

### Sample questions for Quiz 2, 22.101 (Fall 2006)

Following questions were taken from quizzes given in previous years. They are meant to give you an idea of the kind of questions (what was expected from the class in previous times) that have been asked in the past. Percentage credit is indicated for each question based on 100% for a 90-min quiz (closed book). Problem numbering means nothing here.

You will have to decide for yourself what connections, if any, there may be between these questions and the Quiz 2 that will be conducted on Nov. 15, 2006.

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Problem 1 (10%) Among the energy levels of a central force potential is a level labeled 1d. Suppose we now add a spin-orbit interaction term to the Hamiltonian such as in the shell model. Using the spectroscopic notation, label the new levels that evolve from this 1d level. Specify how many nucleons can go into each of the new levels, and explicitly write out the quantum numbers specifying the wave function of each nucleon.

Problem 2 (10%) In an odd-odd nucleus the last neutron and proton go into a 1d<sub>3/2</sub> and a 1g<sub>9/2</sub> level respectively. Use the shell model to predict the spin and parity of this nucleus.

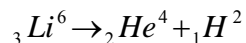
Problem 3 (15%) The binding energy per nucleon of  ${}^6_3\text{Li}$  is about 5.3 Mev while that of  ${}^4_2\text{He}$  is 7.1 Mev. Does this mean that the former is unstable against  $\alpha$ -decay? Explain. (Note: The binding energy of the deuteron  ${}^2_1\text{H}$  is 2.25 Mev.)

Problem 4 (15%)

Explain briefly what is the reason for introducing the total momentum,  $\underline{j} = \underline{L} + \underline{S}$ , in the Nuclear Shell model. How does this affect the labeling of the states?

Problem 5 (15%)

Since the binding energy per nucleon (B/A) of  ${}^4_2\text{He}$ , at 7.07 Mev, is higher than that of  ${}^6_3\text{Li}$ , at 5.33 Mev, it may appear that the former is more stable than the latter and therefore  $\text{Li}^6$  should spontaneously undergo  $\alpha$ -decay,



where the B/A of  ${}^2_1\text{H}$  is 1.11 Mev. Determine whether this is indeed the case.

Problem 6 (20%)

The reaction,  ${}^3_1\text{H} + {}^1_1\text{H} \rightarrow {}^3_2\text{He} + n$ , has a Q-value of -0.764 Mev. Tritium  ${}^3_1\text{H}$  also undergoes  $\beta^-$  decay with end-point energy of 0.0185 Mev. Find the difference between the neutron and hydrogen mass in Mev. Draw an energy level diagram showing the levels involved in the reaction and the  $\beta^-$  decay, then indicate in your diagram the proton separation energy  $S_p$ , the Q-value, and the end-point energy  $T_{\text{max}}$ .

Problem 7 (15%)

Sketch the energy variations of the stopping power (energy loss per unit path) of both electrons and protons in lead (in the same figure). Discuss all the features of these two curves that you know.

Problem 8 (20%)

On the basis of the Bethe-Bloch formula, the stopping power of a material for incident electrons (with kinetic energy  $T_e$ ) can be related to that for incident alpha particles (with kinetic energy  $T_\alpha$ ). Denoting the two by  $-(dT/dx)_e$  and  $-(dT/dx)_\alpha$  respectively, sketch the two curves on the same graph to show how knowing one allows you to find the other.

Problem 9 (20%)

In analyzing the energy loss per unit path length of a charged particle ( $ze$ ) moving with speed  $v$ , an atomic electron in the medium located at an impact parameter  $b$  will gain an amount of kinetic energy equal to

$$T = \frac{2(ze^2)^2}{mv^2b^2}$$

from collision with the charged particle, where  $m$  is the electron mass. Suppose you are told to ignore the binding energy of the atomic electrons, so that all the electron in a collision cylinder of radius  $b$ , thickness  $db$ , and length  $\Delta x$  are ejected. Find the number of electrons per unit path length ejected with kinetic energy in  $dT$  about  $T$ . (Hint: Think of this number as a distribution.)

Problem 10 (15%)

At time  $t = 0$  you are given an atom that can decay through either of two channels,  $a$  and  $b$ , with known decay constants  $\lambda_a$  and  $\lambda_b$ . Find the probability that it will decay by channel  $a$  during the time interval between  $t_1$  and  $t_2$ , with  $t_1$  and  $t_2$  arbitrary. Interpret your result.

Problem 11 (15%)

The basic idea underlying the  $C^{14}$  dating method is that radioactive  $C^{14}$  is produced continuously in the atmosphere by cosmic rays and finds its way into living plants and animals by carbon exchange. When the plant or animal dies the exchange process stops. How would you use this idea to determine the age of an antique wooden chair (when the tree was cut), that is, what activity would you measure and how would you interpret the data? Give a sketch of your method. (Note: The equilibrium concentration of  $C^{14}$  in the air is a known constant.  $C^{14}$  undergoes  $\beta$ -decay to  $N^{14}$  which is stable.)

Problem 12 (40% total, 5% each))

Discuss briefly the significance of each of the following. Give a definition whenever it is appropriate. (If you use the same notation as the Lecture Notes, you may assume the symbols are already defined in the Notes.)

- (a) The asymmetry term in the empirical mass formula.
- (b) Mass parabolas for isobars for even A (give a sketch).
- (c) Mass or energy requirements for electron capture.
- (d) Secular equilibrium in radioactive decay.
- (e) Bragg curve for charged particles (give sketch).
- (f) Bethe formula for stopping power and its relativistic corrections.
- (g) Charge and mass dependence of *bremstrahlung* intensity.
- (h) Mass absorption coefficient for charged particles.

Problem 13 (40% total, 8% each) Give a short and concise answer to each of the following questions.

- (a) Derive using a sketch the asymmetry term in the empirical mass formula.
- (b) Explain what is  $\beta^+$ -decay, then state and justify the energy condition for this process.
- (c) Sketch schematically the curve for the stopping power of a heavy charged particle in a high-Z medium in the energy range, zero to three times its rest mass energy. Label all the characteristic energies that you know, and explain what physical processes are represented in the curve.
- (d) Show that the range of an  $\alpha$ -particle and a proton, both having the same initial speed, will be approximately the same.
- (e) Is *bremstrahlung* an elastic or inelastic process (explain)? Why is this process more important for electron than for proton in problems of interest to the class.

Problem 14 (20%)

Summarize briefly how the shell model is able to predict the magic numbers. State only the most important steps (commensurate with how much time you have). You can quote anything in the lecture notes without further explanation, such as the coupling of two angular momenta, their eigenfunctions and eigenvalues, and the labeling of energy levels (so long as your expressions, figures and symbols are the same as in the lecture notes).

Problem 15 (20%)

- (a) Explain the usefulness of the Mass Parabola.
- (b) Suppose you are asked to analyze the Mass Parabola using the empirical binding energy formula. Show which is more important between the Coulomb term and the asymmetry term. (The coefficients of the two terms are  $a_c = 0.72$  Mev and  $a_a = 23.5$  Mev.)

Problem 16 (20%)

Radioisotope B decays to C (decay constant  $\lambda_B$ ) which can undergo two decay processes, one going to a nucleus D with constant  $\lambda'_C$  and a competing process going to another nucleus E with constant  $\lambda''_C$ . Both D and E are stable. Write down the equations describing the four nuclides  $N_B(t)$ , ...,  $N_E(t)$  in the decay chain. Solve the equations directly using the initial condition  $N_B(t=0) = N_B^0$ ,  $N_C(0) = N_D(0) = N_E(0) = 0$ , and then demonstrate that your solutions are indeed correct.

Problem 16 (25%)

Show that alpha particles and protons having the same initial speed (high speed but nonrelativistic) have approximately the same range in any stopping material. Examine the figure below to see which particle should have a slightly longer range. Justify your answer.

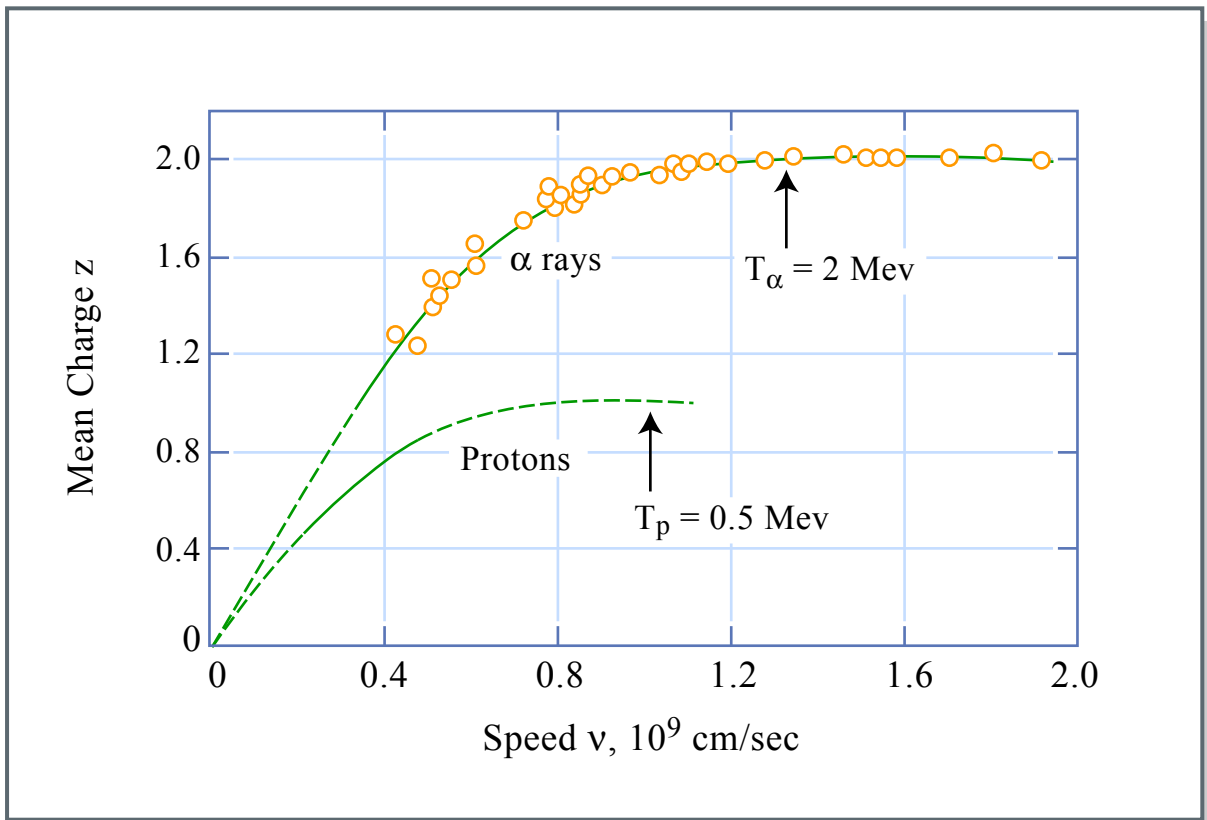


Figure by MIT OCW. Adapted from Evans, 1955.

Problem 17 (15%)

Explain briefly under what conditions energy loss by radiation important is important. Give a sketch of the mass absorption coefficients for electrons in the range (0, 5Mev), comparing two absorbers, Pb and Al. Justify the relative magnitudes you have sketched.