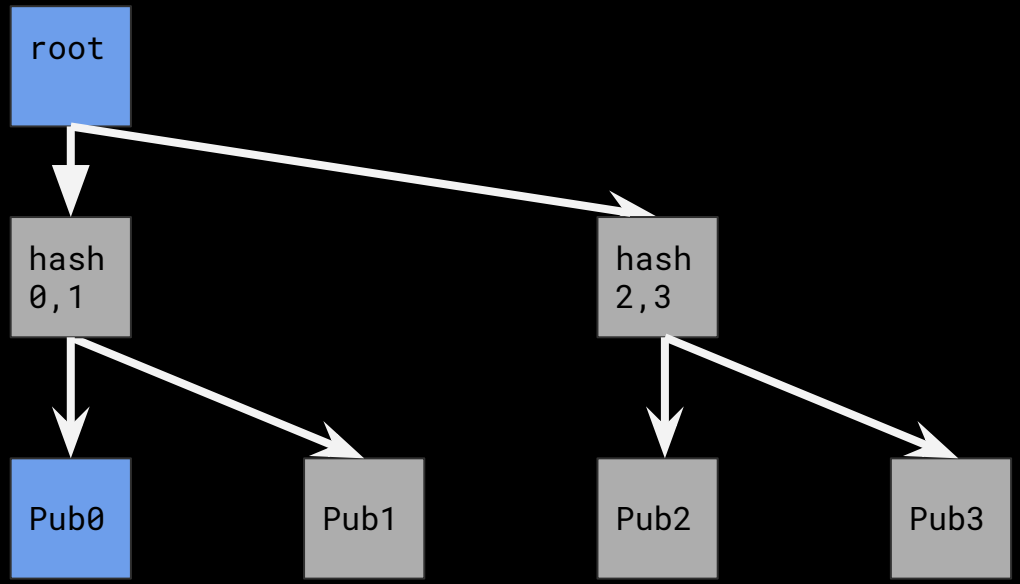
Can do better: commit to many pubkeys

multiple use hash signatures



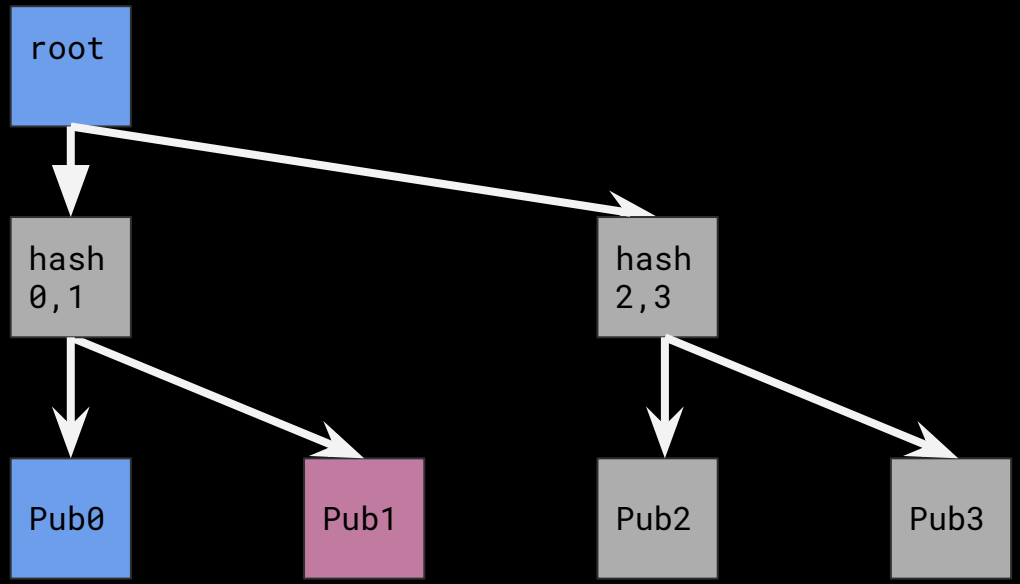
Hash tree, or Merkle tree

multiple use hash signatures



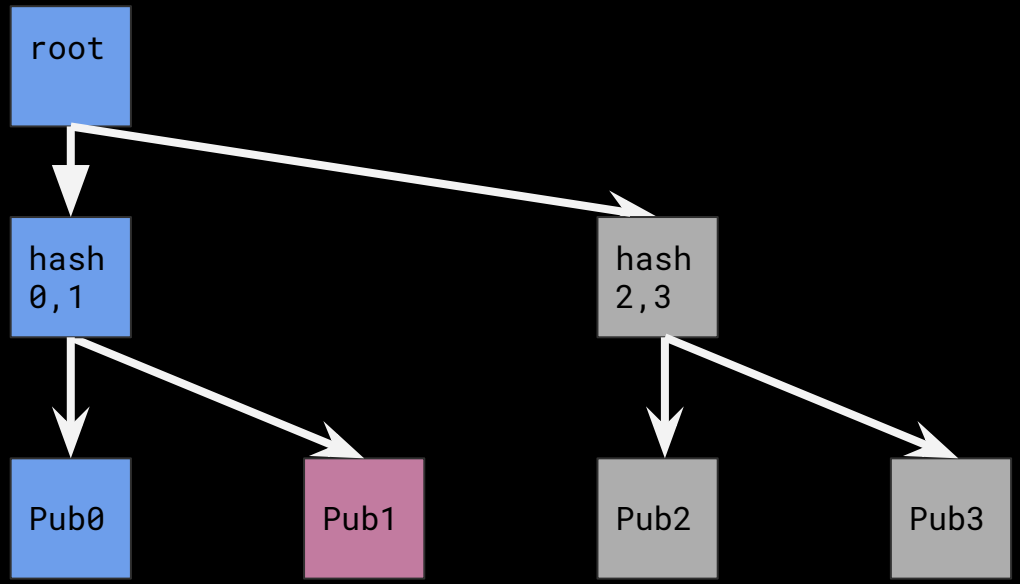
Prove Pub0 inclusion given root

multiple use hash signatures



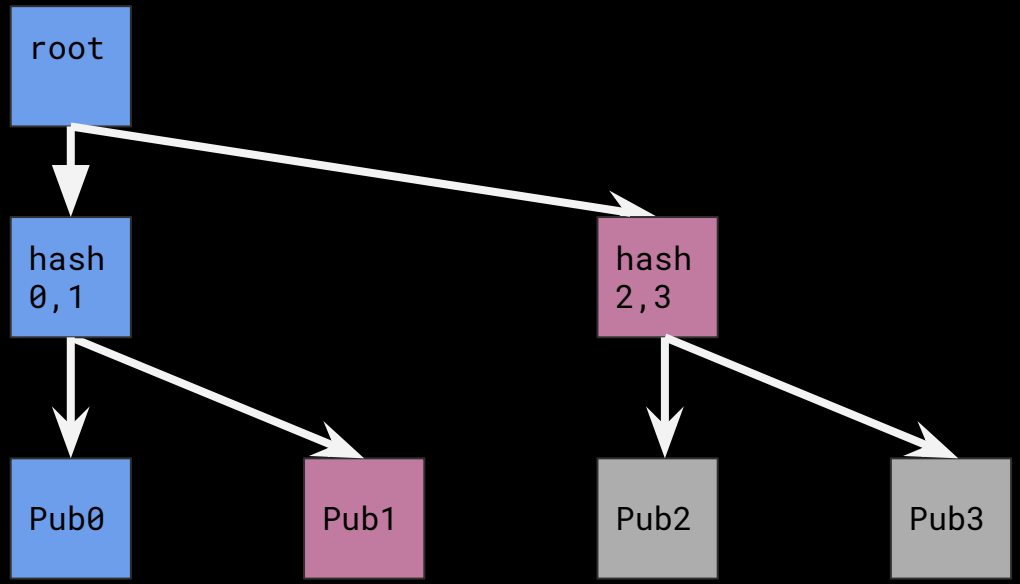
Prove Pub0 inclusion given root

multiple use hash signatures



Prove Pub0 inclusion given root

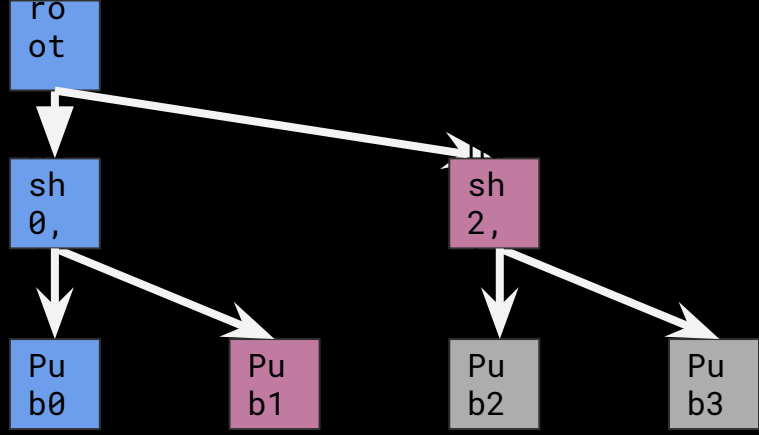
multiple use hash signatures



only need 2 extra hashes, one per row

Merkle trees

Invented for lamport sigs



add O(n) elements in O(1) sized root

prove an element in the set with O(log) n intermediate hashes

Merkle trees

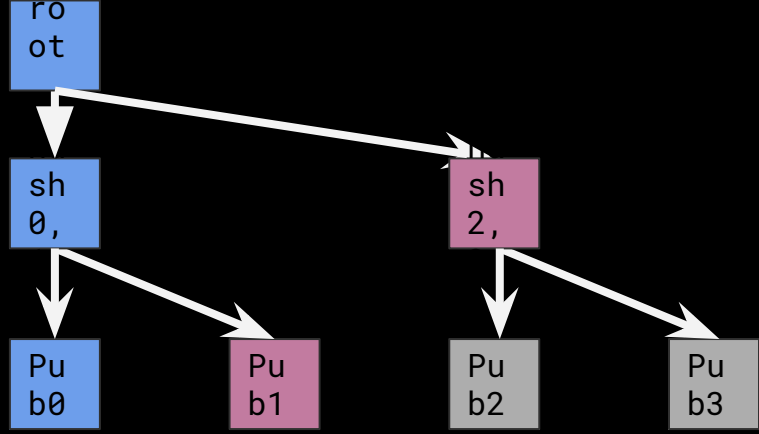
commit to 1024

signing keys

use root as pubkey

signature inclues proof that signing key is a leaf in the tree

10*32 = 320B overhead. cool!



Merkle trees

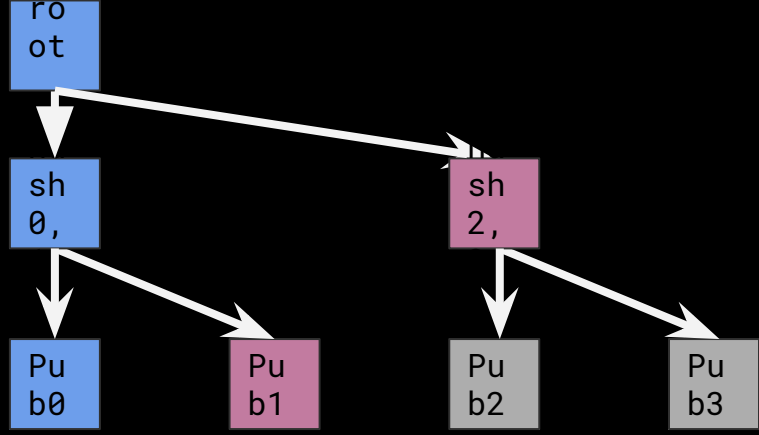
commit to 1024

signing keys

use root as pubkey

signature inclues proof that signing key is a leaf in the tree

10*32 = 320B overhead. cool!



need more power

Hash based signatures are cool.

But we can do better. More powerful signature schemes.

RSA ECDSA ECBN

RSA

Invented by locals :)

Not used in Bitcoin (or any currency)

Used for chaumian blinded cash

Basic setup: make 2 primes: p, q

n = p*q

RSA

Given p,q, computing n is easy.

Given n, finding p,q is hard!

A one way function… but not a hash function.

RSA

Can do some fun math with this.

Set e = 3 (or 65537)

set d = some number you can compute if you know p or q.

$d = e^{-1} \bmod (p-1)*(q-1)$

n is public. d is private.

p, q not needed after setup. e always the same

RSA

Sign: $s = m^d \bmod n$

Verify: $s^e \bmod n == m$

Can sign many times. And do lots of cool stuff.

RSA

RSA key sizes are smaller than hash based signatures; often 2048 bits (256 bytes)

Somewhat tricky to implement! Lots of ways to lose your private key

but Bitcoin (& other coins) uses elliptic curve signatures

Intermission

3 min, walk around, ask questions about pset . . .

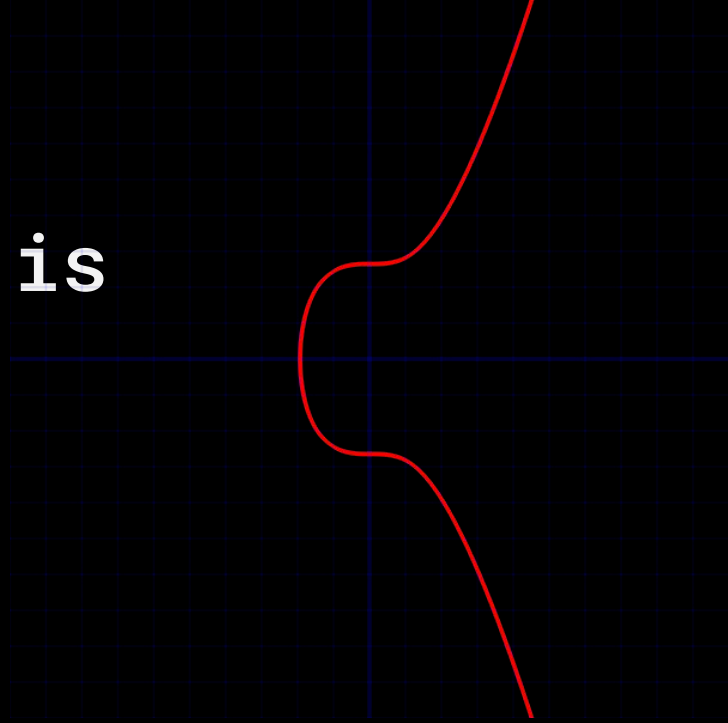
then start on elliptic curves

elliptic curves

Curves. Bitcoin's curve is

$y^2 = x^3 + 7$

Simple, right?



elliptic curves

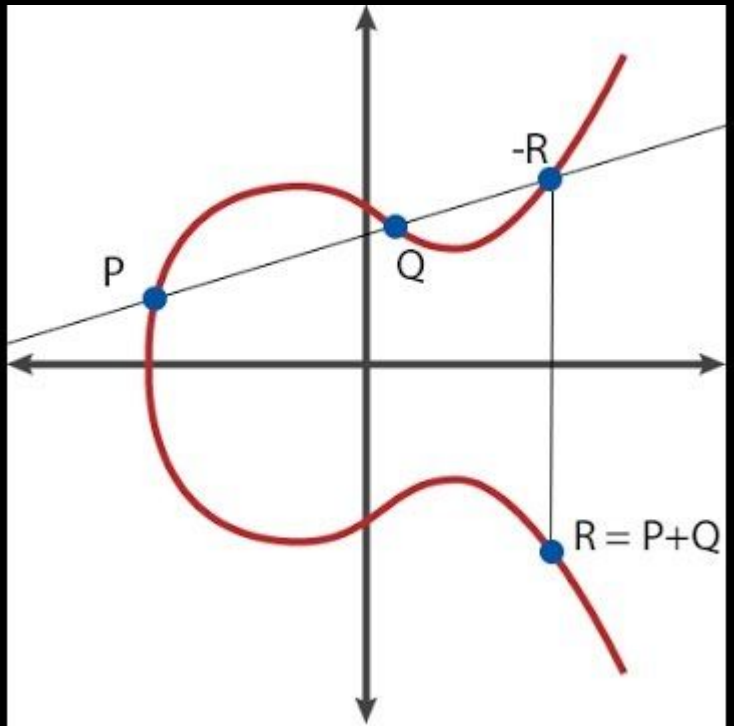
define point addition

line of 3 points

= 0

so P+Q-R=0

P+Q=-R



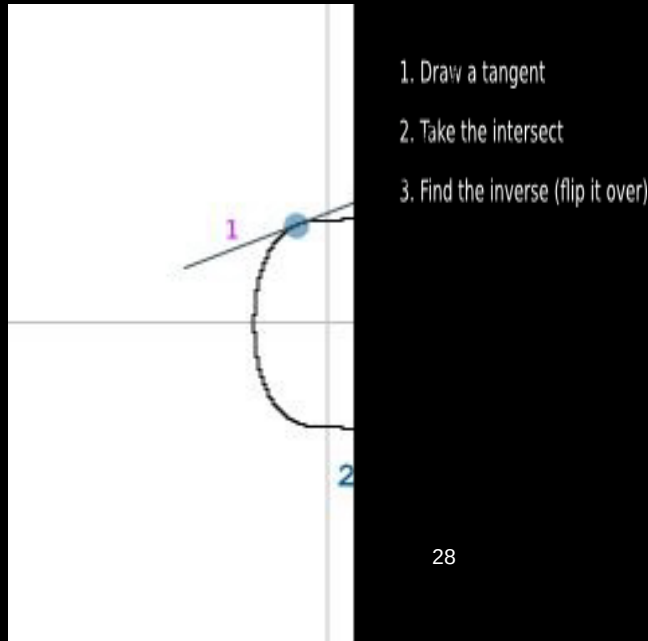
elliptic curves

point "multiplication"

take the tangent

to add a point

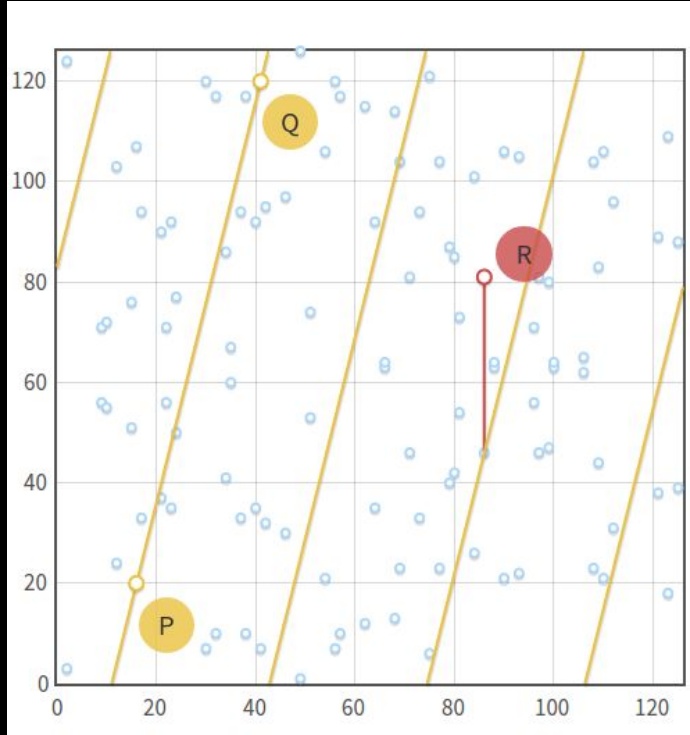
to itself



elliptic curves

note: actually "looks" more like this because it's modulo some big prime number

We don't compute it graphically so it's OK



point and scalar operations

(Note also works on exponents mod n)

a, b lowercase = scalar

A, B uppercase = point

what operations can we do?

point and scalar operations

scalars are regular unleaded numbers

```
a+b a-b a*b a/b
```

everything is OK! just numbers!

point and scalar operations

Points have addition defined… but not multiplication and division

A+B A-B OK A*B A/B NO

add & subtract OK, but can't multiply two points, or divide a point by a point. Not defined.

point and scalar operations

Mixed operations

A+b A-b NO A*b A/b OK

adding points and scalars is undefined

point times scalar OK; repeat the tangent doubling process. Division by scalar also possible.

point and scalar operations

roster of ops: what can we do

a+b a-b a*b a/b (obvious)

A+B A-B A*b A/b

point and scalar operations

roster of ops: what can we do

a+b a-b a*b a/b (obvious)

A+B A-B A*b A/b

Pick some random point G

That's the generator point
Everyone agrees on G

EC private & public keys

private key a = 256 bit scalar

(same as one block from lamport priv)

public key ?

EC private & public keys

private key a = 256 bit scalar

(same as one block from lamport priv)

public key A = a*G

32 byte x coord, 32 byte y coord = 64B

EC private & public keys

private key a = 256 bit scalar

(same as one block from lamport priv)

public key A = a*G

32 byte x coord, 32 byte y coord = 64B

since curve is symmetric about x-axis, can encode x-coord only and 1 bit for y. Down to 33 bytes.

ECDSA

What Bitcoin, other coins use today

It's ugly though…

Come up with another priv key k

r = x-coord of k*G

$s = k^{-1}(hash(m) + a*r)$

ECDSA

Made to avoid a patent on a better signature system

That patent has expired, we are free to use the simpler better algo that must not be named.

ECsig

Have message m, privkey a

make k, a new random private key

R = k*G

s = k - hash(m, R)a

signature = R, s

ECsig

given R,s verify:

s = k - hash(m, R)a

sG = kG - hash(m, R)aG

sG = R - hash(m, R)A

R == sG + hash(m,R)A

ECsig

Make up k and compute s,R? but need a

s = k - hash(m, R)a

without a, can't make a valid s

ECsig

Make up an s, solve for R?

$sG = R - hash(m, R)A$

ECsig

Make up an s, solve for R?

sG = R - hash(m, R)A

R = hash(m, R)A + sG

R = hash(R)! Can't compute, can't cancel out

Fun with points

A = aG

B = bG

aB = bA = (aG)b = (bG)a = (ab)G = C

C is a "Diffie Hellman" point.

Super useful! If you know either a or b, you can compute C.

Fun with points

A = aG

B = bG

aB = bA = (aG)b = (bG)a = (ab)G = C

C is a "Diffie Hellman" point.

Fun with points

A = aG B = bG

D = A+B = (a+b)G

sign with D? Can with d = a+b

make a combined key D = A+B; either party can reveal their side to the other to give signing ability

What do we do with this??

Nothing yet. Hard to program this stuff.

Ground work for cool stuff you can do with keys, transactions, signatures

Fun new area! Non-experts (like me) can come up with new stuff!

Next pset: NameChain

Mine your name; get a high score

(hax0r names also OK)

pls don't DDoS the server :)

MAS.S62 Cryptocurrency Engineering and Design

Spring 2018