Exercise 1. Data fitting

1. Given a set of N values y_i of a function $y(x)$ at the positions x_i, write a short code to fit a polynomial having order one less than N (so there are N coefficients of the polynomial) to the data.

Select one of the supplied sets of (N=) 7 numbers.
These are the values y_i you must take of the function $y(x)$ at the positions

$x_i = 0.0, 0.167, 0.333, 0.5, 0.667, .833, 1.0.$

Run your code on this data and find the coefficients c_j.

Plot together (on the same plot) the resulting fitted polynomial representing $y(x)$ (with sufficient resolution to give a smooth curve) and the original data points, over the domain $0 \le x \le 1$.

Submit the following as your solution:

a. Your code in a computer format that is capable of being executed.

b. The numeric values of your coefficients $c_j,\ j = 1, N$.

c. Your plot.

d. Brief commentary ($<$ 300 words) on what problems you faced and how you solved them.

2. Save your code from part 1. Make a copy of it with a new name and change the new code as needed to fit (in the linear least squares sense) a polynomial of order possibly lower than $N - 1$ to a set of data x_i, y_i (for which the points are in no particular order).

Obtain a pair of data sets of length (N =) 25 numbers x_i, y_i from the same URL. Run your code on that data to produce the fitting coefficients c_j when the order of the polynomial is (M =) (a) 1, (b) 2, (c) 3. That is: constant, linear, quadratic.

Plot the fitted curves and the original data points on the same plot(s) for all three cases.

Submit the following as your solution:

a. Your code in a computer format that is capable of being executed.

b. Your coefficients $c_j,\ j = 1, M$, for three cases (a), (b), (c).

c. Your plot(s).

d. Very brief remarks on whether the coefficients are the same for the three cases, and why.

e. Can your code from this part also solve the problem of part 1?

22.15 Essential Numerical Methods

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