Solution of Problem Set 1

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[redacted]

Problem 1 (20Pts)

(a)

The current can be expressed as I=NZe. N is the rate of protons captured by the Tungsten target; Z is 1 for proton.

$$\boxed{N = \frac{I}{e} = \frac{2.0\times10^{-3}\,A}{1.6\times10^{-19}\,C} = 1.25\times10^{16}\,\sec^{-1}}$$

(b) The force, by using Newton 2nd law, is:

$$F = \frac{dp}{dt} = N(0-p) = Np$$

where p is the linear momentum of individual protons before smashed into the target.

$$p = \frac{1}{c}\sqrt{E^2 - E_0^2} = \frac{1}{c}\sqrt{(E_0+E_k)^2 - E_0^2} = \frac{1}{c}\sqrt{2E_0E_k - E_0^2}$$

$$= \frac{1}{3\times10^8}\times\sqrt{2\times1000\times938.28+1000^2}\times1.6\times10^{-13} = 9.05\times10^{-19}\ \text{Newton}\cdot\sec$$

$$F = -Np = -1.25\times10^{16}\times9.05\times10^{-19} = -1.13\times10^{-2}\,\text{Newton}$$

The force protons exert to the target is the anti-force of the force exert to the proton by the target. So they are equal in terms of magnitude. Thus, the magnitude of the force protons exert to the target is $\boxed{1.13\times10^{-2}\,\text{Newton}}$.

(c) $$\boxed{\frac{E-E_0}{E_0}\times100\% = \frac{E_k}{E_0}\times100\% = \frac{1000}{938.28}\times100\% = 106.6\%}$$

(d) $$\boxed{\#\,\text{neutrons} = N\times n(\text{neutrons each proton produced}) \approx 1.25\times10^{16}\times20 = 2.5\times10^{17}\,\sec^{-1}}$$

Problem 2 (20Pts)

We note that $E_n = 0.025\text{eV} << m_nc^2$. So we calculate the velocity of neutron from its kinetic energy as follows:

$$v_n = \sqrt{\frac{2E_n}{m_n}} = \sqrt{\frac{2\times0.025\times1.6\times10^{-19}}{1.66\times10^{-27}}} = 2.195\times10^2\,m/s = 2.195\times10^5\,cm/s$$

The flux of neutrons can be obtained as:

$$\phi = \rho\left(\frac{\#}{cm^3}\right)\cdot v\left(\frac{cm}{\sec}\right) = \frac{\#}{cm^2\,\sec}.$$

And $1cm^3$ of observation volume includes $N_0 = 9.111\times10^3$ neutrons. The number of neutrons decayed within 1 minute can be calculated as:

$$\boxed{N_{decay} = N_0 - N_0e^{-\lambda_n t} = N_0(1-e^{-\frac{t}{\tau_n}}) = 9111\times(1-e^{-\frac{60}{1.01\times10^3}}) = 525.4}$$

Problem 3 (20Pts)

Assume that the nucleus is static before collision and the collision is elastic.

M_n and M_A are the masses of the neutron and the nucleus; v_n and v_n', the velocity of neutron before and after collision; v_A and v_A', the velocity of nucleus before and after collision. So $v_n' = 0$, the maximum kinetic energy the nucleus can obtain from the collision can be computed from the following two equations:

$$\left.\begin{aligned} \frac{1}{2}M_n v_n^2 + 0 &= \frac{1}{2}M_n v_n'^2 + \frac{1}{2}M_A v_A'^2 \\ M_n v_n + 0 &= M_n v_n' + M_A v_A' \end{aligned}\right\} \Rightarrow E_A = \frac{4M_n M_A}{(M_n + M_A)^2} E_n$$

where E_A is the maximum kinetic energy the nucleus can obtain from the collision and E_n is the kinetic energy of the neutron before collision.

$$\because \text{Hydrogen}: M_H = 1\text{a.m.u.} \quad E_H = \frac{4M_n}{(1+M_n)^2} E_n$$

$$\text{Nitrogen}: M_N = 14\text{a.m.u.} \; E_N = \frac{56M_n}{(14+M_n)^2} E_n$$

$$\Rightarrow \frac{E_H}{E_N} = \frac{1}{14}\left(\frac{14+M_n}{1+M_n}\right)^2 \Rightarrow \boxed{M_n = 1.0655 \text{ a.m.u.}}$$

In compare with the modern value of $\boxed{M_n = 1.0086650 \text{ a.m.u.}}$, the error is $\boxed{\text{about } 5.63\%}$.

Problem 4 (20Pts)

(a) The thermal efficiency of an ideal Carnot engine is

$$\eta_i = 1 - \frac{T_L}{T_H} = 1 - \frac{100 + 273.15}{500 + 273.15} = 0.517$$

So the real thermal efficiency of the system is

$$\eta = 0.4 \times \eta_i = 20.7\%$$

Typical fission can be written as $^{235}U + n \rightarrow {}^{93}Rb + {}^{141}Cs + 2n$. So each fission even will cost $235 \times 1.66 \times 10^{-24} = 3.901 \times 10^{-22}$ gram ^{235}U. In order to provide 1000MW thermal power, ^{235}U used up per day can be determined by

$$\boxed{\frac{1000 \times 10^6 \times 86400}{200 \times 10^6 \times 1.6 \times 10^{-19} \times 20.7\%} \times 3.901 \times 10^{-22} = 5088 \text{ gram/day}}$$

(b) The number of ^{235}U nuclei undergoing fission per second can be determined as

$$\boxed{\frac{1000 \times 10^6 \times 1}{200 \times 10^6 \times 1.6 \times 10^{-19} \times 20.7\%} = 1.51 \times 10^{20} \text{ sec}^{-1}}$$

(c) $$\boxed{\frac{1000 \times 10^6 \times 86400}{7000 \times 4.1868 \times 20.7\%} \times 10^{-6} = 1.42 \times 10^4 \text{ ton/day}}$$

Problem 5 (20Pts)

(a) Activity $A = \lambda N$, where A = activity, λ = decay constant (0.693/half life), and N = number of atoms present. Also, in terms of units, activity is defined as the number of disintegrations (also known as transformations) that occur per unit time.

The initial rate of production (s^{-1}) of activated atoms is $\left(\frac{dN_{act}}{dt}\right)_0 = R$. As the population of active atoms increase, they decay at the rate $\lambda N_{act}(s^{-1})$. Thus the net rate at which they accumulate can be expressed as

$$\frac{dN_{act}}{dt} = \left(\frac{dN_{act}}{dt}\right)_0 - \lambda N_{act} = R - \lambda N_{act} \quad (1)$$

General solution of this differential equation after one transit (t_i)is $N_{act} = \frac{R}{\lambda}\left(x_1 + x_2 e^{-\lambda t_i}\right)$(2)

Differentiating with respect to time gives $\frac{dN_{act}}{dt} = \frac{R}{\lambda}\left(-x_2 \cdot \lambda \cdot e^{-\lambda t_i}\right)$ (3)

Now substituting Eqs. (2) and (3) into Eq. (1), we get

$$\frac{dN_{act}}{dt} = \frac{R}{\lambda}\left(-x_2 \cdot \lambda \cdot e^{-\lambda t_i}\right) = R - \lambda N_{act} = R - R(x_1 + x_2 e^{-\lambda t_i})$$

$$\frac{R}{\lambda}\left(-x_2 \cdot \lambda \cdot e^{-\lambda t_i}\right) = R - R(x_1 + x_2 e^{-\lambda t_i}),\ N_{act}\big|_{t_i=0} = 0$$

$$\Rightarrow x_1 = 1, x_2 = -1$$

$$\therefore N_{act} = \frac{R}{\lambda}\left(1 - e^{-\lambda t_i}\right)$$

The activity A added per cm^3 of the coolant per transit of the target is given by

$$\boxed{A = \lambda N_{act} = R\left(1 - e^{-\lambda t_i}\right)}.$$

(b) At very first, the activity is 0. Then after one time through the target, the activity becomes $R(1 - e^{-\lambda t_i})$. We can do the same as (a). We calculate as above. It is obvious that second time the coolant goes through the external circuit and the target (t_0+t_i), the activity added is $R\left(1 - e^{-\lambda t_i}\right)e^{-\lambda(t_0+t_i)}$. And so on...

So that, after m cycles, the activity per cm^3 of the coolant leaving the target is

$$R(1 - e^{-\lambda t_i}) + R(1 - e^{-\lambda t_i})e^{-\lambda(t_i+t_0)} + R(1 - e^{-\lambda t_i})e^{-\lambda(t_i+t_0)}e^{-\lambda(t_i+t_0)} + \cdots$$

$$+ R(1 - e^{-\lambda t_i})e^{-(m-1)\lambda(t_i+t_0)} = R\frac{(1 - e^{-\lambda t_i})(1 - e^{-m\lambda(t_i+t_0)})}{1 - e^{-\lambda(t_i+t_0)}}$$

(c) The maximum coolant activity at the exit is:

$$\boxed{\lim_{m\to\infty} A_m = R\frac{1 - e^{-\lambda t_i}}{1 - e^{-\lambda(t_i+t_0)}}}.$$