22.314 Fall 2006 Soln to Quiz # 1

Q1

(1.1) As discussed in class, the purely elastic stress distribution in a solid fuel will be:

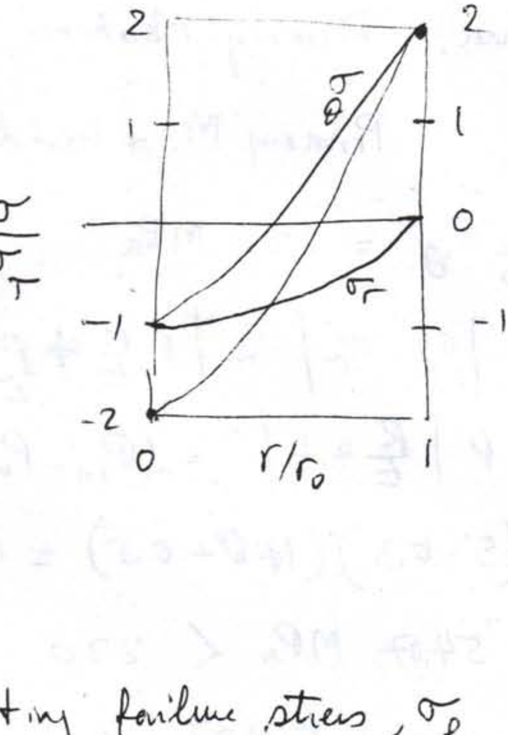
$$\sigma_r^e = \sigma_{r_0}^e + \frac{E\alpha q''' r_0^2}{16k(1-\nu)}\left(\left(\frac{r}{r_0}\right)^2 - 1\right) \quad \text{—} (1)$$

where r_0 is the pellet outer diameter

$$\text{Also:} \quad \sigma_\theta^e = \sigma_{r_0}^e + \frac{E\alpha q''' r_0^2}{16k(1-\nu)}\left(3\left(\frac{r}{r_0}\right)^2 - 1\right) \quad \text{—} (2)$$

$$\text{let } \sigma_T = \frac{E\alpha q''' r_0^2}{16k(1-\nu)}$$

$$\text{Also, } \sigma_z = -P_g + \sigma_T\left(4\left(\frac{r}{r_0}\right)^2 - 2\right)$$



Then, it is clear that the stress intensity is high near the outer edge & is given by

$$\left|\sigma_\theta^e - \sigma_r^e\right|$$

this value will reach the limiting failure stress, σ_f at $r \gtrsim 0.4 r_0$, for a perfectly plastic outer zone.

$$\therefore \left|\sigma_\theta^e - \sigma_r^e\right|_{r=0.4r_0} = \sigma_T\left[2(0.4)^2\right] = \sigma_f$$

$$\therefore \sigma_T \le \frac{\sigma_f}{0.32} = \frac{300\ \text{MPa}}{0.32} \simeq 937\ \text{MPa}$$

$$\therefore q' = \pi q''' r_o^2 = \frac{16\pi k(1-\nu)}{E\alpha}\sigma_T$$

$$= \frac{16(3.14)(\underset{kW/m}{20})(0.7)}{210\times10^9\times10\times10^{-6}}\ 93\cancel{7}\times10^6$$

$$q' = 335\ kW/m$$

(1.2) Since $R/t = 2\cancel{2}/2 = 1\cancel{1}.\cancel{0} > 10$, the thin shell approximation can be used.

ASME criteria: Primary Membrane $P_m < S_m$

Primary M. + Secondary $P_m + Q < 3S'_m$

$$S'_m = \frac{2}{3}\sigma_y = 220\ MPa$$

For $P_m = |\sigma_\theta - \sigma_r| = \left|P\frac{R}{t} + \frac{P}{2}\right|$

$$= P\left|\frac{R}{t}+\frac{1}{2}\right| = (P_{in} - P_{out})\left(\frac{R}{t}+\frac{1}{2}\right)$$

$$= (5-0.3)(1\cancel{1}.\cancel{0}+0.5) = 4.7\times11.5$$

$$= 54.07\ MPa < 220\ ,\ \text{passes ASME}$$

For $P_m + Q = 51.7 + \frac{E\alpha\Delta T}{2(1-\nu)}$ for plain stress ; $\Delta T = \frac{q't}{2\pi r_i k}$

$$= 54.07 + \frac{\cancel{210}\times10^9\times16\times10^{-6}}{2(0.7)}\ \frac{45\times10^3\times2\times10^{-3}}{(2)(3.1415)(\cancel{22}\times10^{-3})(20\times10^3)}\ \frac{1}{10^6}$$

$$= 54.07 + \cancel{26}.05\times10^{-3}\ MPa \ll 660\ MPa$$

Q2 We shall use thin shell approach $\frac{R}{t} = \frac{400}{5} = 80$

Due to the external constraint $u = 0$; $\epsilon_\theta = \frac{u}{R} = 0$

but $\epsilon_\theta = \frac{1}{E}\left[\sigma_\theta - \nu(\bar{\sigma}_r + \sigma_z)\right]$

$$\therefore \quad \sigma_\theta = \nu(\bar{\sigma}_r + \sigma_z) \qquad \text{——} (1)$$

Now, from axial force balance

$$\sigma_z \cdot 2\pi R t = P_{in}\, \pi R^2$$

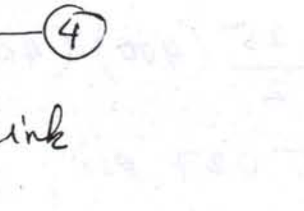
$$\sigma_z = P_{in}\frac{R}{2t} \qquad \text{——} (2)$$

at inner radius $\sigma_r = -P_{in}$

at outer radius $\sigma_r = -P_{out}$

$$\therefore \bar{\sigma}_r = -\frac{P_{in} + P_{out}}{2} \qquad \text{——} (3)$$

Azimuthal force balance



$$\sigma_\theta \cdot 2t = P_{in} \cdot 2R - P_{out} \cdot 2R$$

$$\therefore \sigma_\theta = (P_{in} - P_{out})\frac{R}{t} \qquad \text{——} (4)$$

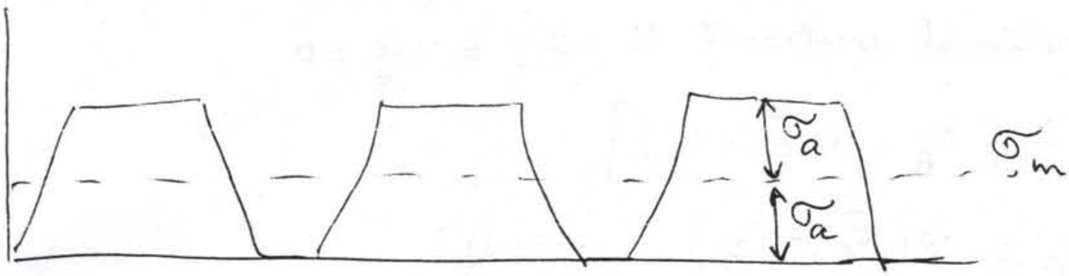
from eqns (1), (2), (3), & (4) we can link

σ_θ, P_{out}, $\bar{\sigma}_r$, σ_z

from (1) & (4) $\quad (P_{in} - P_{out})\frac{R}{t} = \nu\left[-\frac{P_{in}+P_{out}}{2} + P_{in}\frac{R}{2t}\right]$

$$\therefore P_{out}\left[\frac{R}{t} - \frac{\nu}{2}\right] = P_{in}\left[(2-\nu)\frac{R}{2t} - \frac{\nu}{2}\right]$$

Q2, part 2



Using von Mises's effective stress

$$\sigma_{eff} = \frac{1}{2}\left[(\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_3)^2 + (\sigma_3 - \sigma_r)^2\right]^{1/2}$$

Neglecting σ_r as small compared to σ_3 & σ_θ

$$\therefore \sigma_{eff} = \frac{1}{2}\left[\sigma_\theta^2 + (\sigma_\theta - \sigma_3)^2 + \sigma_3^2\right]^{1/2}$$

$$= \frac{1}{2}\left[\frac{v^2}{4}\frac{R^2}{t^2} + (1-v)^2\left(\frac{R}{2t}\right)^2 + \left(\frac{R}{2t}\right)^2\right]^{1/2} P_{in}$$

$$= \frac{1}{2}\left[v^2 + 1 - 2v + v^2 + 1\right]^{1/2} P_{in}\frac{R}{2t}$$

$$= \frac{1}{2}\left[2(1+0.09) - 2(0.3)\right]^{1/2} P_{in}\frac{R}{2t}$$

$$= \frac{1.25}{2}(400)(40) = 10{,}055 \text{ psi}$$

$\therefore \sigma_m = 5{,}027$ psi ; $\sigma_a = 5{,}027$ psi

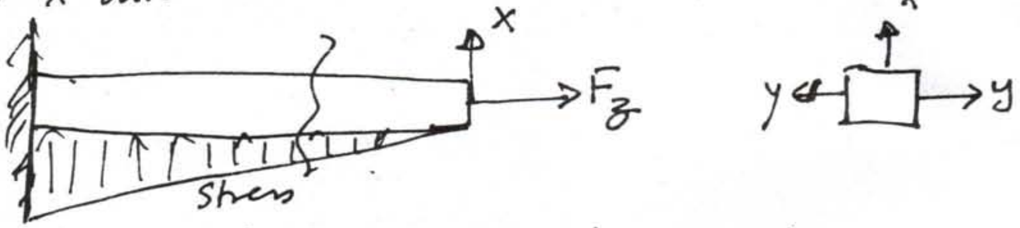
But $\sigma_y = 36$ ksi, $\sigma_N (N = 10^5) = 20$ ksi

$$\therefore \frac{5}{36} + \frac{5}{20} = 0.38 < 1$$

Therefore fatigue will not limit the lifetime as long as it involves 10^5 or so cycles of filling

Q3

Considering the three dimensions and that F_1 is a force in the x directions that will create uniform stresses in the beam.



limit load if $|\sigma_z - \sigma_x|$ or $|\sigma_z - \sigma_y|$ or

$|\sigma_x - \sigma_y|$ reaches σ_y

$\sigma_z = \dfrac{F_z}{A}$ $\qquad \sigma_y = 0 \qquad$ $\sigma_x = \dfrac{F_x}{A}$

Mistake in statement F_1? when $F_2 = 8$ MN

$A = 0.5 \times 0.216 = 0.108 \text{ m}^2$

There are 3 values of F_1 that can lead to limit load condition.

For F_1 being tensile

if $F_1 - F_2 = \sigma_y \cdot A$ $\qquad$ $F_1 = 8 + 345 \times 0.108$
$= 8 + 35.5$
$= 43.5$ MPa

or if $F_1 = \sigma_y A = 35.5$ MPa

If F_1 is compressive

or if $-(-F_1) + F_2 = \sigma_y A$ $\qquad$ $F_1 = \sigma_y A - F_2 = \underline{\underline{+27.5}}$