

The following content is provided under a Creative Commons license. Your support will help MIT OpenCourseWare continue to offer high quality educational resources for free. To make a donation or to view additional materials from hundreds of MIT courses, visit MIT OpenCourseWare at [ocw.mit.edu](https://ocw.mit.edu).

**PROFESSOR:** I wanted to do a quick review of all the photon interactions, because I've released problems at 5:00 for you guys. It involves your banana data. So we're going to be taking a second look at all the banana data and the problem statement for the lab part is simple. Identify all the peaks. Tell me where they came from. And tell me all the peaks that should be there that you don't see and why. And that's like a quarter of the problem set or something.

So just to review the three effects that we talked about sort of in order of what energies they're important in, my spelling I'm sure will be slower than usual today. I spent like 3 and 1/2 days listening to Russian presentations with an English translator microphone. Russian scientific presentation is really similar to English. All the technical words are the same, but as soon as you start trying to talk to a two-year-old, you're just lost. But it's pretty cool. So we went over the photoelectric effect, Compton scattering, and pair production.

And so in addition to knowing what these three mechanisms actually are and how to tell what they would look like on a given detector spectrum, the other two important things we wanted to remember are what are the cross-sections, so what are the relative probabilities of each happening as a function of their energy the photon and the material that they're going in and then filling in this map of if you have energy and  $z$ , where are these effects most prevalent? Does anyone want to kick me off? Does anyone remember the general form of the cross-sections for any of these effects? Or does anyone remember what this map looks like? Yeah? Chris.

**STUDENT:** [INAUDIBLE]

**PROFESSOR:** Indeed.

**STUDENT:** [INAUDIBLE]

**PROFESSOR:** That's right. So pair production-- I think we gave it the symbol kappa to go along with a reading that you guys have-- was around here. That's because it's not going to happen below

1.022 MeV, because you need the energy to make the positron electron pair. And indeed, the more electrons there are in each atom, the more likely pair production is, this happens when the photon gets near the nucleus, which is going to have a higher charge for higher  $z$  and so on.

And so pair production, it was proportional to-- you'll never need to know the exact things that the cross-sections. That's what books are for. But it was proportional to-- let's see, I think it was like  $z$  to the third or fourth, pretty strong like that. What about Compton scattering? Where does that lie on this map?

**STUDENT:** [INAUDIBLE]

**PROFESSOR:** Yup, high  $z$ , low energy. So in this general region. We'll just give it a C for Compton scattering. And in that one, the cross-section was proportional to something like  $1$  over the energy or  $\hbar\omega$  in your reading. That's the same thing as saying photon energy. And then what about photoelectric effect?

Well, that's the only place there is left, right? We'll give it the symbol  $\tau$ . So I'll put these up here. I don't quite know why they chose those symbols. But I'll just stick to the notation in the reading. And then the idea here is this was proportional to something like  $z$  to the fifth over-- what is it, like energy to the like  $7/2$ . So significantly low energy, significantly high  $z$ . And does anyone remember at what energy does the photoelectric effect start to kick in?

Very close to zero. So here, the energy has got to be greater than or equal to, but what is the photoelectric effect physically?

**STUDENT:** [INAUDIBLE]

**PROFESSOR:** Yep. A gamma gets absorbed, or any photon gets absorbed that knocks out an electron. So how energetic does it have to be to knock out the electron?

**STUDENT:** Binding energy of the electrons.

**PROFESSOR:** The binding energy of the lowest bound electron, which we give that symbol  $\phi$  or the work function. The idea here is that as soon as you have enough energy to eject the outermost electron, which is super low for the alkali metals, like sodium, potassium, cesium, then you can exceed the work function and get the photoelectric effect going. And for this one, we said the energy here has to be greater than or equal to  $2$  times the rest mass of the electron  $c$

squared, better known as 1.022 MeV.

Is there a minimum energy for Compton scattering? Photons can scatter. They don't have to have any energy to scatter. Certainly. And let's see the two interesting bits of technology we talked about related to these, one was called a Compton camera, where you could actually use two detectors. Let's say you're looking for a tiny source in a big box somewhere.

You can have one detector. And you can have a second detector, so that this source is sending out gammas in all directions. And let's say one of them interacts with detector one, bounces off, and interacts in detector two. At that point, you've constrained sort of the angle between these detectors, so that you know what energy the gamma came from.

And you know generally where it came from physically, which is a cool piece of equipment. I'm going to try to find pictures of one of these actual things, because I actually haven't seen one myself. I've just heard it described physically and it seems to make sense. And the second one that we touched upon at the very end of last class had to do with this thing right here.

Does anyone remember thermionic devices? Well, the work function for some materials, like for cesium, the work function is a little less than an eV. It's like 0.7 electron volts, which means when you get things up to about 2,000 Celsius or so, the temperature of the atoms themselves exceeds the work function, and the outer electrons just boil off. So if you have two pieces of material, probably in a vacuum, and one of them is like 2,000 C, and one of them is, let's say, room temperature, you end up with this net flux of electrons boiling off the hot one to the cold one.

And this has been one of the methods proposed to directly convert heat to electricity for ultra high temperature applications, like space reactors or other things that can get super crazy hot. So it's one of those energy conversion mechanisms-- did anyone ever hear about this one in high school? Highly, highly doubt that it would ever be mentioned.

One of the professors in our department, Elias Gyftopoulos was one of the folks that came up with this whole idea. And in my senior design course, we actually designed a space reactor that uses thermionics, and he showed up in the audience by surprise. And that's probably the dumbest I've ever looked at a presentation, explaining something that someone invented. They knew every single mistake and everything that was wrong. So since then I've kind of boned up on thermionics knowledge.

But that's enough for the photon stuff. Now we want to start getting into ion nuclear interactions and in today's reading, it started off-- I think the first paragraph went something like, the formula for stopping power can be expressed as follows. Squared times log of-- squared over mean ionization energy.

And I find this explanation to be unsatisfactory. I'm not a fan of the kind of books that just say, here's a formula. For practice, plug things in and use them. So instead, I'm going to skip ahead to a little bit of next week's reading or the [INAUDIBLE] reading and actually derive it. So when I just throw up a formula like this, it's like how the hell do you remember that, right?

Well, it's going to make a lot more sense once we actually derive it. So let's set up this problem. You have a charged particle with charge  $z$  times  $e$ .  $z$ , we'll say, is the number of protons in this nucleus or the charge on an electron if you want it, times the unit charge of an electron. And it's firing at some other electron somewhere else in the material.

So the basis of any sort of ion electron interaction has to start with the ion being either struck or repelled-- I'm sorry, with the electron being struck or repelled by the ion. And so let's say that this ion exists. We'll draw kind of a unit cylinder around this physical situation. And if we draw this distance right here, it's one of those rare cases where the nomenclature is the same in pretty much every reading. There's this distance  $b$ , which we call the impact parameter.

It's kind of a funny name for it, but it just means by how close does your particle get to that electron when it undergoes this single interaction. And so this particle is moving quite fast with some speed  $v$  towards and then away from this electron. And what's going to happen is there's going to be some sort of a Coulomb force between this charged particle and this electron.

So let's say you're firing an electron at an electron. There's going to be some negative repulsion. Or if you're firing an ion at an electron, there might be some positive attraction. But at any rate, there's going to be some deflection. So let's just say it's a negatively charged particle. If we draw its actual trajectory, it's actually going to go off kind of barely in that direction, right? Two charges passing in the night. They know each other's there. And they kind of repel each other.

So what we want to do is figure out what is the total amount of force in the  $x$  and the  $y$  direction. Let's just define our axes to make sure we're all on the same page. And can we resolve that into a total amount of energy lost per unit distance? This quantity right here was

for referred to as stopping power.

Before we launch into it, does anyone know why I put a negative sign on this quantity?

**STUDENT:** This is all [INAUDIBLE].

**PROFESSOR:** Exactly. Yep. If you're changing the energy in the particle, unless it's, let's say, following it to some gravitational field, which we're not covering today or ever then, it's any sort of interaction is going to cause the particle to lose some energy. So this quantity right here is going to be negative, and so this quantity right here is going to be positive. We stick a minus sign in front of it.

But let's get back to the basics then. What is the force between this charged particle and the electron from 802? This Coulomb force.

**STUDENT:** It's a constant.

**PROFESSOR:** There is a constant. Let's just call it  $k_0$ , because the reading calls it  $k_0$ .

**STUDENT:** And then it'd be  $z e$  [INAUDIBLE].

**PROFESSOR:** Yes. Yeah. So this is like your  $q_1$  and your  $q_2$ , right? Your charge 1 and your charge 2 over?

**STUDENT:** The distance.

**PROFESSOR:** The distance squared. Let's call that the distance between the two particles. And so now we can say if this is the distance away in the  $x$  direction, then we know that  $r$  is  $\sqrt{x^2 + b^2}$ . So we can stick that in there, and we know that our Coulomb force would then be this  $k_0 z e^2$  over  $\sqrt{x^2 + b^2}$ . I'm sorry not square root, just  $x^2 + b^2$ .

So like we've done with everything so far in the class-- it's kind of dark in the back. Like we've done with everything in the class, let's split this up into  $x$  and  $y$ -forces. So if we assume that the electron basically doesn't move, what's the net amount of force in the  $x$  direction that this particle is going to feel when it goes from minus infinity, so over here, to plus infinity like over here.

**STUDENT:** Zero.

**PROFESSOR:** Zero. Why do you say that?

**STUDENT:** Because [INAUDIBLE].

**PROFESSOR:** Exactly. Whatever force it feels repelling it from here, as soon as it hits this midpoint, it gets that same amount of propulsion in the other direction. So your net force, if you integrate from minus infinity to infinity, of your  $x$  force as a function of  $t$ , that comes out to zero, which makes our life a lot easier.

All we have to worry about is the total integral of the  $y$  force to figure out how much net deflection do we get in that direction. This integral is also better known as a momentum. Anyone recognize where with this comes from? If you take the integral of a force, it's like the integral of a mass times an acceleration, which is like mass times the integral of acceleration, which is like  $mv$ , which is a momentum this is where some of our particle wave stuff is going to get funky, because we're going to start throwing in expressions for particle momentums in wave equations when we start to determine, well, if this is really an electron. There's some limitations on how we can treat it, where it kind of loses its character as a particle. So I just want to warn you that that's coming up.

So now, let's make an expression for the  $y$  force. If we were to say what is the  $y$  momentum imparted, which is an integral of the  $y$  component of the force  $dt$ , we already have the expression for the force, like you guys derived.  $k_0$  times as little  $ze$  times  $e$  over  $r$  squared, which is  $x$  squared plus  $b$  squared. And then how do we get the  $y$  component of it?

Well, we've got to define an angle. That's our angle  $\theta$ . What's the  $y$  component of that force?

**STUDENT:** [INAUDIBLE]

**PROFESSOR:** Yeah, it's just times cosine  $\theta$   $dt$ . What's the expression for cosine  $\theta$  in this physical situation?

**STUDENT:** [INAUDIBLE]

**PROFESSOR:** Close.  $b$  over  $r$ . And in this case,  $r$  is root  $x$  squared plus  $b$  squared. And the last thing we want is because we have things in terms of  $x$  and  $b$ ,  $b$ 's a constant,  $x$  is a variable,  $t$ 's kind of the wrong variable. So we can do a variable change and say this is equivalent to the velocity of the particle over-- I'm sorry-- to  $dx$  over  $v$ . We're just using this whole like velocity equals, what is it, distance times time, so our whole, what is it-- yeah.

**STUDENT:** [INAUDIBLE]

**PROFESSOR:** Thank you. Distance equals velocity times. Thinks, [INAUDIBLE]. OK, anyway. So luckily, I had the expression right and the explanation wrong. So thank you. Was that Luke or Jared's voice? Awesome. OK. So let's put this whole expression in, keep that little embarrassment behind us.

We have the integral from negative to plus infinity of  $k_0 z e^2$  times  $b$  over  $x^2 + b^2$  times the square root of itself. So let's just say  $x^2 + b^2$  to the  $3/2$ , and there's a  $v$  on the bottom  $dx$ . This is finally valuable. So we're getting closer. Let's take all the constants and shove them outside the integral.

So we have a  $k_0 z^2 e^2 v$  over  $b^2$  times the integral of just  $1$  over  $x^2 + b^2$  to the  $3/2$   $dx$ . Not remembering the formula off the top of my head, I-- yeah?

**STUDENT:** So we can treat the velocity as a constant even though it's losing energy.

**PROFESSOR:** Yes, that's-- well, we'll call it a crude derivation. But if we're assuming that the electron basically doesn't change position, that it changes so little, then we're going to assume that also the velocity basically doesn't change, that one collision for a high enough velocity doesn't lose that much energy. So that's what we're going with for now.

And we'll actually be able to compare this kind of crude derivation to one done from quantum mechanics, and they look pretty similar. There's like an extra factor of two or something. But as I showed you guys in preparing for the test, when I said 9 equals about 10, it therefore follows that 1 equals about 2, and as long as we get the constants and orders of magnitude right, we're going to gain the physical intuition for what we're looking at. I'll leave it up there.

Anyway, evaluated this integral, and it came out to something like  $2$  over  $b^2$ . So this just comes out to  $k_0 z e^2$  with a  $2$  over  $2 v b^2$ . Cancel the  $b$ 's. I don't know where that  $2$  came from. Whatever. Yeah, that's what we have for the stopping power for this sort of one particle hitting one electron.

Now, we have-- well, sorry, that's the momentum equation. But we're interested in the change in energy. So what's that equation we've used before to go from momentum to energy? Our kinetic energy  $t$ .

**STUDENT:** Square root of  $q$  [INAUDIBLE].

**PROFESSOR:** Other way around. So let's do it that way, right, so  $p$  equals  $\sqrt{2} mT$ . OK, so square both sides. Yeah, you got it. And we have our energy  $T$  is  $p$  squared over  $2m$ . So let's take this small little mess, stick it in here, and then we end up with  $4k_0$  squared little  $z$  squared  $e$  to the fourth over  $2mv$  squared  $b$ . Cool.

And so this gives us the little differential energy change from one electron collision. Yeah?

**STUDENT:** [INAUDIBLE]

**PROFESSOR:** I think we cancel one of those, right?

**STUDENT:** Yeah, but then when you square [INAUDIBLE].

**PROFESSOR:** Oh yeah, you're right. Thank you. Comes back.  $b$  squared  $b$  squared. Yep, you're right. Thank you. So now we've only accounted for the ion hitting a single electron as it moves through this hollow cylinder of whatever medium it's going through. So this is when we can kind of take things back from abstract to reality and say, all right, it's moving through some actual material.

And we have to describe its electron density in this cylindrical shell. So the electron density in the cylindrical shell depends on-- well, the number density of the material itself, just how many atoms are there times big  $Z$ , the number of protons in that nucleus and therefore the number of electrons in each nucleus, and the volume of the cylindrical shell.

So what's the expression for the volume of the cylindrical shell? In differential form? Yeah, I started hearing? I heard a  $2$ . That's correct. Keep going. Well,  $2\pi b$  gives us the circumference of the circle on the outside of the cylinder. And if we add a little  $db$  there, some differential thickness element, and we add on a little  $dx$  for some differential distance down the cylinder, we end up with  $2\pi b db dx$ , multiplied by this stuff.

And we get some differential change in energy scales like-- let's say there is a  $4$  and a  $2$  there. So we end up with  $4$ . That's not a  $4$ .  $\pi k_0$  squared, little  $z$  squared, big  $Z e$  to the fourth  $b db dx$  over  $mv$  squared  $b$  squared.

Now those other  $b$ 's cancel. We can divide everything by  $dx$ . And we've already almost got our stopping power expression. We're getting pretty close. Anyone see some similarities between the one I just threw out of my head and what we've got so far? We've almost got the makings



of it.

So now if we want to account for the fact that our charged particle is probably not shooting through the center of a perfect hollow cylinder, but we're just firing it into like actual matter, we have to account for every possible impact parameter in every possible cylindrical shell that it would be moving through. So in this case, we can integrate this.

We've already got our  $db$  right there. That's our integrating variable. And now here's where things get a little tricky. Can we actually integrate this from an impact parameter of 0? And this is not an easy question actually. What do you guys think?

**STUDENT:** [INAUDIBLE]

**PROFESSOR:** So Luke says no, why?

**STUDENT:** [INAUDIBLE]

**PROFESSOR:** We actually have an  $1/b$ . We have a  $v^2$ , but that's not our integrating variable. Yeah, so we have like a  $1/b$  looking--

**STUDENT:** [INAUDIBLE]

**PROFESSOR:** Yeah, that's fine.

**STUDENT:** [INAUDIBLE]

**PROFESSOR:** That's true. There's another more physical reason though. But you're right mathematically. Can you know precisely the location of an electron ever? Now I see a lot of people saying no. Why do you say that?

**STUDENT:** [INAUDIBLE]

**PROFESSOR:** That's right. There's this thing-- the De Broglie uncertainty principle. It's kind of the punchline of a lot of quantum mechanics jokes. You never know where something is going to be or where it's going. We used to say this about some of the older professors in this department. If you call them and say, I'm on my way, I'm getting there as fast as I can, they could be anywhere in the world.

And if they say, don't worry, I'm three miles away. You don't know how long it's going to take

them to get here. Same thing with me and getting here. Although I was on MIT standard time, which means five minutes late. Not bad.

So in this case, we have to ascribe the electron some sort of a wavelength. So in this case, we can't just treat the electron like a particle whose position we know. We're going to go with our original equation for a photon energy, which looks like  $hc$  over  $\lambda$ . Rearrange that so that we'll have some  $\lambda$  wavelength equals  $hc$  over  $E$ .

I'm sorry. This is a momentum thing, not an energy thing. And what's the momentum of the electron? From the classical definition? It's just mass times velocity, right? So we'll just stick in the mass of the electron times the velocity right there. And this wavelength right here, the De Broglie wavelength of the electron is as close as we can specify that impact parameter. And it turns out to be pretty significant, like on the order of 0.1 to 0.2 angstroms. You can't tell where an electron is going to be finer than that.

So we're going to have this  $b$  minimum. I'll just write that in there and some  $b$  maximum, where  $b$  minimum is the same as our De Broglie wavelength of the electron, because we can't define its position any better than that, which is just Planck's constant over its mass times velocity. For  $b$  max, it comes out to something like  $h\nu$  over this quantity,  $I$  bar what's called the mean ionization potential.

What this quantity physically represents is that if your impact parameter is too large, then the electron will-- or the charged particle will feel so little force, that it won't eject an electron and won't really be deflected. And the farthest away it can be corresponds to the minimum amount of energy to create an average ionization in the material.

And this mean ionization potential scales with something like this constant  $k$  times  $z$ , where  $k$  is on the order of like 30 to 35-- think it's like eV. But remarkably tight constant, so picking a mean value like that is no problem. And there we have our  $b$  min and  $b$  max. Those are our limits of integration.

I think I planned this just to fill up the boards today. So let's write out the final integral that we have and see what we get. So we have our stopping power, should be integral from  $b$  min  $h$  over  $mv$  to  $b$  max  $jv$  over  $I$  bar of  $4\pi k_0$  squared little  $z$  squared big  $Z$  to the fourth over  $mv$  squared  $b$  db.

And like I think it was Sarah that you mentioned that we'd have a log. I forget who's mentioned

it. Sorry. That was Luke, OK? You're right. So it ends up just being a natural log. It's like all this junk on the outside times the integral of 1 over b. So this just comes out to  $4\pi k_0^2 z^2 Z^4 / (mv^2 \ln(v_{\max}/b_{\min}))$ .

The h's cancel. We get a v squared. And so all this stuff inside just becomes the natural log of mv squared over mean ionization potential. And we've arrived basically at the same equation that we have over there, that I took care to memorize on the plane. So great that we've gotten here through the math.

Let's actually see what this means, and we're going to go into some of the limits of validity like Luke was saying, where you can't have a natural log of 0. So the stopping power formula isn't quite going to work at 0. Nor will it work at super low energies. So if you want to write what this should be proportional to. I kind of see some constants here that we don't really care. They don't vary at all.

But this looks kind of like a kinetic energy term, doesn't it? Like kinetic energy terms. So it's kind of proportional to this function  $1/t \ln t$ . When you get rid of all the constants and just express it in terms of the variables, it looks a whole lot simpler. And so let's see what this would look like if we started to graph it out.

And this is pretty universal for any charged particle stopping power. So if this was the kinetic energy  $t$ , and this was our stopping power, we've got this  $1/T$  term that's going to look something like this. And we have this natural log of  $T$  term, which is going to look something like this that actually goes down to infinity.

So like Luke was saying, if these two are multiplied by each other, we're not going to have negative infinity as a stopping power, which would physically mean that once the particle hit zero energy, it speeds up to infinite speed, and that doesn't make any sense. But we can start to draw what the curve would look like with this general envelope.

And so at low energies, the stopping power kind of scales like  $1/e$ . Let's now start drawing another graph with a little more physical intuition, the range of the particle. So while stopping power might be kind of a new quantity that represents the differential amount of energy lost as a function of distance-- that's kind of a mouthful-- the range is pretty simple, just how far it goes.

So to get the range from the stopping power, you can integrate-- let's say you fire a particle

into a bunch of matter at some energy  $T$ . So you start off an energy  $T$ , and you want to see how far it gets at distance  $0$ . Well, you can just integrate the stopping power as a function of  $T$ , or you can switch your limits of integration. Let's see.

I'm sorry. I'm not going to switch those limits of integration yet, which is like saying from  $0$  to  $T$  of  $dt dx dt$ , which is like saying from  $0$  to  $T$  of  $dt dx$  to the minus  $1 dx$ . Much simpler expression. And when you forget all the crazy constants, and you just take this kind of form as the variable part of the expression for stopping power, unless your energy is really high, and this natural log counts at all, your range kind of scales like the integral of just  $1$  over  $T$ .

I'm sorry, that to the minus  $1$ , which is like the integral of  $T$ , which is like  $T$  squared, which means that this are pretty interesting intuitive result, that the range of the particle increases with the square of its energy. So this gives you a good hint to say, if I increase the particle by a certain amount, I'll increase the range by the square root of that increase.

So anyway let's start drawing this range curve as a function of  $x$ . What this says right here is that if we start our particle at some high energy, and we're firing into the material, and it's losing energy as it goes, and we track this value of the stopping power to figure out how far it's going to go, change that in a second, for the first little while as this particle loses more and more energy, its stopping power stays mostly constant, and it loses a pretty constant amount of energy as a function of time.

As its energy gets lower and lower, it loses more and more as a function of position. What this actually means is that as the velocity goes down or as the particle's energy goes down, it spends more time in the vicinity of the electron and gets deflected more. It's just that kind of a simple argument. Like the more time it spends near this electron, the more it feels the push.

And so it will lose more and more energy as its energy gets lower and lower until you hit the point where this curve breaks down. Where do you guys think that is? Even mathematically speaking. Well, what happens if your natural log term is negative here? Then you get a negative stopping power, which would be like the particle picks up energy. That's not quite physical at all.

So in reality, we know that at some point it's going to taper off, and the stopping power at  $0$  should be  $0$ . This maximum right here occurs around  $500$  times the mean ionization potential, which is a pretty low energy, but what this actually says is that when the particle's moving really slow, it's moving so slow that once in a while, it will capture one of those electrons, like if

you fire in a proton or a positively charged heavy ion, if it's moving so slow that it can feel the pull, it will just partially neutralize. And that becomes the next mechanism of energy loss.

And so if we keep following this curve, once we hit some sort of a maximum, then it's going to lose less and less and less energy, do less and less damage, and you end up with the same curve, this kind of brag peak curve that we did together when we used the SRIM code. Did I go through the stopping range of ions in matter with you guys? Did I show you this on the screen?

Remember the curve of the-- let's say damage events per distance or number of implanted ions as a function of distance. You end up with the exact same thing. That's what the SR stands for in SRIM is stopping, what is it, stopping power and range or something like that. Something range of ions and matter. Is it the stopping power and range of ions and matter? I don't know, but all SRIM is a gigantic stopping power database and a big Monte Carlo engine.

So it takes an equation just like this one or-- yeah, just like this one, that one, whichever one you want and decides, well, how often is the particle going to lose how much energy depending on where it happens to be? And that's all there is to it. This point here, we would call the range or like the average range at which-- well, that's not quite right.

There'd be some average range around here where the particles actually stop, when their energy goes to zero. And in reality, because every one of these processes is random in nature, the impact parameter is going to be kind of random. Not every particle will stop at the same place, because all the electrons are moving around in the atoms. So there's going to be some sort of a range of ranges, which we call straggling, which is to say that not all of the particles end up at the exact same range, but they end up pretty close.

I think I want to pause here for a seconds and see if there's any questions from this four board derivation or any intuition questions that you guys might have. Yeah.

**STUDENT:** You didn't initially have the negative  $d^2$  over  $dx$ .

**PROFESSOR:** Oh yeah, where'd that go? Let's trace this through.

**STUDENT:** What happened to the negative?

[INTERPOSING VOICES]

**STUDENT:** But we didn't actually derive it.

**PROFESSOR:** Yeah, let's see. So the change in energy should have been a negative change in energy. So if we-- it went missing there. So there we go. That's the only other place that seems to be missing. OK, great. Cool.

Then if you want to start looking at the number of damage events that this particle will incur, we'll call this the number of ion pairs, which might look suspiciously familiar if you guys remember the Chadwick paper, he was talking about this proton of this energy should make this many ion pairs at this distance. Now you guys actually have the tools to find out what that number should be, because it's going to be  $1$  over some ion pair energy, usually around 30 to 35 eV, depending on the material times  $dt/vx$ , which is to say when the stopping power is higher, you're going to have more ion pairs produced as a function of distance.

So the real label for this y-axis here should be like ion pairs or damage or defects or anything like that that refers to the same kind of thing as damages to the material, either by ionization or even by similar nuclear processes. And so that's what results in those SRIM curves that we showed from before, where you have, let's say, a bunch of protons entering a material here at some high energy.

They don't lose very much energy when they go in, but as soon as they get to a low enough energy where they're stopping power reaches the maximum, they dump most of their energy in there. And this is the basis behind proton cancer therapy, which I mentioned to you guys in the first or second day of class.

Now that we know both exponential attenuation and stopping power, we can explain theoretically why proton therapy is a more effective treatment. So let's say this is the person that contains a tumor. Say it's right there. And you have a choice between firing in an X-ray or firing in a proton. What is the dose to this person, not just to the tumor, but through the whole person going to look like for X-rays?

Get another board. So if we look at the number of ion pairs, and let's say this is the thickness of the person, and the tumor is in this range. And if you send in your X-rays, or you send in your protons, what is the number of ion pairs produced from X-ray or from proton going to look like in either case? So first of all, who wants do the X-ray one or tell me what it will be?

You guys-- yeah, Luke?

**STUDENT:** Would it be pretty flat?

**PROFESSOR:** It'll be fairly flat, but there would be some decay to it. So let's say we defined some initial intensity, X-rays just get attenuated exponentially. So you're going to do a whole lot of damage to the person before the X-rays reach the tumor, which is why when you do X-ray therapy, you have to send in x-rays from a bunch of different locations so that the tumor gives out the most, and the rest of the person in any location doesn't get that much dose.

For proton therapy, it's quite different. It looks just like this. So you might do a little bit of damage as you go in, and you tune the energy of those protons, so that they do all the damage in the tumor, and then they stop in the tumor or just beyond it, so that they don't do any more damage to the rest of the person, and they do very little going in.

And so that's why proton therapy centers are popping up all over the world because it's a more effective treatment. It's also more expensive because you need a proton accelerator so then, here's a question for you. This is something they actually do in the lab.

Say, here's your human. There's your tumor. There is your proton gun at a fixed 250 MeV, firing protons out. How do you change the range of those protons without changing their energy?

**STUDENT:** The distance it has to travel?

**PROFESSOR:** Is what?

**STUDENT:** The distance it has to travel the other way.

**PROFESSOR:** The distance it has to travel specifically? I mean, if they travel in a vacuum, do they lose energy? No. So what can you do?

**STUDENT:** [INAUDIBLE]

**PROFESSOR:** You could deflect them and change their direction. But as we'll get into on Tuesday, if you deflect them, they're going to emit lots of X-rays in the form of Bremsstrahlung. So that's probably not what we want to do. You can put stuff-- and I can't be any more specific than that-- in between the proton beam and the patient, because if the stopping power for 250 MeV protons and 50 MeV protons basically doesn't change, then you just put things in the way.

So let's say this is the thickness of the person. You just put some tissue equivalent stuff, or what they'll call a phantom, so some tissue equivalent gel or water or some other stuff to lower the proton energy without deflecting the beam that much. So as you guys saw in the SRIM simulation, if you track the 3D positions of these protons as they enter into the material, they all go pretty straight, and then they start getting funny.

Computer can fly the ions faster than I can. But no matter what it goes through here, while the protons have high energy, they don't get deflected much, they don't lose that much energy. And you can very finely tune the amount of stuff in the way. This would be the stuff section before entering the person.

So this is why it's so useful. So let me check the time, because I haven't checked at all. The clock's broken. Oh, we have like 10 minutes. So now is a good time to stop for questions and see if you guys have any questions from the derivation or the sort of physical meaning of stopping power and matter. Yeah.

**STUDENT:** What was the  $nz$  sort of [INAUDIBLE]?

**PROFESSOR:** Yes, the  $nz$ ,  $n$  is the number density of atoms. So if you're traveling through some actual block of matter, it depends how many atoms are in the way. So if the total stopping power decreases with decreasing density, like if you're going through tungsten, but it happens to be a tungsten gas, that tungsten gas will not have nearly as much stopping power as tungsten metal, because there's just more tungsten than the weight. Or less, I'm sorry.

And the  $z$  right here is the charge per atom, to say if you're firing electrons into something, the strength of the Coulomb force that they'll feel, or let's say, the number of electrons that they can smack into is the same with the number of protons in that nucleus if we're not using ionized materials. And we're typically not firing anything into ionized materials. It's just normal neutral matter.

Does that make more sense? Cool. Yeah, Luke.

**STUDENT:** Where did that  $n$  go?

**PROFESSOR:** It should have been there. Thank you.  $\pi n$ . Should absolutely be there. Anything else? Yeah, Dan.

**STUDENT:** [INAUDIBLE]



**PROFESSOR:** OK.

**STUDENT:** [INAUDIBLE]

**PROFESSOR:** The charge per atom is big  $Z$ . The charge on the particle is little  $z$ , because both of them actually matter. So little  $z$  tells you the strength of the interaction between the particle and each electron. Big  $Z$  tells you how many electrons are there per atom. Big  $N$  tells you how many atoms are in the way. And in that way, you have a complete description of the material.

Curious that the mass of the charged particle is absent from this formula. Isn't it? Yeah. The mass doesn't matter. You will certainly change the momentum of, let's say, a charged particle less. But in the end, it's just non-contact Coulomb forces that determine the energy transfer between the electrons in there and the charged particles slowing down in the medium.

So that is a curious thing to look at, but it is intentional. For the case of this ionization or electronic stopping power, the mass does not enter into it at least in this formula. There is another version derived that's in your reading that they just kind of plop it in front of you that's got the mass somewhere in the natural long term somewhere where it really doesn't change much at all, except for really high energies.

So if you want to think about, well, what do I want you to know, I would want you to be able to go through this derivation again, so that I can know you can go from a intuitive example to an actual equation you can use, graph what that equation should look like and talk about where it really matters and where it breaks down.

Like mathematically speaking, if this natural long term is negative, you're not going to have a negative stopping power. Something else has got to occur, and what's happening here is neutralization. And that's why the stopping power curve diverges for really low energies, because sometimes electrons get captured. Any other questions on stopping power? Cool. This is a good place to stop for now.