PROFESSOR: OK, guys. Welcome back. As you can see, we're not using the screen today. This is going to be one of those fill-the-board lectures. But I am going to work you through every single step. We're going to go through the Q equation and derive its most general form together, which, for the rest of this class, we'll be using simplified or reduced forms to explain a lot of the ion or electron-nuclear interactions as well as things like neutron scattering and all sorts of other stuff.

We'll do one example. For any of you that have looked at neutrons slowing down before, how much energy can a neutron lose when it hits something? We'll be answering that question today in a generally mathematical form. And then a few lectures later, we'll be going over some of the more intuitive aspects to help explain it for everybody.

So I'm going to show you the same situation that we've been describing sort of intuitively so far, but we're going to hit it mathematically today. Let's say there's a small nucleus, 1, that's firing at a large nucleus, 2, and afterwards, a different small nucleus, 3, and a different large nucleus, 4, come flying out. And so we're going to keep this as general as possible.

So let's say if we draw angles from their original paths, particle 3 went off at angle theta and particle 4 went off at angle phi. So hopefully those are differentiable enough. And if we were to write the overall Q equation showing the balance between mass and energy here, we would simply have the mass 1 c squared plus kinetic energy of 1. So in this case, we're just saying that the mass and the kinetic energy of all particles on the left side and the right side has to be conserved. So let's add mass 2 c squared plus T2 has to equal mass 3 c squared plus T3 plus mass 4 c squared plus T4, where, just for symbols, M refers to a mass, T refers to a kinetic energy.

And so this conservation of total mass or total energy has got to be conserved. And we'll use it again. Because, again, we can describe the Q, or the energy consumed or released by the reaction, as either the change in masses or the change in energies.

So in this case, we can write that Q-- let's just group all of the c squareds together for easier writing. If we take the initial masses minus the final masses, then we get a picture of how much mass was converted to energy, therefore, how much energy is available for the reaction, or Q, to turn it into kinetic energy. So in this case, we can put the kinetic energy of the final products minus the kinetic energies-- I'm going to keep with 1-- of the initial products.

And so we'll use this a little later on. One simplification that we'll make now is we'll assume that if we're firing particles at anything, that anything starts off at rest. So we can start by saying there's no T2. That's just a simplification that we'll make right now.

And so then the question is, what quantities of this situation are we likely to know, which ones are we not likely to know, and which ones are left to relate together? So let's just go through one by one. Would we typically know the mass of the initial particle coming in?

We probably know what we're shooting at stuff, right? So we'd know M1. What about T1, the initial kinetic energy?

Sure. Let's say we have a reactor whose energy we know, or an accelerator, or something that we're controlling the energy, like in problem set one. We'd probably know that. We'd probably know what things we're firing at. And we would probably know what the masses of the final products are, because you guys have been doing nuclear reaction analysis and calculating binding energies and everything for the last couple of weeks.

But we might not know the kinetic energies of what's coming out. Let's say we didn't actually even know the masses yet. We'd have to figure out a way to get both the kinetic energies. And what about these angles here? This is the new variable that we're introducing, is the kinetic energy of particles 3 and 4 is going to depend on what angles they fire off at.

Let me give you a limiting case. Let's say theta was 0. What would that mean, physically? What would be happening to particles 1, 2, 3, and 4 if theta and phi were 0, if they kept on moving in the exact same path? Yeah?

AUDIENCE: Is it a fusion event, or [INAUDIBLE]

PROFESSOR: We don't know. Well, let's see. Yeah. If it was a fusion event-- let's say there was one here and one standing still-- then the whole center of mass of the system would have to move that way. So one example could be a fusion event.

A second example could be absolutely nothing. It's perfectly valid to say if, let's say, particle 1 scatters off particle 2 at an angle of 0 degrees, that's what's known as forward scattering, which is to say that theta equals 0.

So this is another quantity that we might not know. We might not know what theta and phi are. And the problem here is we've got, like, three or four unknowns and only one equation to relate them. So what other-- yeah? Question?

AUDIENCE: For forward scattering, when you say theta equals 0, do you mean they just sort of move together forward, kind of like an inelastic collision, and they just keep moving in the same direction?

PROFESSOR: An inelastic collision would be one. And since we haven't gone through what inelastic means, that would mean some sort of collision where-- let's see. How would I explain this? I'd say an inelastic collision would be like if particles 1 and 2 were to fuse, like a capture event, for example, or a capture and then a re-emission, let's say, of a neutron. Yeah. If it was re-emitted in the forward direction, then that could be an inelastic scattering event--

AUDIENCE: Oh, OK.

PROFESSOR: --but still in the same direction. Or an elastic scatter at an angle of theta equals 0 could be like there wasn't any scattering at all. Because really in the end, can matter-- let's say if you have a neutron firing at a nucleus, depends on what angle it bounces off of, in the billiard ball sense. If it bounces off at an angle of 0, that means it missed. We would consider that theta equals 0.

But the point here is that we now have more quantities unknown than we have equations to define them. So how else can we start relating some of these quantities? What else can we conserve, since we've already got mass and energy? What's that third quantity I always yell out?

AUDIENCE: Momentum.

PROFESSOR: Momentum. Right. So let's start writing some of the momentum conservation equations so we can try and nail these things down. So I'm going to write each step one at a time. We'll start by conserving momentum. That's what we'll do right here. And we can write the x and the y equations separately. So what's the momentum of particle 1? How do we express that?

AUDIENCE: Mass times velocity.

PROFESSOR: Yep. So it would be like M1 V1. So we'll have a little box right here for momentum. We could say mass times velocity-- or, how do we express that in terms of the variables that we have here, like we did last week? What about in terms of kinetic energies?

Well, another way of writing mass times velocity would be root 2MT. Because in this case, we would have root 2 times M1 times 1/2 M1 V1 squared. The 2s here cancel.

Let's see. You have M1 squared. You have a V squared. And the square root of M squared V squared is just MV. So this is an equivalent way of writing the momentum in the variables that we're working in already. And so since that doesn't introduce another variable like velocity-- which we do know, but it's kind of confusing to add more symbols-- let's keep as few as possible.

So what's the x momentum of particle 1? Just what I've got up there. 2 M1 T1. What's the x momentum in this frame of particle 2? 0. We're assuming that it's at rest. And now, what's the x momentum of particle 3?

AUDIENCE: [INAUDIBLE]

PROFESSOR: I heard a couple of things. Can you say them louder?

AUDIENCE: Square root of 2 M3 T3?

PROFESSOR: Yep. Root 2 M3 T3. But in this case, if we're defining, let's say, our x-axis here, it also matters what this angle is. So you've got to multiply by cosine theta in this case. And that's the x momentum of particle 3. And we've also got to account for particle 4. So we'll say add 2 cosine phi.

Now let's do the same thing for the y momentum. What's the y momentum of particles 1 and 2? 0. They're not moving in the y direction to start. And how about particle 3? I hear whispers, but nothing vocalized.

AUDIENCE: Sine?

PROFESSOR: Yep. Same thing, but root 2 M3 T3 sine theta. And M4-- I almost wrote the wrong sign there-- has got to be a minus. If the momentum of the initial particle system in the y direction is 0, so must the final momentum in the y direction. So these two momenta have to be equal and opposite. And that's times sine phi.

So now we actually have sets of equations that relate all of our unknown quantities. We have the mass conservation equation, we have the Q equation, we have the x momentum, and we have the y momentum. And from this point on, it's a matter of algebra to express some of these quantities in terms of some of the others. So let's get started with that.

Because angles are kind of messy, and theta should uniquely define phi, let's try and get things in terms of just one angle. So I'm going to start by separating the thetas and the phis on either side of the equals sign, so that hopefully later on we can eliminate one in a system of equations. So all I'm going to do is I'm going to subtract or add the theta terms to the other side of the equation. So let's say we'll separate angles.

So we'll have root 2 M1 T1 and minus root 2 M3 T3 cosine theta. I'll be depending on you guys to check for sign errors here because those will be messy. I do have notes in case, but I'm hoping I won't have to look at them. And all we have left on this side is root 2 M4 T4 cosine phi. So that's the x momentum equation.

Let's do the same thing with the y momentum equation. So all we'll do is take the theta term and stick it to the left of the equals sign. So that would give us minus root 2 M3 T3 sine theta equals minus root 2 M4 T4 sine phi.

Right away, we can see that the minus signs can cancel out, just for simplicity. And what else is common to these that we can get rid of? Yep?

AUDIENCE: Square root of 2.

PROFESSOR: Everything here has a square root of 2. So we'll just get rid of all of the square root of 2s to simplify as much as possible. And now we look a little stuck. But now is the time to remember those trigonometric identities back from high school that I don't think-- has anyone used these since? In 1801 or 1802, anyone used a trig identity?

A little bit? OK. I would hope so. But I don't know what other people are teaching nowadays. At least this way I'll make sure you remember the high school stuff.

We're going to rely on the fact that we already have got a cosine and a sine. We have a set of simultaneous equations. If we can add them together and destroy the angles somehow, that will make things a lot easier.

So for the thetas, we have a cosine, a sine, and an unangled term that looks kind of messy. Here we have a cosine and a sine. Anyone have any idea where we could go next to destroy one of these angles? Anyone remember any handy cosine or sine trig identities?

AUDIENCE: If you squared both terms, you could get square root of cosine squared, square root of-- sorry. You get cosine squared and sine squared and then you factor out the square root of M4T4 and then cosine squared plus sine squared equals 1.

PROFESSOR: Exactly. So we can rely on the fact that if we can square both sides of both equations and add them up, we would have a cosine squared of phi plus a sine squared of phi, which also equals 1. So we can destroy this phi angle and make things a lot simpler. So we'll start by squaring both sides.

Let's start with the x momentum equation. So if we have-- let's see-- root M1T1. So we're going to take that stuff squared. And that squared is not too hard. Neither are those.

So we'll have root M1 T1 squared, which just gives us M1 T1, minus root M1 T1 times root M3 T3 cosine theta. Let's just lump those terms together as root M1 M3 T1 T3 cosine theta. Also, anyone, raise your hand or let me know if I'm going too fast. I'm trying to hit every single step. But in case I skip one, please slow me down. That's what class is for. OK.

And then we've got another one. Let's just stick a 2 in front of there and plus that term squared. So we'll have M3 T3. Let's see. Yeah. Looks like cosine squared of theta. Yep. Equals-- this one's easier-- M4 T4 cosine squared of phi.

OK. Now we'll do the same thing for the y momentum equation. Much easier because there's no addition anywhere. And we have M3 sine squared theta-- over here-- equals M4 T4 sine squared phi.

So this is quite nice. Now if we add these equations together, we get rid of all of the cosine and sine squared terms. So let's add them up. Let's see. We'll add the two equations. Add equations. And let's try and group all the terms together.

So we have M1 T1 minus 2 root M1 M3-- it's getting hard to write over the lip of the chalkboard here-- cosine theta. And we have M3 T3 cosine squared theta plus M3 T3 sine squared theta. Equals M4 T4 cosine squared theta plus sine squared theta-- or phi. I'm sorry. Cosine squared phi plus sine squared phi.

OK. Hopefully that's as low as I'll have to write. And like we saw before, cosine squared plus sine squared equals 1. So that goes away. That goes away. And let's keep going over on this side of the board.

I told you this would be a fill-the-board day. Let's see if we actually get all six instead of just the four visible. But I think we'll finish this derivation in four boards. So let's write what we've got left. Let's see. Remaining.

So we have M1 T1 minus 2 root M1 M3-- so much easier to write standing up-- cosine theta equals M4 T4. Quite a bit simpler.

AUDIENCE: [INAUDIBLE]

PROFESSOR: Did I miss a term?

AUDIENCE: The M3 T3.

PROFESSOR: Ah. Thank you. You're right. You're right. And we had a plus M3 T3. Yeah, that would be important. Thank you. Equals M4 T4.

So we now have a relation between the masses, the energies, and one angle, which is getting a lot better. We still have one more variable than we can deal with. So let's say if we're-- let's see. Which of these variables do you think we can eliminate using any of the equations you see, let's go with, on that top board over there?

Well, what other quantities are we likely to know about this nuclear reaction? Let's bring this back down. Are we likely to know the Q value?

AUDIENCE: Yeah.

PROFESSOR: Probably. Because like you guys have been doing on problem sets one and two, if you know, let's say, the binding energies, or the masses, or the excess masses, or the kinetic energies of all your products, any combination of those can get you the Q value of that reaction. And if you just look up those reactions like, let's say, radioactive decay reactions, on the table of nuclides, it just gives you the Q value. So chances are we can express some of these kinetic energies in terms of Q.

And all we've got left is T1, T3, and T4. So which of these are we most likely to be able to know or measure? T1, we probably fixed it by cranking up our particle accelerator to a certain energy.

T3 or T4, what do you guys think? Let's say we had a very small nucleus firing at a very big one. Which one do you think would be more likely to escape this system and get detected by us standing a couple feet away with a detector? Yep?

AUDIENCE: T3.

PROFESSOR: Probably T3, the smaller particle. We've just arbitrarily chosen that. But for intuitive sake, let's say, yeah. Why don't we try and get T4 in terms of Q T1 and T3? That's not too hard, since it's addition.

So our next step will be substitute. And we'll say that Q equals-- I'm just going to copy it up from there-- T3 plus T4 minus T1. So we can isolate T4 and say T4 equals Q plus T1 minus T3.

And continue substituting. I usually don't like to have my back to the class this much. But when you're writing this much, it can be a little hard. So let's stick this T4 in right here and rewrite the equation as we've got it.

M1 T1 minus 2 root M1 M3 T1 T3 cosine theta plus M3 T3 equals M4 times Q plus T1 minus T3. I anticipate us needing to see this side of the board soon. I also apologize for the amount of time it takes to write these things.

There's another strategy one can use at the board which is defining intermediate symbols. And here's why I'm not doing that. When I was a freshman, back in-- whoa-- 2001.

Who here was born after 2001? Nobody. OK. Thank god. I don't feel so old.

I was in 18023, which was math with applications, which was better known as math with extra theory. And in one class, not only did we fill nine boards, but we ran out of English letters-- symbols-- and we ran out of Greek letter symbols, and we moved on to Hebrew. Because they were distinct enough from English and Greek.

And being, I think, the only Hebrew speaker in the class, I was the only one that could follow the symbols, but I couldn't follow the math anymore. So I am not going to define intermediate symbols for this and just keep it understandable, even if it takes longer to write.

OK. So let's start off by dividing by M4. Our goal now is to try to isolate Q. Because this is something that we would know or measure. And it will relate all of the other quantities, only one of which we won't really know yet.

So let's divide everything by M4. So we have T1 times M1 over M4 minus 2 over M4 times root of all that stuff plus T3 times M3 over M4 equals Q plus T1 minus T3. And we've almost isolated Q. I'll call this step just add and subtract.

And I'm going to group the terms together. So let's, for example, group all the T1s together and group all the T3s together. So if I subtract T1, I get T1 times M1 over M4 minus 1, minus 2 over M4 root M1 M3 T1 T3 cosine theta, plus-- and if I add T3, then I would get M3 over M4 plus 1 equals Q.

So this is a good place to stop, turn around, and see you guys, and now ask you, which of the remaining quantities do we probably not know? So let's just go through them one by one, just to remind ourselves. Are we likely to know what T1 is? Probably.

How about the masses M1 and M4? If we know what particles are reacting, we can just look those up, or measure them, or whatever. We know M4. We know our masses. We know T1.

What about T3? We don't necessarily know yet. So T3 is a question mark.

How about cosine theta or theta? We haven't said yet. And T3 we don't know. And the masses we know. And the Q we know.

So finally, to solve for-- well, we only have two variables left, T3 and theta. So this here-- this is actually called the Q equation in its most complete form-- describes the relationship between the kinetic energy of the outgoing particle and the angle at which it comes off. How do we solve this? How do we get one in terms of the other? Anyone recognize what kind of equation we have here?

It's a little obscure. Well, it's not obscure. But it's a little bit hiding. But it should be a very familiar one. Think back to high school again. Yes.

AUDIENCE: Is it the cosine angle for the triangle [INAUDIBLE]

PROFESSOR: Let's see. Certainly, there's probably some trig involved in here, in terms of, yeah, if you know the cosine, then you know, let's say, the x or the y component of the momentum. But there's something simpler, something that doesn't require trigonometry. Yep.

AUDIENCE: Is it quadratic?

PROFESSOR: It is. It's a quadratic-- so who saw that? It's actually a quadratic equation, where the variable is the square root of T3.

That's the trick here, is you have something without T3, you have something with square root T3, and you have something with T3, better known as root T3 squared. And there. So this is actually a quadratic equation. Despite the fact that it may not have looked that way in the first place, there we go.

So now, someone who remembers from high school, tell me, what are the roots of a quadratic equation? Let's say if we have the form y equals ax squared plus bx plus c, what does x equal? Just call it out.

AUDIENCE: Negative b--

PROFESSOR: Yeah.

AUDIENCE: [INAUDIBLE] square root--

PROFESSOR: Yep.

AUDIENCE: --b squared minus 4ac--

PROFESSOR: Over--

AUDIENCE: 2a.

PROFESSOR: 2a. And in this case, a is that stuff. b is that stuff without the T3. And c is that stuff. Because we have, like, 15 minutes before I want to open it up to questions and I don't think we have to repeat the quadratic formula stuff, I will skip ahead. Skip ahead.

This is when I'd normally say it's an exercise to the reader. But no. It's not the phrase I like to use. It's boring. And I can just tell you guys what it ends up as.

It ends up with root T3 equals-- and this is the one time I am going to define new symbols because it's just easier to parse-- ends up being, we'll call it s, plus or minus root s squared plus t, where s-- let's see if I can remember this without looking it up.

No. I have to look at my notes. I don't want to get it wrong and have you all write it down incorrectly because of me. There we go.

The remaining stuff in the square root, E1 cosine theta over M3 plus M4. And t equals M4 Q-- is it a minus? It is a plus. Over M3 plus M4.

So these are the roots of this equation. This is how you can actually relate the kinetic energy of the outgoing particle directly to the angle. So I want to let that sink in just for a minute, stop here, and check to see if there's any questions on the derivation before we start to use it to do something a little more concrete. Yep.

AUDIENCE: Where did the E come from?

PROFESSOR: The E. Oh, I'm sorry. That's a T. Thank you. Kinetic energy.

Again, we should be consistent with symbols. And I think-- I don't see any other hanging Es. Good. Thank you.

So any other questions on the derivation as we've done it? We managed to do it in less than four boards. There we go.

OK. Since I don't see any questions, let's get into a couple of the implications of this. So let's now look at what defines an exothermic reaction where we say if Q is greater than 0-- which is to say that some of the mass becomes kinetic energy-- if an exothermic reaction is energetically possible, then what is the minimum T1?

Ah. That's why I brought it. What's the minimum T1 up here to make that exothermic reaction happen? We'll put a condition on T1. So if the reaction's exothermic, which means it will happen spontaneously, how much extra kinetic energy do you have to give to the system to make the reaction happen?

Let's think of it in the chemical sense. If you have an exothermic chemical reaction, is it spontaneous or is it not? It is spontaneous.

Same thing in the nuclear world. If you have an exothermic nuclear reaction, do you need any kinetic energy to start with to make it happen? No. OK. There we go. So that's kind of the analogy.

So T1 has to be greater than or equal to 0. It's pretty much not a condition, right? It happens all the time. So if we were to say T1 were to equal 0-- let me get my crossing out color again. If

T1 were to equal 0, then s could equals 0. And T1 is 0 here. And then you just get-- that's an s-- t equals M4 Q over M3 plus M4.

And this just kind of gives you a relation between the relative kinetic energies of the two particles. Another way of writing this relation would just be that E3 plus E4 has to be greater than or equal to E1.

AUDIENCE: T?

PROFESSOR: All this-- hmm?

AUDIENCE: T?

PROFESSOR: Ah. Thank you. Because Es will be used in a different point of this class. So we'll stick with T for kinetic energy. Thank you.

So all that this condition says is that if mass has been converted to energy, then that kinetic energy at the end has to be greater than at the beginning. And that's all it is. So it makes this equation quite a lot easier to solve for an exothermic reaction.

You can also start to look to say, well, what happens as we vary this angle theta? What does the kinetic energy do? Let's take the case of an endothermic reaction. Now we are running out of space.

For an endothermic reaction where Q is less than 0, you would have to have T1 to be greater than 0. Otherwise the reaction can't occur. So you have to impart additional energy into the system to get it going. And it also means that not every angle of emission is possible.

You might wonder, why do we care about the angle, because the reaction still happens anyway? Well, it doesn't happen at every angle. And reactions have different probabilities of occurring depending on the angle at which the things come out. So you could see here that as you vary T1 and as you vary cosine theta, you still have to make sure that this quantity on the inside here-- so, s squared plus t-- always has to be greater than or equal to zero or else the roots of this are imaginary and you don't have a solution.

So it's kind of nice that this came out quadratic. Because it lets you take some of the knowledge you already know and now apply it to say, when or when are nuclear reactions not or are they allowed? Wait. Let me rephrase that. When are nuclear reactions allowed or not allowed? You can now tell, depending on the angle of emission and the incoming energy and the masses, which are all things that you would tend to know.

So is everyone clear on the implications here? If not, let me know. Because that's what this class is for.

AUDIENCE: Yeah. Can you just go over it one more time?

PROFESSOR: Yes. So, for exothermic reactions where Q is greater than 0, all that says from our initial part of the Q equation, if Q is greater than 0, then we have this thing right here, where the final kinetic energies have to be larger than the initial one. Which is to say that some mass has turned into extra kinetic energy.

And the solution to these is pretty easy because you don't need any kinetic energy to make an exothermic reaction happen. So you can just set T1 equal to 0, which makes s equal to 0, because they're all multiplied here. And then it simplifies lowercase t as just a ratio of those masses times the Q equation, which will tell you pretty much how much kinetic energy is going to be sent off to particle 3 right here. Up there. Particle 3.

Because then we have this condition, if root T3 equals s plus square root of s squared plus t, and we've decided that s equals 0, that just means that T3 equals lowercase t, which equals that. So then you've uniquely defined the kinetic energy for an exothermic reaction, as long as you have no incoming kinetic energy.

For the case of an endothermic reaction, first of all, we know that the incoming kinetic energy has to be greater than 0. It's like the excess energy that you need to get a chemical reaction going. Has anyone here ever played with-- what's the one, a striking one here? Well, has anyone ever lit anything on-- no, that's-- yeah. Of course you have. And that's not a good explanation.

Hmm. What's a good, striking endothermic chemical reaction? Can anyone think of one? Yeah?

AUDIENCE: When you put tin foil in Liquid-Plumr and it releases--

PROFESSOR: And it's a hydrogen generator?

AUDIENCE: Let's see. I guess that's an explosion.

PROFESSOR: I think that happen-- yeah. That's more like an explosion. That's, like, the intuitive definition of exothermic. Yeah.

Actually, there's a fun one you can do, too. This is great that it's on video. You do that plus put manganese dioxide in hydrogen peroxide and you have an oxygen generator. And then you have the purest, beyond glacially pure, spring water. You just mix H and O directly. Just don't get near it. Because it tends to be pretty loud.

We do this for our RTC or reactor technology course, where I've got to teach a bunch of CEOs enough basic high school chemistry so they can understand reactor water chemistry. And the way I make sure that they're paying attention is with a tremendous explosion. So folks come here, pay about $25 grand apiece for me to fire water-powered bottle rockets at them. It's a pretty sweet job. So if you guys are interested in academia, you know, these things happen in life. It's pretty cool. Yeah.

All right. Since I can't think of any endothermic chemical reactions off the top of my head, I'll have to keep it general and abstract and say, if you have an endothermic reaction, you have to add energy in the form of heat to get the reaction going. In an endothermic nuclear reaction, heating up the material does not impart very much kinetic energy. You might raise it from a fraction of an electron volt to maybe a couple of electron volts if things are so hot that they're glowing in the ultraviolet.

That doesn't cut it for nuclear. So you have to impart kinetic energy to the incoming particle such that the kinetic energy plus the rest masses is enough to create the rest masses of the final particles. And that's the general explanation I'd give.

I forget who had asked the question. But does that help explain it a bit?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Cool. OK. I'll take five minutes. And let's do a severely reduced case of this, the case of elastic neutron scattering. It's kind of a flash forward to what we'll be doing in the next month or so. Does everyone have what's behind this board here? I know that was, like, three boards ago. So I hope so.

So let's take the case of elastic neutron scattering. Remember I told you that after we developed this highly general solution to the Q equation, everything else that we're going to study is just a reduction of that. And this is about as reduced as it gets.

So in elastic neutron scattering, we can say that M1-- well, what's the mass of a neutron in AMU? And let's forgive our six decimal points' precision for now. What's it about?

AUDIENCE: 1.

PROFESSOR: 1. So we can say that M1 equals 1. And in the case of elastic scattering, the particles bounce into each other and leave with their original identities. So that also equals M3. If we're shooting neutrons at an arbitrary nucleus, what's M2? Yep?

AUDIENCE: A?

PROFESSOR: Just A, the mass number. Same as M4. Now, we don't have M2 in this equation. Whatever.

But the point is, yeah. We're going to use these two. We're going to use these two. So let's substitute that in. Oh, and one last other thing I mentioned. What is the Q value for elastic scattering?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Right. 0. Because the Q value is the difference in the rest masses of the ingoing and outgoing particles. If the ingoing and outgoing particles are the same, M1 equals M3, M2 equals M4, that sum equals 0. Therefore, Q equals 0.

So let's use these three things right here and rewrite the general Q equation in those terms. Which board is it on? Right there. So let's copy that down.

So let's say we have T1 times M1 is 1 over M4 is A minus 1 minus 2 over M4 is A. This is where it gets nice and easy. M1 and M3 are just 1. So 1 times 1 times T1.

We don't know what that is yet. So let's call it the Tn, T of the neutron coming in. How about this? We'll call it T in and T out for ease of understanding. Cosine theta.

What do we have left? Plus T out. And let's make T1 into an in right there. Times M3 over M4.

M3 was 1. M4 is A. Plus 1 equals Q, equals 0. This is quite a simpler equation to solve.

So let's group this all together. There's a couple of tricks that I'm going to apply right now to make sure that everything has A in the denominator to make stuff easier. We can call 1 A over A here. We can call 1 A over A there. That lets us combine our denominators and stick the sine right there.

That becomes an A. Same thing here. I'll just connect the dashes and stick the minus sign there, leaving an A right there. Now we can just multiply everything by A, both sides of the equation. So the As go away there.

We have a much simpler equation. 0 equals T in of-- let's see-- 1 minus A over 1. OK. We'll just call it 1 minus A. Minus 2 root T in T out cosine theta plus T out A plus 1.

And, OK, it's 10 minutes of, or it's five minutes of five minutes of. So I'm going to stop this right here at a fairly simple equation. We'll pick it up on Thursday. And I want to open the last five minutes to any questions you guys may have. Since that request came in on the anonymous rant forum, which hopefully you all know now exists. Yep.

AUDIENCE: So what exactly is forward scattering? I didn't really get that before.

PROFESSOR: So let's look at elastic scattering as an example. So in elastic scattering, two particles bounce off each other like billiard balls. In forward elastic scattering, the neutron, after interacting somehow with particle 2, keeps moving forward unscathed. So in the elastic scattering sense, forward scattering is also known as missing.

AUDIENCE: [INAUDIBLE]

PROFESSOR: You can have other reactions, let's say, where you have a particle at rest, another particle slams into it, and the whole center of mass moves together. I don't know if you'd call that forward scattering as much as, let's say, capture or fusion or something. But in this case, scattering means that two particles go in, two particles leave. Whether it's elastically, which means with no transfer of energy into rest mass, or inelastically, where, let's say, a neutron is absorbed and then re-emitted from a different energy level. And that's something we'll get into in, like, a month.

So you can have forward elastic or inelastic scattering. In this case, I'm talking about elastic scattering, which is the simple case of, like, the billiard balls miss each other. Which is technically a case that can be treated by this. Because all you have to do is plug in theta equals 0 and you have the case for how much energy do you think the neutron would lose if it misses particle 2.

AUDIENCE: [INAUDIBLE].

PROFESSOR: Yeah. It wouldn't lose any energy. Right? It would have the same energy. So that's the case for forward scattering.

A neutron, when it interacts somehow with another particle, can lose as little as none of its energy. If it misses, no one said it had to lose any energy. And by solving this equation here, which we'll do on Thursday, we'll see what the maximum amount of energy that neutron can lose is, which is the basis for neutrons slowing down or moderation in reactors. Yeah.

AUDIENCE: Are T in and T out equal there, in which case that equation is used to solve for theta?

PROFESSOR: T in and T out are not always equal. But in the case of forward elastic scattering, they would be. Because the neutron comes in with energy T in and it leaves with energy T in. For any other case in which the neutron comes off of particle 2 at a different angle, it will have bounced off of particle 2, moving particle 2 at some other angle phi, and giving it some of its energy elastically. The total amount of that kinetic energy will be conserved.

So let's say-- what did we call it? What is it? Yeah. So T1 would have to be the same as T3 and T4 together for this Q equation where Q equals 0 to be satisfied. So what you said can happen. But it's only the case for forward scattering. Any other questions? Yep.

AUDIENCE: In the case of an exothermic reaction, we assume that T1 equals 0. Can you re-explain why we made that assumption?

PROFESSOR: So the question was, in an exothermic reaction, why did we say T1 equals 0? It's not always the case. But it provides the simplest case for us to analyze. So an exothermic reaction can happen when T1 equals 0. It can also happen when T1 is greater than 0. So we're not putting any restrictions on that.

But in the case that T1 equals 0, s is destroyed and the harder part of T is destroyed, making the solution to this equation very simple and intuitive. Which is to say that if you just have two particles that are kind of at rest and they just merge and fire off two different pieces in opposite directions, their energies are proportional to the ratio of their single mass to the total mass. So that's like a center of mass problem.

You'll notice also I'm not using center of mass coordinates. Center of mass coor-- who here has used those in 801 or 802? And who here enjoyed the experience? Oh. Wow. No hands whatsoever.

So center of mass coordinates and laboratory coordinates are different ways of expressing the same thing. Usually you can write simpler equations in center of mass coordinates. But for most people-- and I'm going to go with all of you, since none of you raised your hand-- it's not that intuitive. That's the same way for me. So that's why I've made a decision to show things in laboratory coordinates, so you have a fixed frame of reference and not a moving frame of reference of the center of mass of the two particles.

But the center of mass idea does kind of make sense here. If you have two particles that are almost touching and then they touch and they break into pieces and fly off, the total amount of momentum of that center of mass was 0. And it has to remain 0. And so each of these particles will take a differing ratio of their masses away.

We already looked at this for the case of alpha decay, where if you have one nucleus just sitting here-- let's say there was no T1. There was just some unstable T2 that was about to explode and then it did. Remember how we talked about how the Q value of an alpha reaction is not the same energy that you see the alpha decay at? Same thing right here. So this Q equation describes that same situation.

Notice there's no hint of M1. There was really no M1 in the end. We don't care what the initial mass of the particle that made alpha decay is. All we care about is what are the mass ratios and energy ratios of the alpha particle and its recoil nucleus. So it all does tie together. That's the neat thing, is this universal Q equation can be used to describe almost everything we're going to talk about.

So this is as complex as it gets. And from now on, we'll be looking at simpler reductions and specific cases of each one. So it's five of. I want to actually make sure to get you to your next class on time. And I'll see you guys on Thursday.