22.01 Fall 2016, Problem Set 7 Partial Solutions

December 16, 2016

Complete all the assigned problems, and do make sure to show your intermediate work.

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1 Skill-Building Questions (50 points)

1.1 MIT Reactor Modifications (24 points)

For these questions, consider the MIT reactor in its critical state, and the various experiments that we do with it. >>>Here<<< is a cross section of the relevant parts of the MIT reactor. What would be the effect of each of the following changes on the reactor's criticality, and which of the terms in the two energy group criticality relation would be affected? Explain why, using your knowledge of neutron absorption and leakage, and how they affect criticality.

For all of these problems, start with the two energy-group criticality condition from the blackboard:

$$k_{eff} = \frac{\nu\left(\overline{\Sigma}_{f_f} + \overline{\Sigma}_{f_{th}} \quad \frac{\overline{\Sigma}_{s_{f\to th}}}{\overline{D}_{th}B_g^2+\overline{\Sigma}_{a_{th}}}\right.}{\overline{\Sigma}_{s_{f\to th}} + \overline{\Sigma}_{a_f} + \overline{D}_f B_g^2} \tag{1}$$

Then decide which terms will go up and down as a result, and what will happen to k_{eff} ***as an overall result.***

(a) Passing silicon through the reactor to dope with phosphorus by transmutation ***(this really happens)***

$$k_{eff}\downarrow = \frac{\nu\left(\overline{\Sigma}_{f_f} + \overline{\Sigma}_{f_{th}} \quad \frac{\overline{\Sigma}_{s_{f\to th}}\uparrow}{\overline{D}_{th}\downarrow B_g^2+\overline{\Sigma}_{a_{th}}\downarrow}\right.}{\overline{\Sigma}_{s_{f\to th}}\uparrow + \overline{\Sigma}_{a_f} \quad + \overline{D}_f\downarrow B_g^2} \tag{2}$$

Silicon is just an absorber, so it will increase absorption somewhat. It also somewhat decreases diffusion by virtue of replacing

lower cross sectional materials (air) with silicon, and diffusion is inversely proportional to the total cross section:

$$D = \frac{1}{3\left(\Sigma_t - \overline{\mu}_0 \Sigma_s\right)}; \qquad \mu_0 \approx \frac{2}{3A} \tag{3}$$

(b) Throwing quarters directly in the core of the reactor like a wishing well ***(this actually happened!)***
This actually represents precisely the same physical situation, except for a slight difference in changing the diffusion coefficient due to different scattering/absorption cross sections, and less total mass:

$$k_{eff} \downarrow = \frac{\nu\left(\overline{\Sigma}_{f_f} + \overline{\Sigma}_{f_{th}}\left[\frac{\overline{\Sigma}_{s_{f\to th}}\uparrow}{\overline{D}_{th}\downarrow B_g^2 + \overline{\Sigma}_{a_{th}}\downarrow}\right]\right)}{\overline{\Sigma}_{s_{f\to th}}\uparrow + \overline{\Sigma}_{a_f} \quad + \overline{D}_f \downarrow B_g^2} \tag{4}$$

(c) Replacing the water coolant with liquid sodium
This would completely remove moderation from the core, drastically decreasing the <u>flux-averaged</u> fission cross section:

$$\overline{\Sigma}_f = \frac{\int_{E_{min}}^{E_{max}} \Sigma_f(E)\,\Phi(E)\,dE}{\int_{E_{min}}^{E_{max}} \Phi(E)\,dE} \tag{5}$$

Because this would all but eliminate the thermal flux, it also all but eliminates the thermal flux-averaged fission cross section. It would also increase absorption a bit, and increase diffusion by decreasing μ_0:

$$k_{eff} \quad = \frac{\nu\left(\overline{\Sigma}_{f_f}\uparrow + \overline{\Sigma}_{f_{th}} \quad \left[\frac{\overline{\Sigma}_{s_{f\to th}}\downarrow}{\overline{D}_{th}\uparrow B_g^2 + \overline{\Sigma}_{a_{th}}\uparrow}\right]\right)}{\overline{\Sigma}_{s_{f\to th}} \uparrow + \overline{\Sigma}_{a_f} \quad + \overline{D}_f \quad B_g^2} \tag{6}$$

(d) Closing all the beam ports which let neutrons out for experiments
This would reflect more of the fast neutrons back into the reactor. It would do the same for thermal, but the thermal neutrons' mean free paths are so low that we can neglect this. Interestingly, it doesn't appear on our two-group criticality condition! Nevertheless, we know that fewer neutrons leaking out of the reactor must increase k_{eff}***:***

$$k_{eff} \quad = \frac{\nu\left(\overline{\Sigma}_{f_f} + \overline{\Sigma}_{f_{th}}\left[\frac{\overline{\Sigma}_{s_{f\to th}}}{\overline{D}_{th} B_g^2 + \overline{\Sigma}_{a_{th}}}\right]\right)}{\overline{\Sigma}_{s_{f\to th}} + \overline{\Sigma}_{a_f} + \overline{D}_f {B_g}^2} \tag{7}$$

(e) Raising the temperature of the coolant

This would make all materials sparser (less dense), decreasing their macroscopic cross sections, because density (by way of number density) is included:

$$\Sigma = N\sigma \tag{8}$$

We are ignoring secondary effects of Doppler broadening of cross section resonances, as that's a topic for 22.05. Let's take the case of a properly designed reactor, which should have negative temperature coefficients in all cases:

$$k_{eff} = \frac{\nu\left(\overline{\Sigma}_{f_f} + \overline{\Sigma}_{f_{th}}\left[\frac{\overline{\Sigma}_{s_{f\to th}}\downarrow}{\overline{D}_{th}\uparrow B_g^2 + \overline{\Sigma}_{a_{th}}\downarrow}\right]\right)}{\overline{\Sigma}_{s_{f\to th}} + \overline{\Sigma}_{a_f} + \overline{D}_f B_g^2} \tag{9}$$

(f) Increasing the enrichment of the fuel

This would only serve to greatly increase the thermal cross section, at the expense of just a little bit of ^{238}U***'s fast cross section:***

$$k_{eff} = \frac{\nu\left(\overline{\Sigma}_{f_f}\downarrow + \overline{\Sigma}_{f_{th}}\left[\frac{\overline{\Sigma}_{s_{f\to th}}}{\overline{D}_{th}\downarrow B_g^2 + \overline{\Sigma}_{a_{th}}}\right]\right)}{\overline{\Sigma}_{s_{f\to th}} + \overline{\Sigma}_{a_f} + \overline{D}_f B_g^2} \tag{10}$$

1.2 North Korean Nuclear Weapons (16 points)

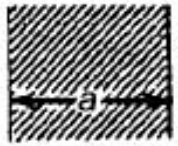
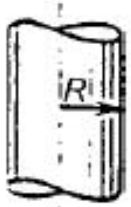
- (12 points) Calculate the radius of a perfectly critical sphere of ^{239}Pu using one-group diffusion theory, assuming it is surrounded by vacuum.

  Here we simply use the one energy-group cross section, choosing an energy of roughly 2 MeV as the most likely fission birth energy. That means all we have to do is write the criticality condition for $k_{eff} = 1$***:***

$$k_{eff} = 1 = \frac{\nu\Sigma_f}{\Sigma_a + DB_g^2} \tag{11}$$

  and we note that the buckling $\left(B_g^2\right)$ ***for a sphere takes a different form than for a slab reactor. We find this table of bucklings in the Neutron Diffusion reading on p. 60 (Table 3.3):***

Table 3.3 Geometric Bucklings and Critical Flux Profiles Characterizing Some Common Core Geometries

	Geometry	Geometric Buckling B_g^2	Flux Profile
Slab		$\left(\frac{\pi}{a_{ex}}\right)^2$	$\cos\left(\frac{\pi x}{a_{ex}}\right)$
Infinite cylinder		$\left(\frac{\nu_1}{R_{ex}}\right)^2$	$J_0\left(\frac{\nu_1 r}{R_{ex}}\right)$
Sphere		$\left(\frac{\pi}{R_{ex}}\right)^2$	$r^{-1}\sin\left(\frac{\pi r}{R_{ex}}\right)$

We now have to find the following cross sections: Σ_f, Σ_a, ***and*** $D = \frac{1}{3(\Sigma_t - \mu_0\Sigma_s)}$. ***All cross sections will also need the number density of plutonium, given as:***

$$N\left[\frac{atoms}{cm^3}\right] = \frac{\rho\left[\frac{g}{cm^3}\right] N_A\left[\frac{atoms}{mole}\right]}{MM\left[\frac{g}{mole}\right]} = \frac{(19.84)\left(6.02\cdot 10^{23}\right)}{239} = 5\cdot 10^{22}\frac{atoms}{cm^3} \tag{12}$$

We then use the JANIS program (I used the Java version, though all the data is the same) to look up microscopic cross section values at 2 MeV, and calculate intermediate values:

Quantity	*Symbol*	*Value*		*Symbol*	*Value* $\left(cm^{-1}\right)$
Total cross section	σ_t	***7.226 b***		Σ_t	***0.3613***
Elastic scattering cross section	$\sigma_{s,el}$	***3.390 b***		$\Sigma_{s,el}$	***0.1695***
Absorption cross section	σ_a	***0.008 b***		Σ_a	***0.0004***
Fission cross section (fast)	σ_f	***1.975 b***		Σ_f	***0.09875***
Neutron multiplication factor	ν	***3.178***			
Average cosine of scattering angle	$\overline{\mu}_0$	***0.00279***			
Diffusion coefficient	***D***	***0.924*** cm^2			
Extrapolation length	***2D***	***1.848*** cm^2			

Plugging these into our criticality condition we get:

$$1 = \frac{\nu\Sigma_f}{\Sigma_a + DB_g^2}; \qquad B_g^2 = \frac{\nu\Sigma_f - \Sigma_a}{D} = \left(\frac{\pi}{R_{ex}}\right)^2 = 0.34 \tag{13}$$

Solving for $\mathrm{R_{ex}}$ ***yields a critical radius of 25.45 cm. This is far larger than the actual value of about 6.3 cm, so clearly something is oversimplified in our one-group, one-energy calculations.***

- (4 points) Assuming some sort of explosive charge compressed the ^{239}Pu to make it go supercritical, why wouldn't it work well as a nuclear weapon? In other words, what would happen as soon as the sphere goes supercritical, and how would it turn the weapon into a dud?
As soon as the fission reaction takes off and goes supercritical, a tremendous amount of heat would be generated. This would cause rapid thermal expansion, lowering all cross sections and making the sphere very quickly go sub-critical. Because absorption has basically nothing to do with it (its cross section is so low), this would only result in far more leakage.

1.3 Power Manipulations (10 points)

Explain, using your knowledge of criticality and feedback, every noticeable feature in your personal manipulation of the MIT reactor. Use your own data from your personal power manipulation for this question. In particular, how does the MIT reactor *not* behave like a more simple feedback system, and what is the physics behind this difference?
I've taken one random student's data as an example, as they all look pretty much the same. Notice first that the chart reads out like a digital strip chart, not a typical graph that you may be used to. Time is on the y-axis, and power is on the x-axis. I've rotated this graph to make it easier to interpret in the usual way:

The graph on the bottom is the reactor power, while the graph on top is the "reactor period," which is the mean time that it would take for the reactor to increase in power by a factor of e. There are a few features to notice here:

1. *The reactor power does <u>not</u> simply increase proportional to the control rods being inserted/removed. This is because there are some delayed neutrons which take much longer to respond.*

2. *The decrease in power is <u>always</u> slower than the increase. This could be due to the buildup of* $^{135}\mathrm{Xe}$ *occurring faster at higher*

power, so it takes a bit more time for it to burn out on the way back down.

3. ***You can see exactly when you inserted/removed the control rod from watching the reactor period increase and decrease.***
4. ***Increasing the reactor power does temporarily raise the fuel temperature, helping lead to a negative fuel temperature coefficient which self-limits the reactor power.***

2 Noodle Scratchers (50 points)

2.1 The Ultracold Nuclear Reactor (20 points, answer not given)

A new reactor concept would use liquid hydrogen as its coolant and moderator, instead of water. One unique feature of this reactor is a new type of moderator, which works so well that a significant fraction of the neutrons are *ultracold*, or have energies well *below* the thermal energy of the surrounding atoms. This means that ultracold neutrons can undergo *upscattering* to the thermal group. In order to fully analyze this reactor, one needs to consider three groups of neutrons: fast (f), thermal (th), and ultracold (uc).

Develop a fully symbolic criticality condition (k_{eff}) for this reactor, in terms of its materials and geometry. Assume the following:

- All fission neutrons ($fraction\ (1-\beta)$) are born fast.
- All delayed neutrons ($fraction\,\beta$) are born thermal.
- The reactor is a homogeneous cylinder, both of height radius H.
- Define any symbols (cross sections, fluxes, diffusion coefficients, etc.) needed to solve this problem.

We begin by writing the balance between gains and losses in each energy group:

$$\frac{(1-\beta)\,\nu}{k_{eff}}\left(\Sigma_{f_f}\Phi_f+\Sigma_{f_{th}}\Phi_{th}+\Sigma_{f_{uc}}\Phi_{uc}\right)=\Phi_f\left(\Sigma_{s_{f\to th}}+\Sigma_{s_{f\to uc}}+\Sigma_{a_f}+D_fB_g^2\right) \tag{14}$$

$$\beta\nu\left(\Sigma_{f_f}\Phi_f+\Sigma_{f_{th}}\Phi_{th}+\Sigma_{f_{uc}}\Phi_{uc}\right)+\Sigma_{s_{f\to th}}\Phi_f+\Sigma_{s_{uc\to f}}\Phi_{uc}=\Phi_{th}\left(\Sigma_{s_{th\to uc}}+\Sigma_{a_{th}}+D_{th}B_g^2\right) \tag{15}$$

$$\Sigma_{s_{f\to uc}}\Phi_f+\Sigma_{s_{th\to uc}}\Phi_{uc}=\Phi_{uc}\left(\Sigma_{s_{uc\to th}}+\Sigma_{a_{uc}}+D_{uc}B_g^2\right) \tag{16}$$

Now we note that to get the criticality condition, we only care about neutrons that are created in or leave the <u>entire system</u>. That means that scattering terms don't represent net gains or losses! We simply add all three equations together and cancel oppositely signed, additive terms to get the total criticality condition:

$$\frac{(1-\beta)\,\nu}{k_{eff}}\left(\Sigma_{f_f}\Phi_f+\Sigma_{f_{th}}\Phi_{th}+\Sigma_{f_{uc}}\Phi_{uc}\right)=\Phi_f\left(\cancel{\Sigma_{s_{f\to th}}}+\cancel{\Sigma_{s_{f\to uc}}}+\Sigma_{a_f}+D_fB_g^2\right) \tag{17}$$

$$\beta\nu\left(\Sigma_{f_f}\Phi_f+\Sigma_{f_{th}}\Phi_{th}+\Sigma_{f_{uc}}\Phi_{uc}\right)+\cancel{\Sigma_{s_{f\to th}}}\Phi_f+\cancel{\Sigma_{s_{uc\to f}}}\Phi_{uc}=\Phi_{th}\left(\cancel{\Sigma_{s_{th\to uc}}}+\Sigma_{a_{th}}+D_{th}B_g^2\right) \tag{18}$$

$$\cancel{\Sigma_{s_{f\to uc}}}\Phi_f+\cancel{\Sigma_{s_{th\to uc}}}\Phi_{uc}=\Phi_{uc}\left(\cancel{\Sigma_{s_{uc\to th}}}+\Sigma_{a_{uc}}+D_{uc}B_g^2\right) \tag{19}$$

We also note that adding the $(1-\beta)$ ***fraction of prompt neutrons and the*** (β) ***fraction of delayed neutrons sums up to a fraction of unity:***

$$\frac{\nu}{k_{eff}}\left(\Sigma_{f_f}\Phi_f+\Sigma_{f_{th}}\Phi_{th}+\Sigma_{f_{uc}}\Phi_{uc}\right)=\Phi_f\left(\Sigma_{a_f}+D_fB_g^2\right)+\Phi_{th}\left(\Sigma_{a_{th}}+D_{th}B_g^2\right)+\Phi_{uc}\left(\Sigma_{a_{uc}}+D_{uc}B_g^2\right) \tag{20}$$

Finally, we express this as a ratio for criticality:

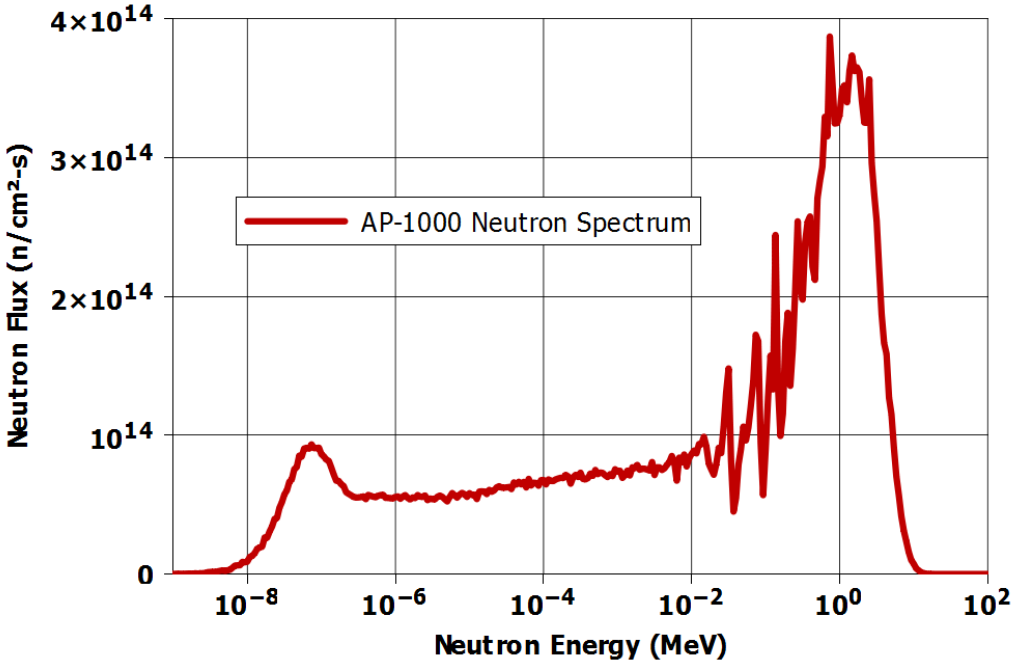
$$k_{eff}=\frac{\nu\left(\Sigma_{f_f}\Phi_f+\Sigma_{f_{th}}\Phi_{th}+\Sigma_{f_{uc}}\Phi_{uc}\right)}{\Phi_f\left(\Sigma_{a_f}+D_fB_g^2\right)+\Phi_{th}\left(\Sigma_{a_{th}}+D_{th}B_g^2\right)+\Phi_{uc}\left(\Sigma_{a_{uc}}+D_{uc}B_g^2\right)} \tag{21}$$

2.2 Will It Blend: AP-1000 Edition (30 points, answer not given)

Read the >>>AP-1000 spec sheet<<<, and answer the following questions:

(a) (25 points) Assuming that the core is completely homogeneous (blended) mix of fuel, cladding, coolant, and structural materials, **calculate the criticality** (k_{eff}) **of the AP-1000 using homogeneous, two energy-group neutron diffusion theory**. You may assume that the core contains only four materials: coolant/moderator (H_2O), fuel (UO_2), cladding $(assume\,pure\,Zr)$, and structural materials $(assume\,pure\,Fe)$. Ignore control rods, assume they are all out of the reactor during normal operation. Also ignore the reactor vessel or any other materials. You will have to calculate averaged cross sections considering each material, each isotope's natural abundance (or uranium enrichment level), for each energy group. ***Hints:*** You will have to take into account:

- Reactor operating temperature in Kelvin and density in $\left(\frac{g}{cm^3}\right)$ of the materials in the reactor
- Different diffusion coefficients for the two energy groups, and their different extrapolation distances for the geometric buckling
- Perform your *microscopic* cross section averages using tabulated data from the ENDF/B-VII.1 cross section database for *incident neutrons*. Note that you can export the data directly, so you can perform the integrals in Excel or something similar. Discretize the energy integral using any method you see fit.
- Neglect photofission, (n,in) reactions, and anything else complicated
- Use the >>>attached AP-1000 tabulated neutron flux profile<<< to compute your averaged macroscopic cross sections. Here is a plot of the flux spectrum:



Answer: First, we note that we'll need to use the two-group criticality relation to figure out what k_{eff} is. We use the relation from the board on November 8th:

$$k_{eff} = \frac{\nu \left[\overline{\Sigma}_{f_f} + \overline{\Sigma}_{f_{th}} \left[\frac{\overline{\Sigma}_{s_{f \to th}}}{\overline{D}_{th} B_g^2 + \overline{\Sigma}_{a_{th}}} \right] \right]}{\overline{\Sigma}_{s_{f \to th}} + \overline{\Sigma}_{a_f} + \overline{D}_f B_g^2} \tag{22}$$

where we note the following relations in calculating each of these:

$$D = \frac{1}{3\left(\Sigma_t - \overline{\mu}_o \Sigma_s\right)} \tag{23}$$

$$\overline{\mu}_0 \approx \frac{2}{3A} \tag{24}$$

$$\overline{\Sigma} = N\overline{\sigma} \tag{25}$$

$$\overline{\sigma} = \frac{\int_{E_{min}}^{E_{max}} \sigma\left(E\right) \Phi\left(E\right) dE}{\int_{E_{min}}^{E_{max}} \Phi\left(E\right) dE} \tag{26}$$

Now let's make a table of all the things we need to calculate for each isotope, and let's list all the isotopes we need. We'll round isotopic fractions of iron and zirconium to the nearest percent for simplicity. We also note that oxygen is found in both H_2O and UO_2 when calculating its number density. Finally, let's also ignore the change in ν due to fission of ^{238}U, because even at its highest it's only a barn or so, compared to the roughly 500 barns for ^{235}U. Note that we are not ignoring the fission of ^{238}U, however. Let's also fill in the μ_0 terms, since they are easy:

Term	$^{1}_{1}$H	$^{16}_{8}$O	$^{54}_{26}$Fe	$^{56}_{26}$Fe	$^{57}_{26}$Fe	$^{90}_{40}$Zr	$^{91}_{40}$Zr	$^{92}_{40}$Zr	$^{94}_{40}$Zr	$^{96}_{40}$Zr	$^{235}_{92}$U	$^{238}_{92}$U
frac.	100%	100%	6%	92%	2%	52%	11%	17%	17%	3%	5%	95%
N												
$\overline{\sigma}_{a_f}$												
$\overline{\sigma}_{a_{th}}$												
$\overline{\sigma}_{s_f}$												
$\overline{\sigma}_{s_{th}}$												
$\overline{\sigma}_{f_f}$												
$\overline{\sigma}_{f_{th}}$												
$\overline{\mu}_0$	0.667	0.0417	0.0123	0.0119	0.0117	0.00741	0.00733	0.00725	0.00709	0.00694	0.00284	0.00280
$\overline{D}_f$												
$\overline{D}_{th}$												
ν	—	—	—	—	—	—	—	—	—	—		—
B_g^2												

Let's now look at the AP-1000 specifications to get the volume of the core, and the masses/volumes of the other materials involved. Starting on page 7, we get the following useful information, and calculate/look up any values that we need. The most critical values are highlighted in **underlined bold**:

Paramter	Value
Avg. Temperature	$303.4\,C$ $(578.1\,F)$
Core Pressure	15.51 MPa (2250 psia)
Fuel Weight as UO_2	96,163kg (211,558 lb.)
Moles UO_2 (calc.)	$\frac{96{,}163{,}000g}{269.85\frac{g}{mole}} = 356,357\,moles$
Clad Weight	19,593kg (43,105 lb.)
Moles Zr (calc.)	$\frac{19{,}593{,}000g}{91.224\frac{g}{mole}} = 214,779\,moles$
Core dia., equivalent	3.04m (119.7 in.)
Core height, active region	4.27m (168.0 in.)
Total Core Volume (calc.)	**30.99** m^3
Core Structure (ID/OD)	3.40/3.50m (133.75/137.75 in.)
Structural Material Volume **(calc.)**	0.74 m^3
Stainless Steel Density	8,000 $\frac{kg}{m^3}$
Stainless Steel Weight	$\left(8,000\frac{kg}{m^2}\right)\left(0.74m^3\right) = 5,920kg$
Moles Stainless Steel (Fe)	$\frac{5{,}920{,}000g}{55.845\frac{g}{mole}} = 106,008\,moles$
Number of Fuel Rods	41,448
Fuel Rod OD	0.0095m (0.374 in.)
Fuel & Clad Volume (calc.)	2.00 m^3
Water Volume (calc.)	28.25 $m^3 = 30.99 - 0.74 - 2.00$
Water Density at 303.4C, 15.51 MPa	720.357 $\frac{kg}{m^3}$
Water Weight (calc.)	20,350 kg
Moles of Water (calc.)	$\frac{20{,}350{,}000g}{18\frac{g}{mole}} = 1,130,556\,moles$

We got the water properties from a steam tables website: http://www.spiraxsarco.com/Resources/Pages/Steam-Tables/sub-saturated-water.aspx

Using the moles of each substance in the reactor and the total core volume, we can calculate the number density of each substance in the core. We use the following relation for each:

$$N = \frac{(moles_{core})\,(N_A)}{V_{core}} \tag{27}$$

Substance	Moles	Core Volume (m^3)	N_a $\left(\frac{\text{atoms}}{\text{mole}}\right)$	Number Density $\left(\frac{atoms}{m^3}\right)$
UO_2	356,357	30.99	$6.02 \cdot 10^{23}$	$6.922 \cdot 10^{27}$
H_2O	1,130,556			$2.196 \cdot 10^{28}$
Zr	214,779			$4.172 \cdot 10^{27}$
Fe	106,008			$2.059 \cdot 10^{27}$

Now we can use these number densities, times their isotopic fractions, to fill in the number densities of each isotope in the main table. The only tricky ones are oxygen, which equals twice the UO_2 number density plus the H_2O number density, and hydrogen, which equals twice the H_2O number density:

Term	$^{1}_{1}$H	$^{16}_{8}$O	$^{54}_{26}$Fe	$^{56}_{26}$Fe	$^{57}_{26}$Fe	$^{90}_{40}$Zr	$^{91}_{40}$Zr	$^{92}_{40}$Zr	$^{94}_{40}$Zr	$^{96}_{40}$Zr	$^{235}_{92}$U	$^{238}_{92}$U
frac.	100%	100%	6%	92%	2%	52%	11%	17%	17%	3%	5%	95%
N $\left(10^{27}\frac{atoms}{m^3}\right)$	42.92	35.80	0.1235	1.894	0.04118	2.169	0.4589	0.7092	0.7092	0.1252	0.3461	6.576
$\overline{\sigma}_{t_f}$												
$\overline{\sigma}_{t_{th}}$												
$\overline{\sigma}_{a_f}$												
$\overline{\sigma}_{a_{th}}$												
$\overline{\sigma}_{s_f}$												
$\overline{\sigma}_{s_{th}}$												
$\overline{\sigma}_{f_f}$	—	—	—	—	—	—	—	—	—	—		
$\overline{\sigma}_{f_{th}}$	—	—	—	—	—	—	—	—	—	—		
$\overline{\mu}_0$	0.667	0.0417	0.0123	0.0119	0.0117	0.00741	0.00733	0.00725	0.00709	0.00694	0.00284	0.00280
$\overline{D}_f$												
$\overline{D}_{th}$												
ν	—	—	—	—	—	—	—	—	—	—		—
B_g^2												

Now we will tackle the microscopic cross sections. For each one, we take the following procedure:

1. The MITR flux spectrum appears to have 30 energy points per decade. Let's use JANIS to export the cross section using the same thirty points per decade over the chosen energy range. **(NOTE: I had to type 33 values per decade to get JANIS to output the exact number of values as the MITR flux data).** We assume the following in this case:
   (a) The cutoff between the fast and thermal regions is 1eV
   (b) Maximum and minimum neutron energies come from the flux spectrum from the MIT reactor
2. Split the energy/flux data into the thermal and fast groups
3. Convert the given flux from $\frac{n}{cm^2 s}$ to $\frac{n}{m^2 s}$ by multiplying by 10,000
4. Copy/paste the JANIS data into the next column for each energy range
5. Numerically integrate the $\sigma(E)\Phi(E)$ and the $\Phi(E)$ terms using a simple Riemann sum, adding the area of a rectangle of width $\Delta E * \Phi(E)$, where ΔE is the difference in energy between this cell and the previous one
6. Divide the integrals for $\int \sigma(E)\Phi(E)$ by $\int \Phi(E)$ to get the value of $\overline{\sigma}$
7. Cut/paste this data into a new sheet for every isotope, and just copy/paste the JANIS data for that isotope to automatically compute the cross section for each.
   (a) Absorption cross sections are calculated as $\sigma_t - \sigma_s$. Fission cross sections are directly gleaned from the JANIS database.

Using this procedure, the following values were found for each isotope:

Term	$^{1}_{1}$H	$^{16}_{8}$O	$^{54}_{26}$Fe	$^{56}_{26}$Fe	$^{57}_{26}$Fe	$^{90}_{40}$Zr	$^{91}_{40}$Zr	$^{92}_{40}$Zr	$^{94}_{40}$Zr	$^{96}_{40}$Zr	$^{235}_{92}$U	$^{238}_{92}$U
frac.	100%	100%	6%	92%	2%	52%	11%	17%	17%	3%	5%	95%
N $\left(10^{27}\frac{atoms}{m^3}\right)$	42.92	35.80	0.1235	1.894	0.04118	2.169	0.4589	0.7092	0.7092	0.1252	0.3461	6.576
$\overline{\sigma}_{t_f}$ (b)	19.6	3.84	90.2	5.66	27.5	5.62	14.2	10.5	18.3	13.2	18.1	17.9
$\overline{\sigma}_{t_{th}}$ (b)	21.4	3.86	2.77	12.7	3.23	5.50	10.2	7.17	8.63	5.71	144	9.95
$\overline{\sigma}_{a_f}$ (b)	0	0	0.1	0	0.1	0.01	0.4	0	0.1	0	6.2	1.3
$\overline{\sigma}_{a_{th}}$ (b)	0.1	0	0.59	0.6	0.63	0	0.34	0.06	0.01	0	130.2	0.75
$\overline{\sigma}_{s_f}$ (b)	19.6	3.84	90.1	5.66	27.4	5.61	13.8	10.5	18.2	13.2	11.9	16.6
$\overline{\sigma}_{s_{th}}$ (b)	21.3	3.86	2.18	12.1	2.60	5.50	9.86	7.11	8.62	5.71	13.8	9.20
$\overline{\sigma}_{f_f}$	—	—	—	—	—	—	—	—	—	—	4.32	˜0
$\overline{\sigma}_{f_{th}}$	—	—	—	—	—	—	—	—	—	—	112	˜0
$\overline{\mu}_0$	0.667	0.0417	0.0123	0.0119	0.0117	0.00741	0.00733	0.00725	0.00709	0.00694	0.00284	0.00280
$\overline{D}_f$												
$\overline{D}_{th}$												
ν	—	—	—	—	—	—	—	—	—	—		—
B_g^2												

*****Please see the attached Excel sheet for my example calculations, which filled in all the numbers in this table.***** Finally, we can now use the following expression to calculate averaged, macroscopic cross sections:

$$\overline{\Sigma} = \sum_{i=1}^{n} N_i \overline{\sigma}_i \tag{28}$$

We then arrive at the following table:

Term	$^{1}_{1}$H	$^{16}_{8}$O	$^{54}_{26}$Fe	$^{56}_{26}$Fe	$^{57}_{26}$Fe	$^{90}_{40}$Zr	$^{91}_{40}$Zr	$^{92}_{40}$Zr	$^{94}_{40}$Zr	$^{96}_{40}$Zr	$^{235}_{92}$U	$^{238}_{92}$U	$\overline{\Sigma}\ (m^{-1})$
frac.	100%	100%	6%	92%	2%	52%	11%	17%	17%	3%	5%	95%	—
N $\left(10^{27}\frac{atoms}{m^3}\right)$	42.92	35.80	0.1235	1.894	0.04118	2.169	0.4589	0.7092	0.7092	0.1252	0.3461	6.576	—
$\overline{\sigma}_{t_f}\ (b)$	19.6	3.84	90.2	5.66	27.5	5.62	14.2	10.5	18.3	13.2	18.1	17.9	116.7
$\overline{\sigma}_{t_{th}}\ (b)$	21.4	3.86	2.77	12.7	3.23	5.50	10.2	7.17	8.63	5.71	144	9.95	122.5
$\overline{\sigma}_{a_f}\ (b)$	0	0	0.1	0	0.1	0.01	0.4	0	0.1	0	6.2	1.3	1.099
$\overline{\sigma}_{a_{th}}\ (b)$	0.1	0	0.59	0.6	0.63	0	0.34	0.06	0.01	0	130.2	0.75	5.573
$\overline{\sigma}_{s_f}\ (b)$	19.6	3.84	90.1	5.66	27.4	5.61	13.8	10.5	18.2	13.2	11.9	16.6	115.5
$\overline{\sigma}_{s_{th}}\ (b)$	21.3	3.86	2.18	12.1	2.60	5.50	9.86	7.11	8.62	5.71	13.8	9.20	116.9
$\overline{\sigma}_{f_f}$	—	—	—	—	—	—	—	—	—	—	4.32	˜0	0.1495
$\overline{\sigma}_{f_{th}}$	—	—	—	—	—	—	—	—	—	—	112	˜0	3.876
$\overline{\mu}_0$	0.667	0.0417	0.0123	0.0119	0.0117	0.00741	0.00733	0.00725	0.00709	0.00694	0.00284	0.00280	0.329 (avg.)
$\overline{D}_f$	0.004237												—
$\overline{D}_{th}$	0.003965												—
ν	—	—	—	—	—	—	—	—	—	—	2.43	—	—
B_g^2	3.045												—

Now we compute the diffusion constants using Equation 23, look up ν for $_{92}{}^{235}$U, and use the expression for the buckling of a finite right cylinder:

$$B_g^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2 = 3.045 \tag{29}$$

FINALLY, using the values from this table, we substitute into the original criticality condition:

$$k_{eff} = \frac{\nu\left[\overline{\Sigma}_{f_f} + \overline{\Sigma}_{f_{th}}\left[\frac{\overline{\Sigma}_{s_{f\to th}}}{\overline{D}_{th}B_g^2 + \overline{\Sigma}_{a_{th}}}\right]\right]}{\overline{\Sigma}_{s_{f\to th}} + \overline{\Sigma}_{a_f} + \overline{D}_f B_g^2} = k_{eff} = \frac{2.43\left[0.1495 + 3.876\left[\frac{115.5}{(0.003965)(3.045)+5.573}\right]\right]}{115.5 + 1.099 + (0.004237)\,(3.045)} = 1.67 \tag{30}$$

(b) (5 points) What are the largest three factors which you believe make your k_{eff} not equal to unity? Be specific about what each would do to the two energy group criticality relation.

Likely sources of error include:

1. ***Slight mismatch between MITR and JANIS tabulated energy values***
2. ***Numerical error from the Riemann sum integration (perhaps a trapezoidal rule would have been better)***
3. ***Not counting the probability of fast scatters into the thermal region (scattering kernel)***
4. ***Ignoring the extrapolation distance in the buckling calculation***
5. ******Homogenizing the core totally messes things up!!!******

6. ***Not doppler-broadening the cross sections, which are at room temperature in JANIS***
7. ***Assuming that stainless steel is pure Fe (Ni has a high absorption cross section) and that ZIRLO is just pure Zr***

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Fall 2016

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