22.02 – Introduction to Applied Nuclear Physics

Problem set # 7

Issued on Sunday Apr. 15, 2012. Due on Friday Apr. 20, 2012

Problem 1:   Spin and Parity assignment

a) *(solved)* Give the spin and parity, as expected from the shell model, of the ground states of ^{173}Yb.

Solution:

In order to determine the spin and parity of these nuclides, we consider the valence nucleon model: only the last unpaired nucleon determine the spin and the parity of the nucleus. Thus we first need to determine which one is the unpaired nucleon. For ^{173}Yb it is a neutron, since Yb has an odd number (103) of neutrons. All the neutrons are stacked in the shell model energy levels, and they occupy completely all the levels, up to the $2f_{5/2}$ shell, as we can read from Fig. 39 (Krane 5.6) of the lecture notes. The unpaired neutron is then in one of the $2f_{5/2}$ eigenstate (there will be 6 of these states, as given by that level degeneracy). Thus we expect the spin of the ^{173}Yb nucleus to be $\frac{5}{2}$. The parity is determined by the angular momentum, in this case $f = 3$ (an odd number) so it will be odd parity, $\frac{5}{2}^-$
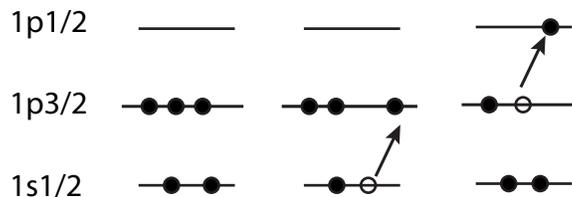
b) Give the spin and parity, as expected from the shell model, of the ground states of
1) ^{7}Li,   2) ^{15}N,   3) ^{29}Si,   4) ^{43}Ca,   5) ^{87}Ru,   6) ^{133}Cs.

Problem 2:   Excited states

a) *(solved)* The first two excited states of Be-9 have energy E=214keV and E=1.1MeV. What are the expected spin-parity assignments of these excited states?

Solution:

The properties of Be-9 will be mainly due to its last unpaired neutron. In the ground state, this is found in the $1p\frac{3}{2}$ state, thus giving $\frac{3}{2}^-$ spin/parity. Possible excited states (see figure) arise if the neutron in that level get excited to the next level ($1p\frac{1}{2}$) or if one neutron from the 1s levels gets promoted to the $1p\frac{3}{2}$ level. Note that we expect this last configuration to be lower in energy than the previous one, because of the pairing force. Thus we assign spin parity $\frac{1}{2}^+$ to the 214keV excited level and $\frac{1}{2}^-$ to the 1.1MeV level.

b) ^{81}Br ground state has spin $I = \frac{3}{2}$ and odd parity ($I^\pi = \frac{3}{2}^-$). The three first excited states have a small energy: 1) $I^\pi = \frac{5}{2}^-$, $E_1 = 275.985$keV;   2) $I^\pi = \frac{9}{2}^+$, $E_2 = 536.20$keV;   3) $I^\pi = \frac{1}{2}^-$, $E_3 = 538.20$keV
Explain these 4 states based on the shell model.
[Note that the valence nucleon model (and the shell model) are good models to predict the ground state properties of nuclei, but often fail to predict the excited states. Thus you can often expect some counter-intuitive results when looking at the excited state ordering of the energy levels.]

Problem 3:   Spin-orbit coupling

Let the energy for a nucleon in the absence of spin-orbit coupling be E_0 and the spin-orbit potential be $V_{so}/\hbar^2 \hat{\vec{L}} \cdot \hat{\vec{S}}$.

a) What is the energy for the nucleon in each of the possible spin-orbit configuration and what is the difference in energy between the two states?

b) (*solved*) Show that the center of gravity of the two configurations (or the average energy weighted by the degeneracy) is given by E_0.

Solution:

The weighted average of the energy is

$$\overline{E} = \frac{\mathcal{D}(l+1/2) \times E_{l+1/2} + \mathcal{D}(l-1/2) \times E_{l-1/2}}{\text{total} \# \text{ of states}}$$

where $\mathcal{D}$ is the degeneracy of each state, which is equal to $\mathcal{D}(j) = 2j+1$ and $E_{l\pm\frac{1}{2}}$ are the energies of the two possible spin-orbit states. Thus we have

$$\overline{E} = \frac{(2l+2)(E_0 + V_{so}\frac{l}{2}) + (2l)(E_0 - V_{so}(l+1)/2)}{4l+2}$$
$$= \frac{(4l+2)E_0 + V_{so}[(l+1)l - l(l+1)]}{4l+2} = E_0$$

c) Consider Bismuth-209 ($^{209}_{83}$Bi). Based only on filling the levels of the shell model, what is the spin/parity assignment of this nuclide?

d) Bismuth-209 is found just outside the shell which gives rise to the magic number 82, although the simple oscillator model would not predict a shell splitting for this nucleon number. What part of the nuclear interaction is responsible for this splitting? Calculate the energy splitting between the Bismuth-209 level and the 82-magic number level.

Problem 4:   Shell model: Anomalous states

The spin parity for ^{195}Pt is $I^\pi = \frac{1}{2}^-$ instead of the expected $\frac{13}{2}^+$. Similarly, ^{205}Tl ground state is $I^\pi = \frac{1}{2}^+$ instead of the expected $\frac{11}{2}^-$. The ground state of ^{87}Sr is instead the expected state $\frac{9}{2}^+$, but this isotope has a very long-lived excited state with spin-parity $\frac{1}{2}^-$ (lifetime $\sim$ 3 hours).

a) What aspect of the nuclear force makes the first two configurations energetically favorable? Based on the different configuration of Sr-87, what does this force depend on?

b) Give the shell-model configurations for these 3 nuclei that are consistent with their spin-parity assignments.

Problem 5:   Oscillator model and binding energy

Consider the oscillator model for Calcium, ^{40}Ca giving the energies $E_N = \hbar\omega N - V_0'$, with $V_0 = 33\text{MeV}$ (for neutrons).
(Notice that here we included the zero energy level of the oscillator into the potential depth).

a) Calculate the nucleon energy E_N for each oscillator level that allows a bound state. What oscillator level corresponds to the first unbound state?

b) What is the average binding energy per nucleon? (The binding energy for one nucleon in the Nth level of the harmonic oscillator is given by $-E_N$)

c) Compare the binding energy for ^{40}Ca found from the oscillator model with the binding energy found as the difference in mass energy between the nucleus and its constituents.

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