# 22.01 Fall 2016, Problem Set 6

October 30, 2016

Complete all the assigned problems, and do make sure to show your intermediate work.

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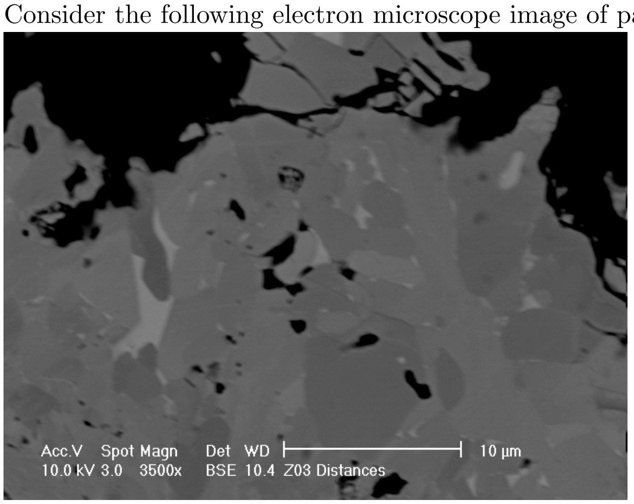
## 1 Skill-Building Problems (50 points)

1. **Short Answers (5 points each)**

   (a) Explain, using stopping power expressions and cross sections, why the energy loss due to ionization drops off so sharply with increasing energy, while radiation loss increases linearly.

   (b) Explain the quantitative differences in stopping power of electrons as they reach relativistic speeds. What energy cutoff do you consider relativistic, and why?

   (c) Consider the following electron microscope image of palladium diffusion into zirconium carbide:

   Where is the palladium in this image, and how do you know, based on your knowledge of electron interaction mechanisms with matter? Back up your answer with a relevant quantitative estimate of electron interactions. Using an image processing program, measure the relative brightnesses of various types of spots in the images. Can you guess the average atomic number of each of the spots? In other words, is brightness linearly proportional to the type of electron interaction(s) that you are interested in?

   (d) Explain, using attenuation and stopping power, why protons are far more effective at damaging a localized tumor. Draw any applicable range relations and/or attenuation graphs to make your point.

2. **Calculation Problems (10 points each)**

(a) Analytically develop a graph of the range of protons in lead vs. their energy, over a range between 100 keV and 100 MeV, with at least ten points on the curve. Check your answer by using SRIM (www.srim.org) to simulate the range of protons in lead at each energy.

(b) Analytically develop a graph showing the ratio of radiated bremsstrahlung energy to the incoming ion energy for the cases above.

(c) Analytically develop a graph of the range of 1 MeV ions in lead vs. their atomic mass, with at least ten points on the curve. Check your answer by using SRIM (www.srim.org) to simulate the range of protons in lead at each energy. What other variable do you have to constrain to make a uniform comparison?

## 2 Noodle Scratchers (50 points)

1. One way of ensuring a uniform dose to a tumor of finite size (not small) is called intensity modulated radiation therapy (IMRT), where the proton beam is modulated in energy and/or angle to shift the Bragg peak to different specific locations. The goal is to maximize dose to the whole tumor, while minimizing the dose to surrounding tissue. For the following questions, assume we are trying to treat a tumor 1cm in diameter, surrounded by 5cm of healthy tissue.

   (a) (30 points) Derive a relationship between the required energy and the atomic mass of a singly charged ion required to reach the center of the tumor. This will tell you how big of an accelerator one needs to use each type of ion. You may want to use SRIM to roughly check your calculations. Use the following formulas of stopping power in your calculations:

$$-\left(\frac{dT}{dx}\right)_{ioniz.} = \frac{4\pi N k_0^2 z^2 Z e^4}{m_e c^2 \beta^2}\left[ln\left(\frac{2m_e c^2\beta^2}{\bar{I}\left(1-\beta^2\right)}\right) - \beta^2\right]; \qquad k_0 = 8.99\cdot 10^9 \frac{N-m^2}{C^2} \tag{1}$$

$$\frac{-\left(\frac{dT}{dx}\right)_{ioniz.}}{-\left(\frac{dT}{dx}\right)_{nucl.}} = \frac{2M}{m_e Z}\frac{ln\left(\frac{\gamma_e E_i}{\bar{I}}\right)}{ln\left(\frac{\gamma E_i}{E_d}\right)} \tag{2}$$

$$\frac{-\left(\frac{dT}{dx}\right)_{rad.}}{-\left(\frac{dT}{dx}\right)_{ioniz.}} = \left(\frac{m_e}{M}\right)^2\left(\frac{ZE_i}{1400 m_e c^2}\right) \tag{3}$$

   (b) (15 points) Now derive a relationship between the amount of ionization of each of these ions at their starting energy and within the Bragg peak. This gives ratio of the amount of damage to the tumor compared to the surrounding tissue directly from the ions themselves.

   (c) (5 points) Consider the cases of electrons, protons, carbon ions, and iron ions. Which type of ion is most suitable for use in IMRT, and why? Hint: Consider other mechanisms of ion energy loss in tissue, and quantitatively compare how intense they would be in a relative sense.