Exercise 6. Iterative Solution of Matrix Problems.

1. Start with your code that solved the diffusion equation explicitly. Adjust it to always take timesteps at the stability limit $\Delta t = \Delta x^2/2D$, so that:

$$\psi_j^{(n+1)} - \psi_j^{(n)} = (\frac{1}{2}\psi_{j+1}^{(n)} - \psi_j^{(n)} + \frac{1}{2}\psi_{j-1}^{(n)} + \frac{s_j^{(n)}}{2D}\Delta x^2).$$

Now it is a Jacobi iterator for solving the steady-state elliptical matrix equation. Implement a convergence test that finds the maximum absolute *change in* ψ and divides it by the maximum absolute ψ, giving the normalized ψ-change. Consider the iteration to be converged when the normalized ψ-change is less than (say) 10^{-5}. Use it to solve

$$\frac{d^2\psi}{dx^2} = -1$$

on the domain $x = [0, 1]$ with boundary conditions $\psi = 0$ at $x = 0$, $\frac{\partial\psi}{\partial x} = 0$ at $x = 1$, with a total of N_x equally-spaced nodes. Find how many iterations it takes to converge, starting from an initial state $\psi = 0$, when

(a) $N_x = 10$
(b) $N_x = 30$
(c) $N_x = 100$

Compare the number of iterations you require with the analytic estimate in the notes. How good is the estimate?

Now we want to check how accurate the solution really is.

(d) Solve the equation analytically, and find the value of ψ at $x = 1$, $\psi(1)$.
(e) For the three N_x values, find the relative error[1] in $\psi(1)$.
(f) Is the actual relative error the same as the convergence test value 10^{-5}? Why?

Enrichment only, optional and not for credit. Turn your iterator into a SOR solver by splitting the iteration matrices up into red and black (odd and even) advancing parts. Each part-iterator then uses the latest values of ψ, that has just been updated by the other part-iterator. Also provide yourself an over-relaxation parameter ω. Explore how fast the iterations converge as a function of N_x and ω.

Note. Although in Octave/MATLAB® it is convenient to implement the matrix multiplications of the advancing using a literal multiplication by a big sparse matrix, one does not do that in practice. There are far more efficient ways of doing the multiplication, that avoid all the irrelevant multiplications by zero.

[1] the difference between the "converged" iterative $\psi(1)$ and the analytic $\psi(1)$ normalized to the analytic value.

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