PROFESSOR: So Tuesday, we developed the largest equation that you'll probably ever use at MIT. Thursday, we destroyed it down to a rather manageable size.

And today, we are going to solve it and actually show you guys how to work out different reactor problems all the way from simple one-group homogeneous reactors to expanding intuitively, not directly, from the mathematics to solve, or to pose and solve, two-group reactor problems like the one they did on the AP 1000 reactor where they separate the neutrons into a fast and a thermal group.

And I wanted to put up where we left off. We have some equation describing fission, n, n reactions, photo fission, absorption, and leakage or diffusion as the big balance equation for how many neutrons are there in some reactor.

And I think last time, I had some example reactors up on the board ranging from an infinite slab with a thickness a, to a cylinder with-- we'll call it a thickness a and a height z.

And the question that we want to be able to answer is, if we draw a graph of flux versus x through this reactor, and in this case, it would be r and z, what do these functions look like? What is the form of this flux?

And so today, from this equation, we're actually going to solve it. So bear with me. There'll be maybe 20 minutes of remaining derivation, and then I'm going to teach you how to use it. So today's class is going to be more like a recitation of how do you actually use this equation instead of getting to it.

First of all, I want to simplify things, which is going to make it-- de-escalate. If we simplify things and don't worry about these NIN reactions and photo fission, we have the equation that you'll actually see in your reading. I'm going to drop them for now-- and I realize I'm missing one little gamma-- because they're just extra terms.

They were instructive in writing the neutron transport equation because all the terms looked similar, but now they're just kind of extra things for us to write. And the last thing that isn't in units of flux is this Laplacian operator, for those who don't remember.

And this Laplacian operator takes different forms in different dimensions and different coordinate systems. For 1d Cartesian, it's pretty easy. It's just double derivative, let's say in x.

For cylindrical, it's significantly uglier, plus d squared over d r squared. So this is what the Laplacian operator looks like in the case of a finite cylindrical reactor.

So first we're going to focus on the infinite slab case right here because it's a lot easier to solve, analytically. And then we'll show you how it would go for the cylindrical reactor which looks a lot more like the reactors you'll see everywhere else in the world.

So let's start doing a little bit of rearrangement and isolate that Laplacian, so we can then subtract d del squared phi from each side. OK. And that cancels those out.

And then we've got something on the left hand, is in del squared phi, and on the right side has all in units of flux. So then we can divide everything by the flux. And if we then cancel out all the flux terms, all two of them, we're left with something awfully simple.

And the last thing we can do is divide by d on both sides. And I'm sorry. I had to cancel that flux because that's the way that goes. The d's cancel, and I'll redraw what we have right here. So we have minus del squared flux over flux equals constants.

And what are they actually? It'd be 1 over k times new sigma fission minus sigma absorption over d. Everyone remember why we're putting these bars on the cross sections in d and everything? Can someone just tell me?

AUDIENCE: Averaged?

PROFESSOR: That's right. We've averaged overall energy, so some average cross-section would be the average from our minimum to our maximum energy of that cross-section as a function of energy times flux, over the range itself.

So by this, we're saying, we're averaging the cross-section somehow over the whole energy range. Now, I ask you guys, what sort of functions in Cartesian space happen to have its double derivative over itself equal to constant?

AUDIENCE: Exponentials.

PROFESSOR: Is what?

AUDIENCE: Exponentials.

PROFESSOR: Exponentials, or sine and cosine. You've basically said the same thing two different ways, yeah, because the plus and minus exponentials can be rewritten in sines and cosines.

So if we now assume that our flux solution has to take the form of, let's call it a cosine bx plus c cosine, what's the next constant? Fx. I'm sorry, that's a sine.

I don't want to use d again for obvious reasons, and e stands for energy. So that's the next letter that we haven't really used. The flux has got to take this shape or else this condition is not satisfied. So this is the easy way of solving differential equations, which is guess the solution from previous knowledge or experience or something.

So based on this, if we were to draw a flux profile, let's say, right here was x equals 0, one of those terms has got to go away by reasons of symmetry. What do you think it'll be? Which of those halves right there is symmetric about the x-axis right here? Sine or the cosine?

AUDIENCE: Cosine.

PROFESSOR: Cosine. Yep.

The sines, it can be inverted around your coordinate system, but it's not symmetric. It's not a mirror image. So actually, the sine term goes away and we've got to have some solution which looks like a times cosine b of x because right here our d flux dx should equal 0 at x equals 0.

Now, I've intentionally drawn the flux not to go to zero at the very edge of the reactor. If the flux automatically went to zero at the edge of the reactor, there'd be no need to shield it, right? So there are some neutrons streaming out of this reactor. And this distance right here is actually equal to two times the diffusion constant.

Let me get rid of the stuff we don't need anymore. This is what we call the extrapolation length. And now I should also mention, I'm not going to derive it because I think we've done enough deriving for one week, but I'll just give you the form.

This diffusion constant can actually be expressed in terms of other cross sections where this mu nought is what's known as the average scattering angle cosine. And it approximately equals 2/3 times the average atomic mass of whatever it's scattering off of.

So this d that I've just introduced from some physical analogy actually has an expression from cross sections and the material properties that you can look up. So we've now turned it into some sort of condition where everything can be looked up in the Janis library, or whatever cross section library you have, plus whatever number densities you actually have in your reactor.

So the picture's starting to get very real and very physical. So now, if we assume that phi takes the form of a cosine bx, let's plug it in. So if we rewrite this expression-- I think I'll need another color for a substitute-- so we'll take a cosine bx and stick it in there and stick it in there.

And we'll end up with, minus del squared cosine is going to look like a times b squared cosine bx over a cosine bx equals those constants. It's starting to get very simple. Keep the bars on there because they're quite important.

And if we cancel things out, the cosine bx's go away, the a's go away, and we're left with whatever is inside that cosine. B squared equals a bunch of material properties.

There's no information in here about the geometry of our reactor. There's only the material properties, whereas over here, this constant b, I'm going to add a little g to it which stands for geometry. And we've now set up a condition where if you know the geometry and the materials in the reactor, you can solve for the final unknown which is its k effective.

How critical or not is this reactor? So what would b have to be to make this cosine valid? So what would bg equal if we have the form of this flux like so? I'll give you a hint.

If the cosine goes to zero right here at some reactor length, a over 2, plus some extrapolation length, 2d, then how do we make the cosine equal to zero at this point? What does bg have to equal?

AUDIENCE: Pi halves over 2d?

PROFESSOR: Quite close. Bg would have to equal pi over a over 2 plus 2d, such that when you substitute-- or over 2, right, yeah. Oh, yeah, OK-- so that when you substitute x equals a over 2 plus 2d into there, this cosine evaluates out to zero. Does that makes sense? Cool.

So now, we have bg. There's nothing here but geometry plus a little bit of extrapolation length right here. Yeah?

AUDIENCE: Shouldn't there be an over 2?

PROFESSOR: Oh, yeah. There should be an over 2, so that when you plug in x equals a over 2 plus 2d, you get pi over 2, and cosine of pi over 2 is 0. Yep. Cool.

So now we have our bg. So we substitute that back in there. Let's just continue to call it bg since we have it over there. And we have this much simpler expression, bg squared equals 1 over k mu sigma fission plus sigma absorption over d.

Then, where's our rearranging color? We can multiply everything by d. And let's see. That should have been a minus. Copied myself over wrong. The d's go away. You can then add sigma absorption to each side. And then those go away. And then, you can multiply everything by k, and those go away.

And finally, divide everything by sigma a plus d bg squared. And what we're left with is our criticality condition. Our k, our criticality, is pretty simple, nu sigma fission over absorption plus leakage.

So finally, after all that derivation, we've arrived at some intuitive result. Remember we said that k, the criticality, or the k effective, is the ratio of gains to losses of neutrons. So that's exactly what we have.

The only gain mechanism right here is by fission, and the loss mechanisms are either absorption by the stuff in the reactor or leakage outside of the reactor. And so this is actually how you tell when the reactor's in perfect balance, is if this condition is satisfied, and if it equals 1, then the reactor is critical.

So now, we can start to play with this. And let's say we started off with a critical reactor and all of a sudden, we were to boost the absorption. What should happen to k effective? If you start observing more, right, it should go sub-critical.

Now, mathematically speaking, that means this denominator gets larger. So this ratio gets smaller. And therefore, k effective has to go down, as well. Now that's an easy case.

Let's start exploring some of the more interesting ones. Let's say you raise the temperature in the reactor. Do we necessarily know what's going to happen next? Let's work it out.

So most cross sections, when you raise the temperature, will actually go down in value due to a process called Doppler broadening that you'll learn about in 22.05. But suffice to say for now that cross sections tend to go down with temperature.

The most important reason why is because a cross section is a number density times a microscopic cross section, and if the temperature goes up, then the density goes down, and the number density goes down.

And if the number density goes down, the macroscopic cross-section goes down. The atoms just spread out from each other. So regardless of what happens at the microscopic cross-section, which I'll leave to Ben and Cord to teach you next year, we know that the macroscopic cross-section goes down because it gets less dense.

So let's try and work out now, what would happen to k effective? So what will happen to nu if we raise the temperature? Nothing, let's hope. What happens to sigma fission? This goes down a bit. But what about sigma absorption? Sigma absorption is going to go down.

Does bg change? Has the geometry of the reactor changed? Probably not. Might have thermally expanded by a few nanometers, but let's just say, it doesn't change at all.

What about the diffusion constant? Let's work that out. Sigma total is going to go down. Sigma scattering is going to go down. So probably what's going to happen is, this diffusion constant is going to go up, which means that if the atoms spread out more, neutrons will move farther, on average.

Hopefully, that makes intuitive sense, because if the cross-sections go down, then a neutron can move farther before an average interaction.

So the diffusion constant is probably going to go up. Which way does k effective go? You're correct. No one said anything because you can't really say anything.

So it depends on the relative amounts that these increase or decrease. So depending on what you choose for your materials, you can have what's called positive or negative temperature feedback, which means in some conditions or scenarios, what you want to happen is that if the temperature goes up, k effective should go down, but not necessarily so.

Depending on what you use, you can actually have situations where raising the temperature raises k effective, and that is some seriously bad news and is actually outlawed. You can't design a reactor with positive temperature coefficients.

So this is the first little taste of reactor feedback is, now that we've written this criticality condition, we can start to explore what happens when you start probing the reactor.

So let's say, what happens if you just add more reactor? In this case-- where's my green? All the way over there-- without changing the materials, what happens when you make the reactor bigger? What increases, decreases or stays the same?

Let's just work it through. Does nu change? Sigma fission? No. Sigma absorption? D? How does bg change? Bg decreases, and as you'd expect, if you add more reactor to your reactor, the k effective should increase.

And so this, hopefully, is starting to follow some intuitive pattern. With a given criticality condition, in some situations, you can work out, will the reactor gain or lose power? Speaking of, where's the power? Where'd it go? Yeah?

AUDIENCE: The kinetic energy of neutrons?

PROFESSOR: So yes, the power comes from the kinetic energy of neutrons, but where did the power go in our expression? Yeah?

AUDIENCE: Power's not dependent on criticality.

PROFESSOR: That's right. That's exactly right. And it follows directly from the math. This a got canceled away. It doesn't matter. You can actually have what's called a zero power reactor. So the power of the reactor and its criticality are not necessarily linked.

You can have a reactor that is critical while producing tons of power. You can also have a reactor that is critical while producing-- I won't say zero, but an infinitesimally small amount of power. And they actually have built these.

They're great test systems for testing our knowledge of neutron physics because you've got a reactor that's producing maybe 10 watts of power. It's easy to cool by blowing a fan on it, let's say. But you can still measure the neutron flux in different places and test how well your codes are working with a much safer configuration than sticking probes into a gigawatt commercial reactor. Yep?

AUDIENCE: So [INAUDIBLE] steady state reactor, how are you [INAUDIBLE] if it's not really at the steady state?

PROFESSOR: That would push the reactor out of steady state. Indeed, so on Tuesday, we're going to start covering transience, and if k effective become something other than one, the reactor is no longer in steady state.

It's not in equilibrium because the gains and the losses are not equal to each other. And at that point, the power will start to change, what you guys all saw when you manipulated the reactor power.

So since you brought it up, does anybody remember, if we draw as a function of time, let's say the reactor power was cruising along, and right at the time is now, you withdrew a control rod. What happened when you guys did that? Anyone, because you all did it.

It went up. OK. And then what? When you stopped withdrawing the control rod, did it level out? So everyone, tell me what happened.

AUDIENCE: It slowed down.

PROFESSOR: It slowed down the increase, but it didn't stop going up. Kind of freaky.

So this is why I had you guys do that power ramp because just controlling a reactor is not as simple as, remove the control rod, you remove a certain amount of reactivity because there are time-dependent effects due to delayed neutrons, neutrons that aren't immediately released after fission that can have a large effect on how you control your reactor.

And then if you wanted to decrease it again, let's say you put the control rod back into its original position, the power would not come back to its original position. But then, eventually, it would start to coast down and probably go beneath its original position at which point you have to constantly be controlling those control rods to keep it in what I'll call dynamic equilibrium.

You never really hit static equilibrium unless it's off. As I went to a seminar a couple weeks ago and said, I don't study biological organisms in static equilibrium because that's better known as a dead organism. They're not very interesting. But dynamic equilibrium sure is, for them and for us.

So with this process of getting the single-group balance equations, I'd like to generalize this to the two-group balance equations. And this is something you can actually use.

In every case, we're going to say, let's put our gains on the left and put our losses on the right, if we want to have this reactor in equilibrium. And now we'll separate our equations into the fast and thermal regions of neutron energy.

So we'll call those f, and we'll call the thermal ones th. So using this model of our neutron diffusion equation, what are the gains of neutrons into the fast spectrum?

AUDIENCE: Straight from fission.

PROFESSOR: Yeah, straight from fission. So how do we write that? This process that we're going through now, this is where recitation really begins because this is how I want to show you guys how to approach a problem, let's say, a one-sentence statement like, give me the flux anywhere in a two-group reactor.

This is how we go about it. So how do you equationally put the neutron gains from fission? What terms do we have up there right now?

AUDIENCE: Your neutron multiplication factor and your cross section.

PROFESSOR: Yep. Yep. You'll have your nu, your neutron multiplication factor. And now, we're actually going to split every cross section into its fast and thermal energy ranges because now we're actually splitting that energy, like we did when I drew that crazy cross section.

Let's see, we had log of e versus log of sigma, and they all follow roughly that formula. And we split it and said, if we want to draw an average cross section, it would look something like this. And that would be our sigma thermal and this would be our sigma fast.

So that's what we're doing here. So now it gets a little more complicated because both fast and thermal neutrons can contribute to fission. So how do we write this in terms of equations?

AUDIENCE: [INAUDIBLE]

PROFESSOR: We only want the neutrons that are born into the fast region, the fast gains. That doesn't mean you don't have to consider where are the thermal neutrons, because it's mostly those thermal neutrons that, when they get absorbed and make fission, create fast neutrons.

So what we'd really need is sigma fission fast times our fast flux because we're going to split every variable into its fast and thermal parts, plus-- let's put a parentheses there-- sigma fission thermal phi thermal.

So do you guys see what I've done here? We're assuming that every neutron is born in the fast group, where we're cutting this off at around 1 ev. And we are assuming that no neutrons are born below 1 ev, which is a very good assumption.

So in this case, both the fast and the thermal fluxes contribute to creating fast neutrons. Is there any other source of fast neutrons? Good, because I don't know of one either.

OK what about losses? By what mechanisms can neutrons leave the fast group? Yeah?

AUDIENCE: Aren't they absorbed?

PROFESSOR: Yeah. They can be absorbed. So how do I write that?

AUDIENCE: Sigma af--

PROFESSOR: af--

AUDIENCE: --times the flux fast.

PROFESSOR: Times the fast flux. So only neutrons in the fast flux group will leave the fast flux group by absorption. And what's the other mechanism that we had in our neutron diffusion equation?

AUDIENCE: Scattering.

PROFESSOR: Yeah, actually, so that's not in the diffusion equation, but you are right. That's the missing piece that is going to be the hard part, so. Let's add that in now.

So there's going to be some scattering from the fast to the thermal group, times our fast flux. So not every scattering event will cause the neutron to leave the fast group, but some of them will.

So we have to figure out, what is the proportion of those neutrons that will scatter from the fast group to the thermal group? For the case of hydrogen, it's pretty easy because the probability of a neutron landing anywhere from zero to e, starting off at energy ei, if we had our scattering kernel, is a constant. So that's not too hard.

And then last, what other way can we lose neutrons from the fast group?

AUDIENCE: Leakage.

PROFESSOR: Yep. Leakage. They can leave the reactor, and we can write that as a d fast bg squared flux. Make sure everything has bars that needs them. OK. Now, using the same sort of logic, let's-- Yeah, Luke?

AUDIENCE: What's the bg? How is that different from [INAUDIBLE]?

PROFESSOR: It's not. It's the same. It is the same bg that describes the geometry of the reactor.

AUDIENCE: I guess what's the subscript b of g?

PROFESSOR: G means geometry. Yep. And you had a question?

AUDIENCE: Yeah, just in the last flux, [INAUDIBLE] fast flux.

PROFESSOR: Yes, thank you. That is a fast flux. Yep. But it's important to note that this flux right here is not a fast flux. We'll get back to that soon.

Now, using the same sort of logic, let's write the gains and losses in the thermal group. So what is the only source of neutrons into the thermal energy group? I want to hear from someone who hasn't said anything yet. So Jared, what would you say? Either Jared because I haven't heard from either of you.

AUDIENCE: [INAUDIBLE]

PROFESSOR: You did. OK, then you. I'm sorry. All right. Yeah, you said the no power thing. Thank you.

AUDIENCE: So could you, like if something is absorbed in the fast spectrum, jump down to the thermal?

PROFESSOR: Close. I want to replace one word in what you said. If something is blank in the fast spectrum, it goes down to the thermal spectrum.

AUDIENCE: Scattered.

PROFESSOR: Yes, scattered. Every neutron that leaves the fast group by scattering enters the thermal group also by scattering. And in this case, we want to have the fast flux appear here because the number of neutrons entering the thermal group depends on how many scatter out of the fast group. Yeah, Luke?

AUDIENCE: Would you ever scatter up into the fast group?

PROFESSOR: You'll see. Yes. Yeah, great. I just gave something away. Yes. You can, but no, you usually don't. So we would consider that once neutrons enter the thermal group, they're at thermal equilibrium with the stuff around them, and up scattering is rarely a possibility.

You'll see. Yeah, quite soon actually. Don't worry. Not like quiz you'll see, but you'll see. Yeah. I've already got some stuff planned out. It's going to be a part of the homework question.

So now what loss mechanisms do we have?

AUDIENCE: Leakage.

PROFESSOR: Yeah, leakage. So we're going to have some separate thermal diffusion coefficient because that diffusion coefficient depends on the cross sections which depends on the groups you're in, times the same geometry, times phi thermal.

And what's the only other mechanism of loss?

AUDIENCE: Absorption.

PROFESSOR: Absorption. We've got to clear a. Why is there no scattering from the thermal group?

AUDIENCE: Didn't you say it was very rare to have it scatter up to the fast group?

PROFESSOR: I'd say even simpler. Once you're at the bottom, there's no more lower you can go. So in neutronics, when you hit the bottom, you don't say, throw me a shovel. You say, you're at the final energy group.

So now, what we'd like to be able to do is, last thing we want to stick in is our k effective, our criticality, because in reality, this is kind of what we want to know in terms of the geometry and the materials in the reactor.

So if we know what we make it out of and how big to make it, we should be able to get those in balance such that k effective equals 1. So the only really unknown here besides the flux unknowns is k.

And the reason I don't care about the flux unknowns is, they're going to go away soon. Yeah?

AUDIENCE: Does the thermal also have the [INAUDIBLE] over k?

PROFESSOR: Absolutely, because the k effective is on the bottom of the total original sources of neutrons. Just like, let's see. That was one group. Yeah. So I'd say right now, this accounts for the production of all neutrons, and everything else down the chain is losses. Yeah, Monica?

AUDIENCE: Do we assume that all neutrons [INAUDIBLE]?

PROFESSOR: We know, experimentally, that they tend to be born between 1 and 10 mev, but since you asked, let's escalate the problem. And then we will de-escalate very quickly, just to say, let's do a thought experiment, right? Let's say some of the neutrons were born thermal.

What would we have to add to this expression? There's one variable missing that's not anywhere on these boards, but was there on Thursday and Tuesday.

AUDIENCE: Spectrum?

PROFESSOR: That's right. Chi, the birth spectrum. So if some neutrons are born thermal, then we would have to add a Chi fast here, and we would have to add a Chi thermal to say, this is the proportion of neutrons born fast or thermal, times nu times sigma fission fast, phi fast plus sigma fission thermal, phi thermal.

And I'm not writing nice because I'm just going to erase it in a second, but to go with your thought experiment, this is what it would look like if some of the neutrons were born thermal. Perfectly fine thing to model. Doesn't happen much in real life, but great exam question for next year. Thank you.

AUDIENCE: Next year.

PROFESSOR: Next year. I'm not going to give you your own exam question. That's just too easy for you. So for now, let's forget about that stuff and stick with the most realistic situation.

Ah, running out of room already. OK. That's for next week. So let's forget about that. What do we do next? We have two equations and three unknowns. Interesting. Or do we really?

Well, for one thing, if we can get that top equation all in terms of one of the fluxes, either fast or thermal, then every term is in terms of a flux and they can all be divided out.

So let's take one of these equations and substitute in so that we get everything in terms of only one flux. So let's say, the top one, which has got the k in it, has one instance of phi thermal.

So let's isolate phi thermal in terms of everything else. So we have that thermal equation right there. So we have sigma scattering from fast to thermal times fast flux equals two things times phi thermal, which is d thermal bg squared plus sigma absorption thermal.

We're actually not that far away. So all we do is, we divide each side-- where's my simplifying color-- substitute. That's not it. Rearrange.

Divide everything by this stuff, and those cancel out. And we're left with an expression for phi thermal which we can now plug into that top equation. So we're like one step away from the final answer. There, everything's still visible.

And so now we end up with 1 over k times nu sigma fission fast, fast, plus sigma fission thermal times this expression, sigma scattering fast to thermal, phi flux over d thermal.

I don't usually spend this much of the class with my back to you, but this is pretty mathematically intense, so I apologize for that. And equals sigma absorption fast, fast flux, plus d fast bg squared fast flux.

That's not a bar. That has one. That has one. That does. That's good. OK. Now every single term here is in terms of fast flux. So we can just cancel them from every single term here.

And now we're left with an expression for k effective that's just in terms of material properties and geometry for the two-group problem. We're only one step away.

So if we multiply everything by k and divide everything by this stuff, we'll just have a sigma absorption plus d fast bg squared. That would equal k. And just like that is the criticality condition for a two-energy group homogeneous reactor of any geometry.

All that matters to define the geometry is, what's this bg squared? So this case works for an infinite slab reactor. It works for an actual right cylindrical reactor. You just have to sell for or look up the correct buckling term, this bg squared, which I'll tell you now, we refer to as buckling or geometric buckling, and you've got the solution to this.

Let's just check to see what we actually have here. We have nu material property, material property. All of those are material properties except for the bg's. So this tells you how to design a reactor, physically, and in terms of which materials to make sure that it's critical.

And if we look at what this looks like here, again, it's a ratio of gains to losses because eventually, the losses right here, these are the losses from the fast group. These are the losses from the thermal group. These are the gains in the fast group, noting that some of the neutrons born in the fast group scatter out of the thermal group, but don't leave the reactor.

So again, it turns out to a gains over losses ratio. And there you have it. So I want to stop at 10 of-- Yeah.

AUDIENCE: Did we drop the scattering term from the fast equation?

PROFESSOR: It should be-- did we? Yeah. Let's stick it in right here. So we'll just also stick it here. There. And that flux goes away because it was in terms of everything. Yeah. There we go. Thank you. OK.

But again, this represents losses on the bottom, gains on the top, just like any other k effective. So I wanted to stop here at 10 of, 5 of, and answer any and all questions you guys have about going from the neutron transport equation all the way to something that you could solve, and then start to play around with to say, what happens if I switch isotopes?

What happens if I raise the temperature? What happens if a chunk falls off of the reactor and it gets smaller? Yeah.

AUDIENCE: We got the equation for [INAUDIBLE].

PROFESSOR: Yes.

AUDIENCE: [INAUDIBLE]?

PROFESSOR: Somewhat. We can assume that for considerably long enough times, and to a neutron, a long time could be like seconds, that the time and the spatial form of the flux are separable which is something that we'll talk about on Tuesday.

But, if you remember, one of the major assumptions we made in the neutron transport equation was steady state. We got rid of any transient effects. We'll bring them back, now that we have a way simpler case, on Tuesday. Yeah, Luke?

AUDIENCE: [INAUDIBLE] step, the plus scattering--

PROFESSOR: From fast to thermal.

AUDIENCE: Is that also supplied by the sigma [INAUDIBLE]?

PROFESSOR: Where is that going?

AUDIENCE: It must be in the denominator, right, because it was over on the right side?

PROFESSOR: Let's see. Oh, yeah, we divided by all the stuff on the right side, didn't we? OK. So that shouldn't be there. But it should be there because we divided by everything on the right side.

Let's just check that really carefully. So it should have been-- no, that's the thermal one. So we're not worrying about that. Yep. So it would just end up here. Yeah. Good point. Cool.

Let's talk a little bit about what I'd want you guys to be able to do with this. So what would I want you to be able to do on the homework and on an exam? With the neutron transport equation, recite it from memory. Well, not really.

But if I were to give you the neutron transport equation, I'd maybe want you to explain what some of the terms mean, or tell me how you would get the data, or explain one of the simplification steps and justify why you think it's OK because we actually wrote out the justification for every step on the board.

Or explain, for example, what's the physical reason that we can solve the neutron transport equation with this diffusion approximation? And in which regions does that approximation break down?

So can anyone tell me, from yesterday, where is the diffusion equation a bad approximation of the flux? Yep.

AUDIENCE: Near the control rods or the fuel.

PROFESSOR: Near the control rods or the fuel, or anywhere else where cross sections change all of a sudden because diffusion describes long distance steady state solutions across places, and where things change drastically, diffusion breaks down.

Because we assumed here that the neutrons behaved like an ideal gas or some chemical species with no neutron to neutron interactions, because the mean free path length for those interactions is like, what did we say, 10 to the 8th centimeters, so a megameter? Yeah. I love using those sorts of terminologies.

1,000 kilometers before a neutron would hit another neutron. Or I might ask you to, let's say, reduce the neutron diffusion equation and come up with a simple criticality condition. Or let's say, if you were to make a physical change to the reactor, tell me if you think it would go more or less critical, and what would happen next?

Or I could give you a different physical situation, like the up scattering scenario, which I will, and ask you to pose and maybe solve these equations, or at least get forms of the criticality condition.

I'm not going to ask you to get tons of flux equations because that's all 22.05 is about, is doing this sort of neutron physics. But I want to make sure that you walk into that class prepared.

Plus we've been kind of heavy on the-- you know, this class, the name of it is Intro to Nuclear Engineering and Ionizing Radiation. And so far, we've been pretty heavy on the ionizing radiation and physics.

So this is where the engineering comes in. Assuming you have some material properties, you can now pick them to create a reactor in perfect equilibrium. Yeah, Kristin? No?

So did anyone else have any questions about the material or about what I might ask you to do with it? Yeah.

AUDIENCE: You said this equation would hold for any geometry just based on [INAUDIBLE] neutrons.

PROFESSOR: Yep. So all that you would do differently is, right here, when we had that Laplacian operator, we took the one-dimensional case of an infinite reactor in finite and one dimension, which meant the Laplacian operator is just double derivative of x.

But you could pose the equation in cylindrical coordinates and say, well, let's say now you had an infinite cylinder reactor, you wouldn't necessarily have sines and cosines that would satisfy this relation. Anyone happen to know what you'd have?

The sines and cosines of the cylindrical world, called Bessel functions. So these are the sorts of, in cylindrical geometry equations, that behave similar to sines and cosines with kind of regular routes and that you can describe in a similar way.

But I'm not going to get you guys into that. I'll just say, OK, there exists solutions that you can look up in the cylindrical case. And I would not make you derive them by hand because, what's the point?

Again I'm not here to drill your-- can you do the same math over and over again? I want to make sure that you can intuitively understand, what's a k effective? In a sentence, it's gains over losses.

What happens when you push that out of equilibrium? Or what physical situations could push that out of equilibrium? So any other questions for you guys? Yeah.

AUDIENCE: Just curious on the cylindrical graph we have there, what would the graph in flux look like?

PROFESSOR: It would look pretty similar. In r, it would kind of come down like that. It would always be symmetric about the center for symmetry arguments, and in z, it would look kind of like that.

And so actually in the end, the form of flux in r and z comes out as the first Bessel function. Let's say, that's a times the-- what would you call it -- times cosine of this distance is z, so pi is z over-- I'm going to have to add a subscript.

And there'd be some constant a in there. So what you can assume for multi-dimensional reactors is that the dimensions are separable. So the r part is solved separately from the z part. And that comes right from the Laplacian operator right here.

If you assume that some flux in r and z can be written like the r part as a function of r times the z part as a function of z, then the solution gets a lot easier to deal with. But this is not something I'd ask you to do in any coordinates but Cartesian because those are more intuitive, and you'll get plenty of the other stuff later.

AUDIENCE: What's the name of those functions again?

PROFESSOR: They're called Bessel functions. So if you want to look up, there's a little bit about them in the reading, but it's one of the more advanced topics that I'm not going to have you guys responsible for.

Much rather it be you'd be able to tell me what happens if you compress the reactor or raise its temperature, or pull out a control rod, or raise the pumping speed and cool down the water, or something like that.

So there'll be plenty of those kinds of questions on the homework to help reinforce your intuition, as well as some of the noodle scratches will be developing a criticality reaction or equation for a more complex system than the one I've just shown you here, but using the same methodology and the same ideas.