22.01 Fall 2016, Problem Set 4

October 7, 2016

Complete all the assigned problems, and do make sure to show your intermediate work.

Part I

Skill-Building Problems (50 points)

1 Successive Decay Chains

The molten salt reactor (MSR) could be used not just for power, but to produce two incredibly valuable gases from its ^{7}Li-enriched salt coolant: Tritium $\left(^3H\right)$ and helium-3 $\left(^3He\right)$. Tritium is extremely useful as fusion reactor fuel, while helium-3 is the most effective gaseous neutron detector available. The latter is valuable enough that NASA has considered mining it on the Moon.

1. Starting with the capture of a neutron by ^{7}Li, write a complete set of nuclear reactions that describe the production and destruction of ^{3}H and ^{3}He. Consider that both gases can also capture neutrons to be destroyed, with microscopic cross sections $\sigma_{H,n}$ and $\sigma_{He,n}$.

2. Develop a set of differential equations, similar to the Bateman equations, describing the production and destruction of ^{3}H and ^{3}He.

3. Graph the solutions to this set of differential equations, showing the relative levels of ^{7}Li, ^{3}H, and ^{3}He in the reactor. Use the following data to make your graphs:

$$\sigma_{c,7-Li} = 10^{-4}b \qquad \sigma_{c,3-H} = 10^{-10}b \qquad \sigma_{c,3-He} = 10^{-5}b \qquad \lambda_{^3H} = 1.8{\cdot}10^{-9}\ \frac{1}{s} \qquad \Phi = 1{\cdot}10^{14}\,\frac{n}{cm^2-s}$$

4. Now assume that ^{3}H costs \$30,000/g and ^{3}He costs \$53,000/g. Graph the value of the gases in the MSR as a function of time, assuming an initial ^{7}Li number density of $10^{22}\,\frac{\text{atoms}}{\text{cm}^3}$.

2 Statistics and Certainty

1. Since we know that smoking is a major source of radioactivity, smoke shops should also be major sources of radioactivity. How long would you have to count in a smoke shop to be 95% sure that you can distinguish your count from the background? Assume a new background activity of $A_{Smoke-Shop}$ inside the store.

   (a) ***<u>Bonus Question (25 points):</u>*** ***Go do this. Tell us how radioactive a local smoke shop is, state/calculate your uncertainty, and how long you had to count. You should get the shop owner's permission before doing this, to avoid arousing suspicion.***

3 Radioactive Dating with Confidence

For this problem, consider the methods used to radioactively date the Shroud of Turin (← this is a hyperlink), thought to be the burial cloth of Jesus of Nazareth.

1. Why did the investigators use carbon as the dating isotope? Consider what other isotopes could have been present, and give at least three reasons that carbon was chosen.
2. How did the investigators use statistics to prove beyond a reasonable doubt that the Shroud of Turin was *not* the burial cloth? What is a χ^2 test, and how did they arrive at the χ^2 values in the paper?
3. Why did the investigators send so many unknown control samples to so many laboratories, and why did they use different cleaning procedures?

4 Generating Cobalt-60 and Profit

How many days should one irradiate a 100g source of ^{59}Co in order to maximize profit from the reactor? Assume a fully homogeneous, thermal reactor, with the following parameters:

$$\Phi = 10^{14}\frac{n}{cm^2 s} \qquad Reactor\,Cost = \$1000/day \qquad {}^{60}Co = \$100/\mu Ci$$

Start by writing down what is physically happening (the nuclear reactions), model them using a system of differential equations, solve the system of equations for the amount of ^{60}Co as a function of time, and construct & solve an equation to maximize the profit of the reactor. Look up any nuclear data that you need from the JANIS cross section database and the KAERI Table of Nuclides.

Part II

Take-Home Lab: Estimating the Radioactivity of One Banana (50 points)

Using the banana ashes accumulated from last year's 22.01 class, estimate the radioactivity of one banana. Make the following assumptions:

1. The ashes were created from 50 pounds of peeled, fresh, ripe bananas.
2. No other sources of contamination are present in the bananas

Use the high purity Germanium detector (HPGe) in the Nuclear Reactor Laboratory (NRL) to collect background and banana spectra. Devise a way to determine the total radioactivity of your sample based on any of the available features of the spectrum (peak height, number of counts, area under peak, etc.) of the most appropriate peak of the most appropriate isotope. Compare your answer to any reputable source from the literature, and make sure to cite your source:

- Journal articles need the authors, title, journal, volume, pages, and year.
- Books need the authors, editors, title, publisher, pages used, year, and ISBN number.
- Other articles (like those online) need the author, URL, date that you accessed it, and date of publication. These should be used as a last resort.

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