22.312 ENGINEERING OF NUCLEAR REACTORS

Tuesday, October 9th, 2014, 1:00 – 2:30 p.m.

OPEN BOOK QUIZ 1 (solutions)

Problem 1 (50%) – Loss of condensate pump transient in a LWR condenser

i)
Consider the seawater in the condenser as the control volume for this first analysis. The conservation of energy for steady-state yields the following equation:

$$0 = \dot{Q} + \dot{m}_w h_{wi} - \dot{m}_w h_{wo} \qquad \Rightarrow \qquad \dot{Q} = \dot{m}_w (h_{wo} - h_{wi}) = \dot{m}_w c_w (T_{wo} - T_{wi}) = 2137 \text{ MW}$$

where $\dot{m}_w$, T_{wi}, T_{wo}, and c_w, are, respectively, the seawater mass flow rate, inlet and outlet temperatures, and specific heat, all given in the problem statement. Kinetic and gravitational terms were neglected; seawater was treated as an incompressible fluid; seawater flow was assumed to be isobaric, per the problem statement. Note that the same result could have been obtained by applying the conservation of energy to the steam side of the condenser.

ii)
Now consider the steam side of the condenser as the control volume. During the transient the amount of energy entering the control volume is constant (i.e. the mass flow rate and enthalpy of the incoming wet steam is constant); however, the heat transfer rate removed by the seawater is lower (i.e. 90% of its steady-state value), and the condensate is no longer pumped away by the condensate pump. As a result a net accumulation of mass and energy is expected on the steam side of the condenser, which thus results in a higher pressure and temperature.

iii)
Again the steam side of the condenser is the control volume for the analysis. Conservation of mass

$$\frac{dM}{dt} = \dot{m}_i \tag{1}$$

Where $\dot{m}_i$ = 1,231 kg/s is the mass flow rate of the incoming wet steam, given in the problem statement. Integrating between initial time (t_1=0) and final time (t_2=30 s), we get:

$$M_2 - M_1 = \dot{m}_i t_2 \tag{2}$$

Note that the initial mass in the control volume, M_1, is easily found from the initial masses of condensate and steam, which in turn are found from the initial volumes of condensate and steam given in the problem statement, and the specific volumes of saturated water and vapor given in the property table:

$$M_1 = M_{f1} + M_{g1} = V_{f1} / v_f(T_1) + V_{g1} / v_g(T_1) = 86{,}150 \text{ kg}$$

Here T_1 = 25°C is the initial temperature of the steam and condensate. Therefore, Eq. (2) can be used to get M_2 = 123,094 kg.

The conservation of energy equation is:

$$\frac{dE}{dt} = -0.9\dot{Q} + \dot{m}_i h_i \quad (3)$$

where $0.9\dot{Q}$ is 90% of the heat transfer rate to the seawater calculated in Part 'i', and h_i = 1,840 kJ/kg is the enthalpy of the incoming wet steam. Integrating Eq. (3) between the initial and final times, we get:

$$M_2 u_2 - M_1 u_1 = (-0.9\dot{Q} + \dot{m}_i h_i) t_2 \quad (4)$$

Expanding the LHS of Eq. (4), we get:

$$M_2[u_f(T_2) + x_2 u_{fg}(T_2)] - M_1[u_f(T_1) + x_1 u_{fg}(T_1)] = (-0.9\dot{Q} + \dot{m}_i h_i) t_2 \quad (5)$$

where T_2 and x_2 are the final temperature and steam quality in the condenser, respectively. Note that the initial steam quality is found from the initial masses in the condenser: $x_1 = M_{g1} / M_1$ = 0.000199.

We can also write the following volume equation for the final conditions:

$$V = M_2[v_f(T_2) + x_2 v_{fg}(T_2)] \quad (6)$$

where the total volume of the condenser is $V = V_{f1} + V_{g1}$ = 864 m^3.

Equations (5) and (6) are two implicit and coupled equations in the unknown T_2 and x_2, thus the problem can be solved by iterations. The results are T_2 = 37. 4°C (corresponding to a pressure P_2 = 6.56 kPa) and x_2 = 0.000261. Therefore the temperature and pressure increase, as expected, and the steam quality also increases. Finally the mass of condensate $M_{f2} = M_2(1 - x_2)$ increases (M_{f2}= 123,060 kg vs M_{f1}= 86,133 kg), which is expected because the condensate pump has stopped working.

Problem 2 (50%) – Thermal parameters in the core of a helium-cooled fast reactor

i)
The local peaking factor is defined as the ratio of the maximum pin power to the average pin power in the hot fuel assembly:

$$P_{loc} \equiv \frac{\dot{q}_{\max}}{\bar{\dot{q}}} = \frac{16}{(1\times16+6\times14+12\times15)/19} \approx 1.086$$

ii)
Let us first calculate the axial peaking factor, defined as the ratio of the maximum linear power to the average linear power in the hot pin:

$$P_{ax} \equiv \frac{q'_{\max}}{\bar{q}'} = \frac{q'_{\max}}{\frac{1}{L}\int_{-L/2}^{L/2} q'(z)dz} = \frac{q'_{\max}}{\frac{1}{L}\int_{-L/2}^{L/2} q'_{\max}\cos(\frac{\pi z}{L_e})dz} = \therefore \frac{\left(\frac{\pi L}{2L_e}\right)}{\sin\left(\frac{\pi L}{2L_e}\right)} \approx 1.209$$

The average linear power in the hot pin is $\bar{q}'$ = 16 kW/ 1.2 m ≈ 13.3 kW/m. Then the maximum linear power in the hot pin is $q'_{\max} = \bar{q}' P_{ax}$ ≈ 16.12 kW/m. And finally the maximum heat flux is:

$$q''_{\max} = \frac{q'_{\max}}{\pi d_{co}} \approx 570 \text{ kW/m}^2$$

where d_{co} = 9 mm is the pin diameter.

iii)
The mass flow rate can be found from the inlet coolant velocity, density and flow area as:

$$\dot{m} = \rho_{in} V_{in} A \approx 110 \text{ kg/s}$$

where V_{in} = 38 m/s, A = 0.75 m^2 and the coolant at the inlet density is given by the equation of state:

$$\rho_{in} = \frac{P_{in}}{RT_{in}} \approx 3.86 \text{ kg/m}^3$$

where T_{in} = 623 K (350°C) and P_{in} = 5.0 MPa.

iv)
The coolant velocity at the outlet of the core is easily found as follows:

$$V_{out} = \dot{m}/(\rho_{out} A) \approx 62.9 \text{ m/s}$$

where $\rho_{out} = \dfrac{P_{out}}{RT_{out}} \approx 2.32$ kg/m^3 and T_{out} = 973 K (700°C) and P_{out} = 4.7 MPa.

v)
Conservation of energy applied to the coolant in the core yields:

$$0 = \dot{Q} + \dot{m}(h_{in} + \frac{v_{in}^2}{2} + gz_{in}) - \dot{m}(h_{out} + \frac{v_{out}^2}{2} + gz_{out})$$

$$\dot{Q} = \dot{m}[c_p(T_{out} - T_{in}) + \frac{v_{out}^2 - v_{in}^2}{2} + g(z_{out} - z_{in})] \approx 200 \text{ MW}$$

where we have used the constitutive relation for enthalpy of an ideal gas $h_{out} - h_{in} = c_p(T_{out} - T_{in})$; and $c_p = c_v + R$=5193 J/kg-K. Note that in this case the kinetic and gravitational terms actually can be calculated (since we have the velocities and elevations), and turn out to be completely negligible with respect to the enthalpy term, as usual.