22.38 - PS #10

<A&T : 8.7, 8.8, 8.15>

8.7) a) $$P(\theta) = \frac{P(E|\theta)P(\theta)}{P(E)}$$

1st evidence:

$$P(\alpha=2) = \frac{\frac{2}{\alpha}\left(1-\frac{1}{\alpha}\right)\ \mathrm{e}\ (.5)}{\sum_{\alpha=2}^{3} \frac{2}{\alpha}\left(1-\frac{1}{\alpha}\right)\ \mathrm{e} \cdot (.5)}$$

$$= \frac{.25}{.25+.111} = .69$$

$$P(\alpha=3) = 1-.69 = .31$$

2nd evidence:

$$P(\alpha=3) = \frac{\frac{2}{3}\left(1-\frac{2}{3}\right)(.31)}{\sum_{\alpha=2}^{3} \frac{2}{\alpha}\left(1-\frac{2}{\alpha}\right) P_\alpha} = 1$$

based on the evidence, $\alpha=3$, because for $\alpha=2$, we cannot obtain the evidence $e=2$.

8.7) b) $f'(\alpha) = 1$

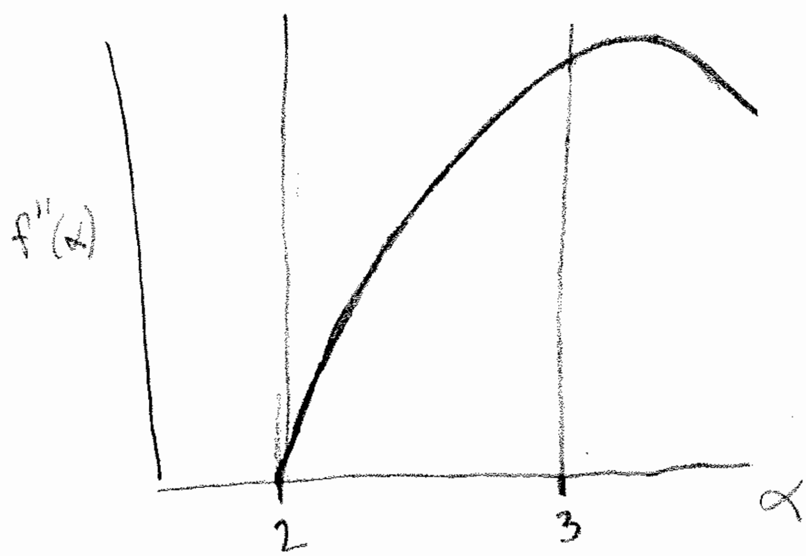
$$f''(\alpha) = K^{-1} L(\alpha) f'(\alpha)$$

$$L(\alpha) = \left[\frac{2}{\alpha}\left(1-\frac{e_1}{\alpha}\right)\right]\cdot\left[\frac{2}{\alpha}\left(1-\frac{e_2}{\alpha}\right)\right]$$

$$= \frac{2}{\alpha^2}\left(1-\frac{1}{\alpha}\right)\left(1-\frac{2}{\alpha}\right)$$

$$K = \int_2^3 \frac{2}{\alpha^2}\left(1-\frac{1}{\alpha}\right)\left(1-\frac{2}{\alpha}\right)\cdot 1 \, d\alpha = \frac{11}{324} = .0339$$

$$\boxed{f''(\alpha) = \frac{60}{\alpha^2}\left(1-\frac{1}{\alpha}\right)\left(1-\frac{2}{\alpha}\right)}$$



$\alpha_{Baysien} = 3$

8.8) Probability survival of proof load distribution : $p = .9 \rightarrow .7$
$p = .5 \rightarrow .25$ (8.1)
$p = .1 \rightarrow .05$

i) a) $f'(p) = \frac{1}{.9} = 1.111$

b) $f''(p) = k \cdot L(p) \cdot f'(p)$

$L(p) = p^3$

$k^{-1} = \int_0^{.9} p^3 (1.111)\, dp = 1.1 \left[3p^2\right]_0^{.9} = 1.1\,(2.43)$

$f''(p) = \frac{p^3}{2.43} = .411 p^3$

c) $p = .9$

ii) a) $f'(p) = \frac{1}{.1} = 10$

b) $L(p) = p^3$; $k^{-1} = \int_{.9}^{1} p^3 (10)\, dp = (10)\, .57$

$f''(p) = \frac{p^3}{.57} = 1.75\, p^3$

c) $p = 1$

iii) a) $f(p)$ is discrete : $p > .9 = .7$; $p < .9 = .3$

b) $P(p) = \frac{P(E|p)P(p)}{P(E)}$

$p > .9$: $P(E|p)$: $\frac{\int_{.9}^{1} p^3\, dp}{\int_0^1 p^3\, dp} = .19$; $p < .9$ $P(E|p) = \frac{2.43}{3} = .81$

$P(p > .9) = (.19)(.7) / (.19)(.7) + (.81)(.3) = \underline{.722}$

$P(p < .9) = 1 - P(p > .9) = \underline{.28}$

c) $p > .9$ is the estimated p

8.15) $\mu_\lambda = .5$; $COV = .2 \Rightarrow \sigma_\lambda = .1$; $f_x(x) = \lambda e^{-\lambda x}$

failures @ 12 & 18 months

the conjugate distribution of an exponential distribution (as given for $f(x)$) is a gamma distribution:

$$f_\Lambda(\lambda) = \frac{\nu(\nu\lambda)^{k-1} e^{-\nu\lambda}}{\Gamma(k)}$$

where $E(\lambda) = \frac{k}{\nu}$; $Var(\lambda) = \frac{k}{\nu^2}$

finding k' & ν' : $.5 = \frac{k'}{\nu'}$; $(.1)^2 = \frac{k'}{\nu'^2}$ $\Rightarrow$ $k' = 25$, $\nu' = 50$

for the posterior distribution:

$$k'' = k' + n = 25 + 2 = 27$$

$$\nu'' = \nu' + \Sigma x_i = 50 + 12 + 18 = 80$$

updated mean & COV:

$$\mu''_\lambda = \frac{k''}{\nu''} = \frac{27}{80} = .34$$

$$COV'' = \frac{\sqrt{var(\lambda)}}{\mu''_\lambda} = \frac{.34\sqrt{k''}}{\nu''} = .022$$