

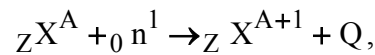
Problem 1 (20%)

(a) 10% Show by theory of electrostatics that the repulsive energy due to a uniform distribution of Charge Ze over a sphere of radius R is given by $U_c = \frac{3Z^2e^2}{5R}$.

(b) 10% Using the empirical relation for nuclear radii, $R = r_0 A^{1/3}$, show that the electrostatic repulsion contributes a negative term in the binding energy of the form, $a_c \frac{Z^2}{A^{1/3}}$. From the empirical nuclear mass formula, one gets a value $a_c = 0.595$ MeV. Use this information to obtain the best value of the constant r_0 .

Problem 2 (25%)

When a thermal neutron is captured by a fissionable nucleus ${}_Z X^A$, leading to a reaction:



the resultant Q-value can be considered as the excitation energy available for inducing a fission reaction of the compound nucleus ${}_Z X^{A+1}$.

(A) 5% Show that the Q-value can be calculated in terms of the binding energy by a formula:

$$Q = B(A+1, Z) - B(A, Z).$$

(B) 10% Use the semi-empirical formula

$$B(A, Z) = a_v A - a_c \frac{Z^2}{A^{1/3}} - a_s A^{2/3} - a_a \frac{(A - 2Z)^2}{A} + \delta$$

where

$\delta = a_p A^{-3/4}$ for an e-e nucleus, $\delta = -a_p A^{-3/4}$ for an o-o nucleus, $\delta = 0$ for an e-o or o-e nucleus

and the constants in the semi-empirical formula having values (in unit of MeV):

$$a_v = 14.1, \quad a_c = 0.595, \quad a_s = 13.0, \quad a_a = 19.0, \quad a_p = 33.5,$$

calculate the excitation energy available for the case of ${}_{92}U^{235}$.

(C) 10% Do the same thing for the case of ${}_{92}U^{238}$ and show that the excitation energy available for this latter case is approximately 1 MeV less than the former case. State the origin of this difference of the excitation energy between the two nuclei.

It is known that thermal neutrons can induce fission in ${}_{92}U^{235}$, but cannot do so in ${}_{92}U^{238}$. It is

also known that in the latter case one needs to use high energy neutrons of energies greater than 1 MeV to induce the fission reaction. Your calculation should confirm this experimental fact.

Problem 3 (20%)

It is known that the ground state of n – p system in a deuteron is a triplet state(3S_1 state), with a potential parameters of $b_t = 1.93$ fm and $V_{ot} = 38.5$ MeV. The radial wave function of the ground state is given by:

$$\begin{aligned} u_I(r) &= 0.163 \sin(0.938r) \text{ fm}^{-1/2} & r \leq b \\ u_{II}(r) &= 0.249e^{-0.232r} \text{ fm}^{-1/2} & r > b \end{aligned}$$

- (a) 10% Calculate the probability that the n-p system is separated by a distance greater than b. What does this mean physically?
- (b) 10% From the fact that a deuteron has the potential parameters, $b_t = 1,93$ fm , $V_{ot} = -38.5$ MeV., show that the potential well can have only one bound state. To show this, you can quote a theorem that I asked you to prove in HW-3 about the number of possible bound states and the range of the value V_{ot}, b_t^2 .

Problem 4 (10%)

Given that the ground state wave function and energy of a hydrogen-like atom are:

$$\psi_0(x) = \sqrt{\frac{Z^3}{\pi a_b^3}} e^{-\frac{Zr}{a_b}} ; \text{ where } a_b = \frac{\hbar^2}{me^2} ; E_0 = -\frac{Z^2 me^4}{2\hbar^2} .$$

Show that the expectation value of the kinetic energy of the electron is equal to the negative of one half of the potential energy. and also to the negative of the total energy.

Problem 5 (30%)

Consider scattering of a neutron by a proton where the n-p interaction can be represented by a potential well. The potential well has a range b and a depth of $-V_0$.

In the scattering problem, the total energy is the center of mass neutron kinetic energy,

$$E = \frac{\hbar^2 k^2}{M} .$$

In solving the Schrödinger equation, the consistency relation resulting from

matching the radial wave function at the edge of the well is

$$k \cot(kb + \delta) = K \cot Kb , \tag{1}$$

where $K^2 = K_0^2 + k^2$, and $K_0^2 = \frac{M}{\hbar^2} V_0$.

(a) 10% By expanding the cotangent function on the left-hand side of Eq.(1), one obtain

$$k \cot \delta = \frac{K \cot Kb + k \tan kb}{1 - \frac{K}{k} \tan kb \cot Kb} . \quad (2)$$

From this relation, show that one can obtain the Fermi scattering length a by taking the limit,

$$\lim_{k \rightarrow 0} k \cot \delta(k) = -\frac{1}{a} = \frac{K_0 \cot K_0 b}{1 - K_0 b \cot K_0 b} . \quad (3)$$

Write Eq.3 in the form

$$a = b \left(1 - \frac{\tan K_0 b}{K_0 b} \right) . \quad (4)$$

and give the ranges of parameter $K_0 b$ for which the value of the scattering length is positive or negative.

(c) 10% For n-p scattering in the triplet state, the potential well parameters are:

$$b_t = 1,93 \text{ fm} , \quad V_{0t} = -38.5 \text{ MeV} .$$

Calculate the scattering length and its sign.

(d) 10% For n-p scattering in the singlet state, the potential well parameters are:

$$b_s = 2.50 \text{ fm} , \quad V_s = -14.3 \text{ MeV} .$$

Calculate the scattering length and its sign.

