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MICHAEL SHORT: Want to give a quick review so that we can launch into some more technical stuff today. We started talking about this reaction for boron neutron capture therapy was the focus of today's lecture. And I want to get this one up on the board since I'm going to move to different slides later.

We had boron 10 captures a neutron and becomes lithium 7, helium 4. There's a gamma ray. And there's going to be some q value, or energy either released or consumed, in this case, released in the form of the kinetic energy of the recoil products as well as the gamma ray energy.

So let's also give a quick review of the Q equation since I think we covered that last week. If you remember, if you have a general system of, let's say, a small initial nucleus i firing into a larger nucleus I , after the reaction, off comes some small final nucleus and some large final nucleus. If I were to draw these arrows to scale, the little one would probably be moving faster.

Then we can figure out how much mass and kinetic energy each of these nuclei are by just conserving everything. So we can write, let's say, the mass of nucleus i , c squared plus the kinetic energy of i plus the mass of big I c squared. Plus the kinetic energy of big I has to equal the mass of little f c squared plus the kinetic energy of little f plus the mass of big F c squared plus its kinetic energy.

And what this tells us is if the total amount of mass, or the total kinetic energy change, they've got to exchange energy kind of equally. So we can write the difference in mass or energy by just, let's say, taking all of the mass terms and putting them on one side. So we can say that-- let's just say everything here is multiplied by c squared. We have M_i plus M_I minus M_f minus M_F has got to equal the sum of the final kinetic energies minus the initial ones, which we also call this Q value.

And so by getting the difference in the masses or the kinetic energies at the end, you can

figure out whether this reaction is exothermic or endothermic. If you remember, we said if Q is greater than 0, it's exothermic. If Q is less than zero, it's endothermic. In a nuclear reaction like a chemical reaction, you've got to input extra energy into the system beyond just the rest masses of the particles to make an endothermic reaction happen.

For example, since we found out that this BNCT reaction is endothermic, you can make it go the other way, but you have to impart kinetic energy to one or both of those nuclei to overcome the Q value that-- let's see-- yeah, to overcome the Q value that you'd get.

And I want to put up a couple of other terms just to quickly review. We started looking at the table of nuclides. We learned how to read it specifically to find things like excess mass and binding energy, the definitions for which I want to leave up here on the board. So we had the excess mass, which, again, doesn't have a physical significance. It's the difference between the actual mass of a nucleotide and the integer approximation of its mass just from the number of protons and neutrons, and the binding energy. Since some of you asked for a nucleus uniquely defined by its total mass number a and number of protons z .

So let's say this is a functional quantity because you can have any nucleus with a certain number of protons and a certain number of protons plus neutrons would be the sum-- let's see-- the sum of its individual nucleons minus its actual mass, which also is a function of z and a , or its proton number and its total atomic mass number. So let's leave these things up on the board while we move into some new stuff.

So back to where we were on Tuesday. We wanted to figure out, well, how do we calculate this Q value? You can do it any of three ways. You can use the masses, like we have over here. So the masses in amu, or atomic mass units, times c squared gives you the Q value in MeV. Or the difference in kinetic energies gives you the Q value in MeV, although you don't usually just know these off the bat, especially for the final products. Or you can use the binding energies. Because the binding energy is directly related to the mass additively, you can substitute in binding energies here and you'll get the same thing.

And so if we have the table of nuclides, you can either use the atomic mass of any nucleus or its binding energy and just look it up directly. The thing with the fewest steps is just to use the binding energies because those are already given in MeV or keV. And then it's just an addition subtraction problem. If you use the masses, don't forget to multiply by c squared. And remember our conversion formula, which should not be rounded, which is this right here.

Again, 931.49, not 931, not even 931.5, or else you're off by 10 kiloelectron volts.

And so then the question is, all right, let's try to calculate the Q value from this reaction. I have the individual energies and kinetic energies up here. But just so we have a worked out example, let's actually do this. So the Q value of this reaction should be the binding energy of lithium 7 plus the binding energy of-- what do we have-- helium 4 minus the binding energy of boron 10 minus the binding energy of a neutron.

Now first of all, the easy one. What's the binding energy of a lone neutron? Let's call it out. 0, yeah. A lone nucleon is not bound to anything, so that's easy. That's 0. And we can just use the table of nuclides to look up the other three. Luckily I've got it live over here.

So let's just punch these in. We'll have boron 10. And our binding energy right here is 64.75 MeV. Let's look up helium 4. Good. That's showing up on the screen. I think I'll make it bigger so it's easier for everyone to see. The binding energy is 28.296 MeV. And notice everything's already in MeV, so this is nice and easy to deal with. And lithium 7. 39.245 plus minus minus 0 MeV.

Let's see. Actually, I think my signs for exo and endo are backwards. Let me fix that right now because the idea is if you release energy-- there we go. Sorry about that. No, wait a minute. Let's calculate this out, figure what we get. And again, I can't do six digits in my head, but I will do it as fast as I can here. 245 plus 28.296 minus 0.75. Ah, indeed. 2.79 MeV. I had it right the first time. So good. Shouldn't second guess myself. Cool.

So now we know the total Q value of this reaction. And I'll bring the reaction back up here. We also know, which you can find from measurements, that the gamma ray comes off with an energy of 0.48 MeV, leaving-- let's see how much-- leaving 2.13 MeV for the sum of the kinetic energies of the lithium and the helium nucleus. So let's say T_{Li 7} plus t for helium.

Now the question is how did I get to those numbers for the split between the kinetic energies of those two? Anyone have any idea? Yeah.

AUDIENCE: Related to their mass?

MICHAEL SHORT: Yeah. So it's definitely related to their relative masses. More specifically, we have this conservation of energy equation, but we've still got two variables and one equation. We need a second equation. That certainly does relate the mass. Yep.

AUDIENCE: Conservation momentum.

MICHAEL SHORT: You can relate their momentum. So we can say that if this initial kinetic energy of boron and of the neutron was approximately 0, then it's like these two nuclei, lithium and helium, were kind of standing still and all of a sudden moved off in opposite directions, which means they've got to have equal and opposite momenta.

So let's say the absolute value of the momentum of lithium has got to equal the absolute value of the momentum of helium. This is our second equation, where we'll use the first one to figure out what are the relative kinetic energies. And so we're going to use a quick trick to say if momentum equals mass times velocity, we can also say it equals the kinetic energy, which is $\frac{1}{2} mv^2$, and then multiply by things in order to make it mv . So we can multiply by 2. Let's make a little bit of space here.

If we take the kinetic energy, multiply by 2, multiply by m , and take the square root, we have t . Let's see, that would give us $2 m^2 v^2$ inside the square root gives us mv . So now we can take these expressions and we can say that the root 2 mass lithium, t lithium, equals root 2 mass helium t helium. And they both have a square root of 2 . We can square both sides of both equations. And we end up with mass of lithium t lithium equals mass of helium t helium.

Now we take this equation, rearrange it a little bit. Let's just call that Q to keep things in variable space. And we can say that t helium equals Q minus t lithium. We can take this t helium, stick it in here. We end up with m lithium t lithium equals the mass of helium times Q minus t lithium. There's a missing h right there. There we go.

And then from here-- oh good. We've got some blank space right here. Let's see. So we'd have-- I'll do out all the steps-- equals mass of helium times Q minus mass of helium times t lithium. And we're solving for t lithium, so we can put this term over on the other side. So let's say t lithium times the mass of lithium plus the mass of helium equals Q times the mass of helium. Then we can just divide both sides by the sum of those masses

Those two terms cancel, and we're left with the expression for the kinetic energy of lithium, which is the mass of helium over the sum of the masses times the total Q value. Looks like the two of those might actually be backwards, huh? Because the lithium one should be smaller. I'll correct that for the notes when I put them up online because this ratio should be smaller than $0.5 Q$. But at any rate, this is how you actually get them.

And I want to point out something a little flash forward to some of the decay that you're going to be looking at in terms of nuclear decays. Let's talk for a second about alpha decay. In alpha decay, you have a low nucleus sitting around. And then all of a sudden it emits a helium nucleus as well as a recoil nucleus. And one of the questions that came up a lot last year is why isn't the Q value of an alpha decay reaction-- let's just write one up on the board, possibly one that you'll be dealing with very soon, hands on.

Uranium 235 can spontaneously go to a helium nucleus. That's 92 plus looks like 90 to 31. What comes before uranium? I think that is thorium, although I wouldn't-- yeah, I think that's thorium. And the idea here is that there is some Q value associated with the kinetic energy of both of these. But it's not the alpha kinetic energy because the thorium nucleus would have to take away some of that kinetic energy.

And I want to show you this on the diagrams. So let's look at U 235. And we can see that it has an alpha decay to thorium 231. Awesome. Got the symbol right. And it has a decay energy of 4.679 MeV. Let's take a look at the diagram, which actually lists all of the possible alpha decay energies, of which there are many. Had to zoom out a little bit there.

Yeah? Anyone have a question? OK.

So notice that the difference in energy levels is 4.676 MeV. And if we look at the highest energy alpha ray, it's less than that. And that's because just like we showed up here, the thorium nucleus has to take away some of that kinetic energy to conserve both energy and momentum. So this is a question that came up quite a few times last year, and I want to make sure you guys don't get tripped up like this.

So again, I think I've said it every single day, and I'll say it again because it's the next day, is make sure to conserve mass energy and momentum. That's the whole theme of this class.

Yes.

AUDIENCE: How do you get that value for [INAUDIBLE] mega electron volts?

MICHAEL SHORT: Should have been 2.79 minus-- oh, did I do a little mental math mistake? That should be-- oh. Yeah, no. A little dyslexia thing. 3, 1. Yeah. There we go. Thank you. Cool.

OK. Is everyone clear on how to calculate Q values from nuclear reactions using either kinetic

energies, which you won't typically know, or masses or binding energies, which you can look up directly from the table of nuclides? Yeah.

AUDIENCE: So what did you do down there in the bottom right corner of the chalkboard?

MICHAEL SHORT: Of this one?

AUDIENCE: Yeah.

MICHAEL SHORT: Yep. So I took this expression right here, which is to say the Q value has got to be the sum of the kinetic energies of the lithium and helium nuclei, rearranged it thusly to isolate-- ah, there we go-- yep, to isolate the helium kinetic energy, and then substituted that expression in here to get this one right there.

AUDIENCE: OK.

MICHAEL SHORT: Yeah. So this way, we have-- in this case, we had, let's say, two variables and three unknowns. But because we have this equation relating them, we're left down with two variables, two unknowns, and we can actually solve this thing. Yep. Yeah, good question. Yes.

AUDIENCE: The energy of the gamma, is that just a known? Like the .48 MeV?

MICHAEL SHORT: That's something either I tell you or you would measure, let's say. That's just for completeness to say all right, this reaction actually gives off a gamma, and I want to give the right value for the kinetic energies. And we'll get into what gamma transitions are allowed and then how you measure them in the next couple of weeks, actually. Yes.

AUDIENCE: So do we refer to Q as the Q you calculated up there, or that 2.31?

MICHAEL SHORT: They're the same one, actually.

AUDIENCE: The 2.7

MICHAEL SHORT: Oh, I see. That's a good point. So this wouldn't really be Q, would it? But it is the sum of the kinetic energies. This is like Q minus the gamma ray energy. Let me stick that in there. L, i. Yep. Yeah. Good point.

OK. Any other questions before I move on? We're going to get into a universal formula to predict in a so-so way what the binding energy of any given nucleus will be and start looking at stability trends so you can predict, just from the number of protons and the number of

neutrons, how stable a nucleus will be with a few exceptions, which we will go over.

And this is what's referred to as the semi-empirical mass formula. So I'm going to erase some stuff. Has everyone got the notes on this bottom board right here? OK. Let me know when you're ready, and let's see. Yeah. I want to make sure I move at your guys' pace.

Let's say going to have a graph of binding energy per nucleon on versus nucleons. Well, anyway, I'll leave that up there and I'll do the work on this board right here. So let's say we wanted to figure out a weighted graph or to predict the binding energy per nucleon. So I have this binding energy term over A , where A is the total number of nucleons, as a function of the number of nucleons-- in a generalized way. Not accounting for magic numbers or anything else that we'll get into pretty soon.

And I don't like the term magic numbers, but that is the parlance that's used in this field so I'm going to stick with it. Let's try and think about if you imagine the nucleus as a kind of drop of liquid-- and one of the other words for the semi-empirical mass formula is called the liquid drop formula or the liquid drop model.

It assumes that the nucleus takes the shape roughly of a liquid drop, and you can kind of treat some of the energy terms accordingly. This is why I have all the different colors of chalk out for this. Makes it a little visually easier to see.

So let's start writing a general expression for the binding energy as a function of A and Z , and start thinking about what sort of terms would add to or decrease the stability of a given liquid drop nucleus, where all the nucleons are just kind of there in some sort of floating, crazy, coulombic, strong nuclear force soup.

First of all, as you add nucleons to a given nucleus, what tends to happen to the binding energy, in general? Without knowing anything else. Assemble more nucleons, you convert more mass to energy. And you end up increasing the binding energy. So let's call this the volume term. As you increase the volume of this liquid drop, its total binding energies start to increase.

So let's say there's some term that's going to be proportional to A , the number of nucleons that's in this liquid drop. And we'll draw this. Let's see if I can do the trick right. Yes. I love doing that. If we were to graph binding energy per nucleon as a function of number of nucleotides for this term, it would just be a flat line because it's related to A . And there's going

to be some constant, which we're going to call the volume constant, that says, well, there's going to be some relation between the actual amount of stability gained and the number of nucleons. We don't know what it is yet, but what we're really concerned with is the functional form of this thing. It's proportional to A .

Next up, what also happens to a liquid drop as you increase its volume? What other parameters do you increase?

AUDIENCE: The surface area?

MICHAEL SHORT: Exactly. The surface area. The idea here is that if this liquid drop is made of all sorts of different nucleons-- and let's pretend that they're like atoms in a crystal and they're all binding to each other-- the ones on the outside aren't bound to as many nucleons as the ones on the inside.

And so the more nuclei there are near the surface as opposed to inside the liquid droplets, say, inside some little radius where all it sees around it are other nucleons, then they're not quite as bound. And how does the surface area of a liquid drop scale with its volume? To what function or to what exponent?

$2/3$. I mean, let's take a quick look at the volume of a sphere is $\frac{4}{3} \pi r^3$. And the surface area of a sphere is $4 \pi r^2$. So if you want to get some expression for how does area scale as volume, I said cube, then I wrote squared. It's going to end up looking like something times r to the 2-- let's see. Oh, yeah. I'm sorry, that's not the expression I want to write. But the idea here is it's going to scale with r to the $2/3$.

So let's pick a different color and say we're going to have some surface term times number of nucleons to the $2/3$. And if we then adjust this formula to also take into account this surface area term-- which is to say for very small nuclei, there's a lot of nucleotides near the surface, and as the nucleus gets bigger and bigger, more and more of them are in the juicy center and don't know they're near the surface-- we'd have some modification that looks like that.

Now I'm going to erase the stuff over here because I'm running out of room. Now these nucleons aren't just untagged, anonymous nucleons. They're either protons or neutrons. And what happens when you try and cram a lot of protons into one space?

AUDIENCE: They want to repel each other.

MICHAEL SHORT: Yep. They want to repel each other by coulombic forces. And so every proton-- let's pick a different color for the coulombic forces-- and that should be a minus if I want to stick with all the notation. There's going to be some other term to account for the fact that the nuclei, specifically the protons, are trying to repel each other. So in this case, it's going to be proportional to, let's say, the number of protons that we have. And every proton should feel a repulsive force from every other proton.

So let's say it's times z times z minus 1, so that every proton feels the force of every other proton except for itself. And that's going to be mediated by the total number of nucleons. So if there are more neutrons in the way, it won't be quite as bad. And there's going to be some other-- we'll call it a C for the coulombic term-- and that will say that as you make a bigger and bigger nucleus, you start to get more and more coulombic repulsion trying to rip it apart.

So if we were to then modify the purple curve-- oops. Trying to get it to go the same as the nucleus gets bigger. I want to make sure it's really to scale-ish. As the nuclei get bigger and bigger, it's going to be a little less stable. And already we're starting to get a curve that is getting close to looking like the binding energy curve from the reading. But there are a couple more terms to reckon with. So let's pick a fourth color.

What other sort of trends that you notice in the reading about the stability of different nuclei? Let's say you were to take a common nucleus like carbon-12. It's got six protons and six neutrons, and it's exceptionally stable. What about carbon 6? A nucleus of just 6 protons? Doesn't exist. Exceptionally unstable. What about carbon 24? 18 neutrons, six protons. Sound stable or not? Not at all.

So there's some sort of asymmetry term going on. When the number of protons and neutrons is roughly in balance, especially for light nuclei, the nucleus tends to be more stable. So we can write some sort of term-- let's call it an asymmetry term-- that relates to the number of neutrons minus the number of protons.

And in this case, for reasons I'm not going to get into, but are derived in a reference in your reading, there's a squared on it. But suffice to say if the number of neutrons and number of protons are equal, then the nucleus is predicted to be pretty stable.

And this works out quite well for light nuclei. It starts to break down a little bit for heavier nuclei. And then divide by the number of nuclei that there are. I also see a missing $1/3$ because let's say this nucleus has a volume that scales with roughly the number of nucleons. Then the

distance of that coulombic force is going to be like A to the $1/3$ or the radius of this nuclear drop.

So let's take the asymmetry term. That's going to give us a further modification slightly downward. And finally, there's what's called the pairing term. What's the last color I haven't used? This pairing term δ . And this is not a smooth function. It's a piecewise function that depends on whether you have an odd or an even number of each type of nucleon, protons or neutrons.

And so what this means, it's going to add a little bit of jaggedness to the beginning of the curve and equal out in the end because this δ term can be something like plus, let's call it an A pairing. And it scales with the square root of A , or minus, or 0, depending on if the nuclei are odd odd, like odd number of protons, odd number of neutrons, even even, or odd even.

Now I know that the derivation is a little hand wavy. That's why we call it semi-empirical. We're taking each of these additive terms and saying it kind of comes from a fairly OK, a little poor approximation of the nucleus. But what we end up with is a formula whose constants are fit, whose terms, the actual variables, are derived somewhat from physical intuition. These constants were then fit later by some other folks, and the references for this are in the reading. They're all in MeV, and this gives you a binding energy in MeV for a given nucleus.

Now it works some places and it doesn't work in other places. Yeah.

AUDIENCE: So the lower case a [INAUDIBLE] no matter what the nucleus looks like?

MICHAEL SHORT: That's right. So this is the universal, semi-empirical, usually works formula for the binding energy of a nucleus. So the constants don't change because the variables here are z , [INAUDIBLE] a , and n . And don't forget-- because you'll need to remember this on the homework-- that A equals z plus n .

So for example, if you want to express what is the most stable nucleus, you could take the derivative of this formula with respect to A or z . And don't forget that you can substitute this expression into there. That's giving you guys a hint for the homework.

And let's look at what this actually looks like as far as theory compared to experiment. So the red points are theoretical predictions. The black points are experimental predictions. And all of the different nuclei are shown here. First of all, the curve looks quite a bit like the one that we

just hand wavy made on the board right here.

And second of all, there aren't too many exceptions. It's hard to see what the exceptions are, so it's a little easier to draw them in terms of relative error. So you can see where does this formula work and where does this formula not? So if you notice, for the small nuclei, approximating it as a liquid drop is not a very good approximation because you can't treat this as a homogenized, smeared liquid drop. It's much more-- well, there's either two or three nucleons, and very few protons and neutrons in each.

But then as you get to larger and larger nuclei, it starts to hit very close with a few exceptions that I want to point out right now. If you zoom in on that part, you can actually see that at certain neutron numbers, or certain proton numbers, there is an exceptionally high stability of a lot of those nuclei. And that's as you start to approach these what's called magic numbers, or numbers of nucleons which, say, fill all energy levels at a certain level.

And again, it's not for every nucleus as a function of neutron number. But even drawing an envelope around this curve, you can see that the nuclei around 82, around 50, around 28, are a whole lot more stable than the ones in between. And this pattern kind of repeats with larger and larger periodicity. And it kind of looks like right here, at the edge of our knowledge of nuclei, we haven't quite gotten to the next peak yet. This is something we're going to talk about Friday on the quest for super heavy elements, or SHE's, as you'll see in the reading. Yeah.

AUDIENCE: So the most stable nuclei peaks were closest to 0.

MICHAEL SHORT: Closest to 0 is the closest agreement between experiment and theory. So the ones that are exceptionally stable, which are not predicted by this very simple formula, are up here at the peaks of these magic numbers. And actually, I want you to take a look right here at some of these very small nuclei.

Like helium 4 is probably way up here somewhere, all the way over on the right. It's an exceptionally stable nucleus that is not very well approximated by liquid drop model because it's got four nucleons. They're all on the surface. There's nothing on the inside of a helium nucleus, let's say.

And then if you look at stability trends in terms of are the nuclei more stable if they have odd numbers or even numbers, you can graph the two separately and look at the number of stable

nuclei that have an odd total mass number or an even total mass number. And there's a few things to note here.

One of them is the even numbers tend to have a lot more stable nuclei. This is something I mentioned on the second day of class. If you look at the [INAUDIBLE] table of nuclides-- let's go to their home page-- and just look at it sort of in a color way. The blue colors are stable nuclei, and you notice that every other row of pixels here has a whole lot more stable ones. And that's the same thing that we're seeing right here, is that there's a lot more even nuclei that are more stable.

If I jump back to our semi-empirical maths formula, notice that this binding energy goes up for even, even nuclei. So when there's an even number of protons and an even number of neutrons, the semi-empirical mass formula does predict an increase in stability, which you can actually see on the table of nuclides, and on this sort of stability trend.

And so let's look a little closer and see how many nuclei for each proton number or each neutron number are actually stable. And we graphed the odd and the even ones separately. And what's important to note here is one, the odd is way lower than the even. There's usually either 2, 1 or 0 stable nuclei at that number. And what other sort of features do you guys notice about this?

It's not smooth, first of all. Where are those peaks? Where do you tend to find that most stable nuclei? 4? Where do you tend to find the least? What about these two right here? No stable nuclei at these proton numbers. And remember, proton number uniquely defines an element.

Anyone know what these two might be? What sort of elements? Look to the back of the room if you want. There's a periodic table in the back wall. And you can see, except for the super heavy things down at the bottom, there's a couple of elements that have no stable isotopes. These are technetium and promethium, which are relatively light isotopes-- I say relatively light compared to things like uranium-- with no stable isotopes.

They're also fairly far away from these so-called magic numbers or other regions, where you tend to have a spike in the number of stable nuclei due to-- well, things that you'll learn in 22.02, in terms of nuclear shell occupancy and stability. But you see the same thing when you graph the neutron number. You can see a couple of sudden spikes right here at 2, 8, 20, 28, 50, 82, and 126. When everything gets really stable, all of a sudden you've got one last gasp of a stable isotope before you go off into nowhere land.

So let's start looking at relative stabilities of nuclei, let's say, for a given mass number or a given proton number. Anyone mind if I cover this board because you can't roll it up. You all got the notes from here? Cool. That one has less to erase. And I want to keep these formulas up for our reference.

Let's say I pose this problem. I want to find out, make sure I solve the right one-- actually, I'm going to check my notes real quick-- that what is the-- for a given A, or for a given mass number-- what is the most stable number of protons? for A, given A.

This is the question that I like to answer here. How would you approach this question using the semi-empirical mass formula that-- well, you can't see here, so I will bring it up back on the screen. Here. How would you find this out?

Well, let's say for a given mass number A, for a given poor approximation of the total mass of a nucleus, the more binding energy it has, the more stable it is. Therefore, if you want to find the minimum of a mass for A and Z, given a fixed A, that will give you the most stable nucleus because it will tell you which value of Z gives you the smallest m, or rather, the most tightly bound nucleus that has the most binding energy.

So let's start writing this out. First of all, we can use one of the two equations we already have up here, a relation between the mass and the binding energy. The second one, well, we have it right here. So let's substitute in our binding energy equation and express it in terms of mass.

So let's say our mass of a nucleus A and Z is equal to Z times the mass of hydrogen plus A minus Z times the mass of a neutron minus the binding energy because in this case, what I've done right here is I've added mass to each side of the equation, subtracted binding energy from each side of the equation, and we can just take negative that expression and write it all out together.

So we have $m = Zm_H + (A - Z)m_n - B$ plus A surface A to the 2/3 plus AC z times z minus 1 over A to the 1/3, and plus AA symmetry and minus z squared over A and minus delta. So we've got one expression for the total mass. We've fixed the value of A because we're going to take some fixed-- we're going to choose some fixed value of A.

Let's say A equals 93. And that's the example that I've kind of worked out in my head. It so happens that niobium has a stable isotope that a mass of A equals 93. And we just found out

in some of our research that niobium doesn't stick to chromium very well. That's why I've got it on the brain. So this is what I was thinking about this morning.

So for a fixed A equals 93, we want to find what is the most stable A . How do we do that? Anyone have an idea? Yeah.

AUDIENCE: Differentiate?

MICHAEL SHORT: Differentiate. Sure. Let's take, we'll just say the derivative of-- oh, no, it is a partial derivative because we have two variables here. Take the derivative as a function of z , set it equal to 0. This will give us the z number that gives us the minimum mass for a fixed A . So let's actually do this right now. So let's see. This gives us m_h . And this term expanded out is AMN minus ZMN . So we have a minus MN .

I'm going to make one quick correction right here. I want to make sure everything's in the same units. All of these semi-empirical mass terms are in MeV. These right here are in AMU, atomic mass units. What do we have to add in order to get these all in MeV?

AUDIENCE: [INAUDIBLE] the conversion factor.

MICHAEL SHORT: Conversion factor, yeah. Or in this case, we'll just stick a C squared. I'll make a little bit of room so I can stick the C squared in there. C squared. Now everything's in MeV. We're all in the same units. So let's say we have m_h minus mn .

These are in C squared. And minus VC plus $\frac{2}{3} AS$ times A to the negative $\frac{1}{3}$ plus-- I'm going to expand this out to call it z squared minus z . I'm also going to stick in n equals A minus Z so that this expands out to A minus $2Z$ squared. Is everyone with me here? Yeah.

AUDIENCE: Shouldn't the A terms, like the AVA and the AS , stay at $\frac{2}{3}$?

MICHAEL SHORT: Oh. You're right. I'm deriving with respect to the wrong variable. Thank you. Yep. So we want to do this as a function of z . So that term disappears. That term disappears. Thank you. Let's work on these ones right now. So we have AC time z squared over A to the $\frac{1}{3}$. So that will give us A plus AC over A to the $\frac{1}{3}$ times $2z$. And then we have-- let's see-- minus AC over A to the $\frac{1}{3}$. That's it, actually. Oh, times plus 1, OK?

And then we have the A minus $2z$. So let's expand this out just so we can see it all on the board. So we have AA over A times A minus $4z$ squared plus 4 minus $4AZ$ plus $4z$ squared.

So let's take the derivative with respect to z of that. That term goes away. That becomes $4A$. So we have plus AA over A times $4A$. And there's a minus sign there. And that becomes $8z$.

So we have plus AA symmetry over A times $8z$. And the delta term goes away because there's no z dependence. And what we end up here is the solution for what is the most stable z as a function of A ? This is a linear equation. There's only one solution for it. If we actually want to graph this m as a function of A and Z , we end up with what's called a mass parabola, which is to say you can graph the binding energy per nucleon, or the mass, or pretty much similar things, of a nucleus, of all nuclei with A given as a function of z .

Think I can do this on the remaining space right here. So let's say $4A$ equals 93 if this is Z and this is m as a function of A and Z . Let's actually look at a concrete example. So let's go live to the chart of nuclides and start looking at things with a mass number of 93. Looks like I clicked a little too high. There it is. Let's see. Moly 93 I was looking at, and that becomes niobium 93, which is the stable isotope I was thinking.

So let's put niobium right here. I haven't given an actual scale to this because I just want to show you in sort of relative terms. So let's say niobium is the stable one. So it's going to have the lowest actual mass, even though it's got an A number of 93.

If you look here on the chart of nuclides, you can see it could have come from a couple of different places, either from zirconium 93 or from molybdenum 93. And now is a good time to start introducing these different modes of decay so you can figure out, well, how would a nucleus decay to get to the most stable place?

Let's say it came from zirconium 93 and-- let's see, niobium has a proton number of 41. So if we go to zirconium, it beta decays to niobium 93 with an energy of .091 MeV. Very, very close. So we'll draw it slightly higher. That's about 91 keV. Zirconium 93 could have come from the beta decay of yttrium 93 you can see right here. So let's go up the mass parabola and keep exploring. And now we see that yttrium can decay by beta decay with 3 MeV.

So if we put yttrium on this graph, it would be way higher. Yttrium itself could have come from-- well, let's see, strontium 93 with the decay energy of 4.1 MeV. So let's put strontium here. And I think 4 MEV would be, like, off the chart. But whatever. That's the way we drew it. Already we've got the makings of a parabola. And each one of these can decay by beta decay, or does decay by beta decay, in order to get to the most stable nucleus.

So let's write the nuclear reaction for beta decay of one of these, let's say, from zirconium to niobium. So we'd have ${}^{93}_{40}\text{Zr}$ spontaneously goes to ${}^{93}_{41}\text{Nb}$ plus a beta and plus an electron anti-neutrino. That's the part I don't expect you to know yet. But that's the whole energy conservation thing.

A little bit of a flash forward. The beta decay energy is not necessarily the energy of the electron that you will measure because some of that energy is taken away by the anti-neutrino. But we'll get into how those relate probably next week.

So let's now look on the other side of the parabola and confirm that the semi-empirical mass formula, which predicts something parabolic here and here with respect to Z , actually checks out. So let's back up to niobium 93 and notice that it could also have come from electron capture from molybdenum 93. So let's put molybdenum right here. And it decays with an energy of 0.4 MeV into niobium, which let's say it's around here.

Let's keep going through the chain. Anyone have any questions so far while we keep going? Cool. Let's trace it back up the chain. Technetium 93 can beget molybdenum 93 by electron capture with a much higher energy, 3.201. I'm going to extend our graph because we need the space. Technetium, another 3 MeV. And let's go one more back. Technetium can be made by electron capture from rubidium 93 with an even higher energy, which means a higher difference in mass between these two.

So so far-- let's see, what's 6 MeV for rubidium here? It would be like there, I guess. There's our mass parabola, right from the data. So I like doing this better than just showing you a diagram because you can actually try it for yourself. Pick a fixed A , change Z , and construct the mass parabolas yourself.

Now the question is how could these decay into niobium 93, which is the stable isotope? I have negative one minutes left, so I'm very quickly going to tell you for large energy changes, it can either be positron decay or electron capture. And we'll go over what these modes of decay are next week. This can be, again, positron or electron capture.

And for small amounts of decay energy, it can only be electron capture because in order for positron decay to happen, you have to be able to create the positron. And the positron plus the extra electron ejected to balance charge has got to be 1.022 MeV, or same thing as what's known as two times the rest mass of the electron.

I'm going to stop there, and we'll pick up with lots of examples and questions tomorrow. The last thing-- well, I'll go over the next problem set tomorrow. I want to make sure everyone's seen it seven days before it's due. And the best way to do that is to show it on the board.