22.38: Solutions PS#2

10/4/05

① +20

a) what is the probability that each child will get an A?

+5 $P(T \cdot J \cdot S) = (.5)(.4)(.3) = 0.06$

+5 b) if two students were to recieve grades of A, what is the probability Jim will be among them?

$$P(H_1/E) = P(E/H_1)\,P(H_1) \Big/ \sum P(E/H_1) \cdot P(H_1) = P(E/H_1)\,P(H_1) \Big/ P(E)$$

Evidence: 2 As

H_1 = Jim gets an A ; H_2 = Jim does not get an A

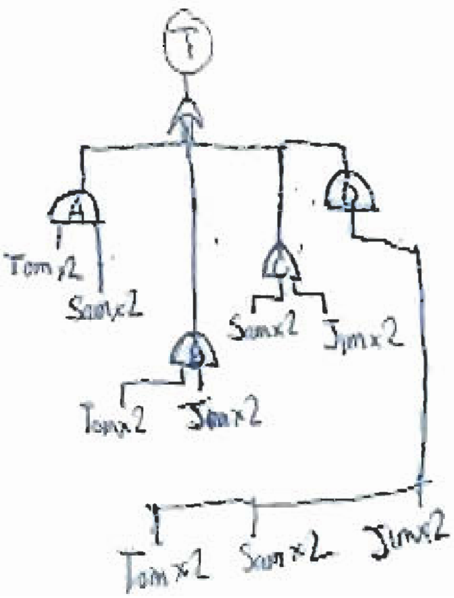
$$P(E/H_1) = \underbrace{(.5)(.7)}_{\text{Tom gets A, but not Sam}} + \underbrace{(.5)(.3)}_{\text{Sam gets A, but not Tom}} = .5$$

$P(H_1) = 0.4$

$$P(E) = \sum_1^2 P(E/H_1) = .4(.5) + \underbrace{\left([(.5)(.3)]\right)}_{\text{Sam and Tom get As}} .6 = .29$$

$P(H_1/E) = .5(.4)/.29 = 68.97\%$

+5c) what is structure function of success? T = success of system



$$Y_T = 1 - (1-Y_A)(1-Y_B)(1-Y_C)(1-Y_D)$$

$$= 1 - (1 - P_T P_J (1-P_S))(1 - P_T P_S (1-P_J))(1 - P_S P_J (1-P_T))(1 - P_S P_T P_J)$$

where P_T = prob. Tom gets 2 successive A's $= .5^2 = .25$
P_S = Prob. Sam " " $= .3^2 = .09$
P_J = Prob. Jim " " $= .4^2 = .16$

$P(T) = 6.8\%$

Note: There is an alternate interpretation to the question (~~any~~ 2As on 2 exams by any 2 children, not the same 2 children);
then:

$$Y_T = [P_T P_J (1-P_S) + P_T P_S (1-P_S) + P_S P_J (1-P_T) + P_J P_T P_S]^2 = .1225$$

where P_i = Prob. of A on 1 exam.

(2.) $Y_T = Y_E Y_C Y_{CO} (Y_{Train})$

$= Y_E Y_C Y_{CO} (1 - (1 - Y_{Train\,1})(1 - Y_{Train\,2}))$

$= Y_E Y_C Y_{CO} (1 - (1 - Y_{P_1} Y_{T_1} Y_{V_1})(1 - Y_{P_2} Y_{T_2} Y_{V_2}))$

Y: 1 = success
0 = failure

Ang and Tang - 2.2

a) sample space

129	14579	
12579		
12589	14589	14689
13579	13589	

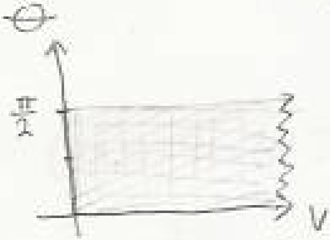
assuming no "backtracking", else there would be an ∞ number of routes

b) $1 \to 2 \to 9$: $(1,3)$; $(1,4)$; $(1,5)$; $(2,3)$; $(2,4)$; $(2,5)$

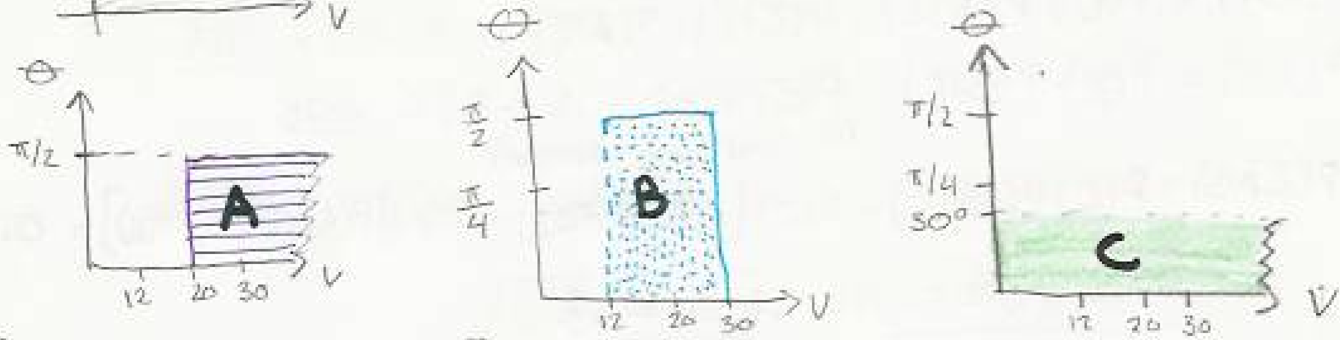
$1 \to 4 \to 6 \to 8 \to 9$: $(1.5, 1.5, .5, 2)$; $(1.5, 1.5, .5, 3)$; $(1.5, 2.5, .5, 2)$
$(1.5, 2.5, .5, 3)$; $(2, 1.5, .5, 2)$; $(2, 1.5, .5, 3)$
$(2, 2.5, .5, 2)$; $(2, 2.5, .5, 3)$

Ang & Tang - 2.5

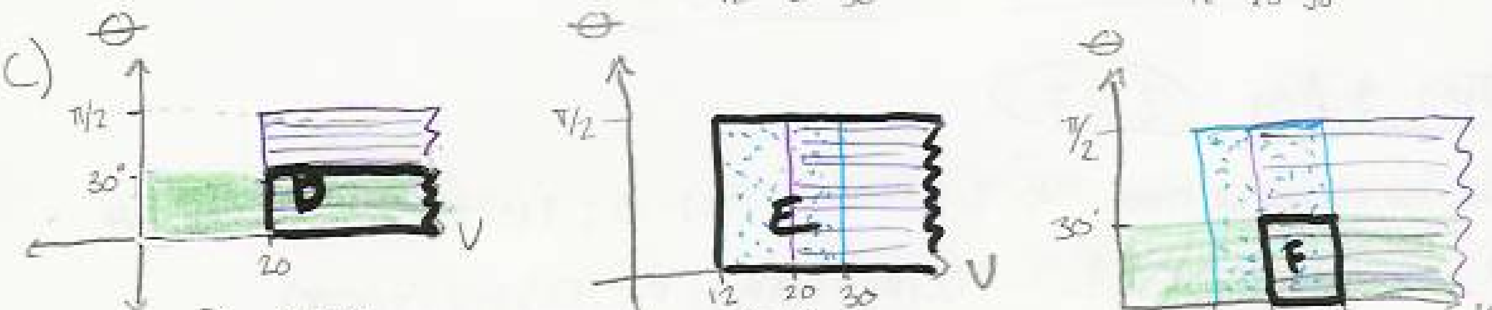
a)



b)



c)



$D = A \cap C$

$E = A \cup B$

$F = A \cap B \cap C$

d) D and E are not mutually exclusive, either is A and C

Ang and Tang 2.9

a) S: $\bar{H} \cap \bar{E}_1 \cap \bar{E}_2$ U: $\overline{S \cup M}$

m: $(H \cap E_1 \cap \bar{E}_2) \cup (H \cap \bar{E}_1 \cap E_2)$

b) $.01(.98) + .02(.99) = P(1 \text{ fail in a week}) = 0.0296$

c) $P(S) = (.9)(.99)(.98) = 0.87318$

$P(m) = (.1)(.99)(.02) + (.1)(.01)(.98) = 0.00296$

$P(U) = 1 - P(S) - P(m) = 0.12386$

Ang & Tang - 2.18

$P(E_1 / E_2) = P(E_1 E_2) / P(E_2)$

Probability of water shortage: $P(WS) = P(\bar{C}A) + P(\bar{C}B) - P(\bar{C}AB)$

- $P(\bar{C}A) = P(\bar{C}) P(A/\bar{C}) = P(\bar{C})(1 - P(\bar{A}/\bar{C})) = .5(.25) = \underline{.125}$
- $P(\bar{C}B) = P(\bar{C}) P(B/\bar{C}) = P(\bar{C}) P(B) = .5(.15) = \underline{.075}$

b/c C and B are independent

- $P(\bar{C}AB) = P(\bar{C}) P(AB/\bar{C}) = P(\bar{C}) [P(AB)/P(\bar{C})] = P(\bar{C}) [P(A) P(B/A) .5] = 0.25$

$$\boxed{P(WS) = .125 + .075 - .025 = .175}$$

Tang & Ang 2.27

$\Rightarrow$ from problem we know: $P(A) = .2$, $P(B) = .1$; $P(C) = .5$; $P(B/\bar{A}) = .04$; $P(C/\overline{AB}) = .4$

a) $P(\text{no parking}) = P(\bar{A}\bar{B}\bar{C}) = P(\bar{C}/\overline{AB}) P(\overline{AB}) = P(\bar{C}/\overline{AB}) P(\bar{A}) P(\bar{B}/\bar{A})$

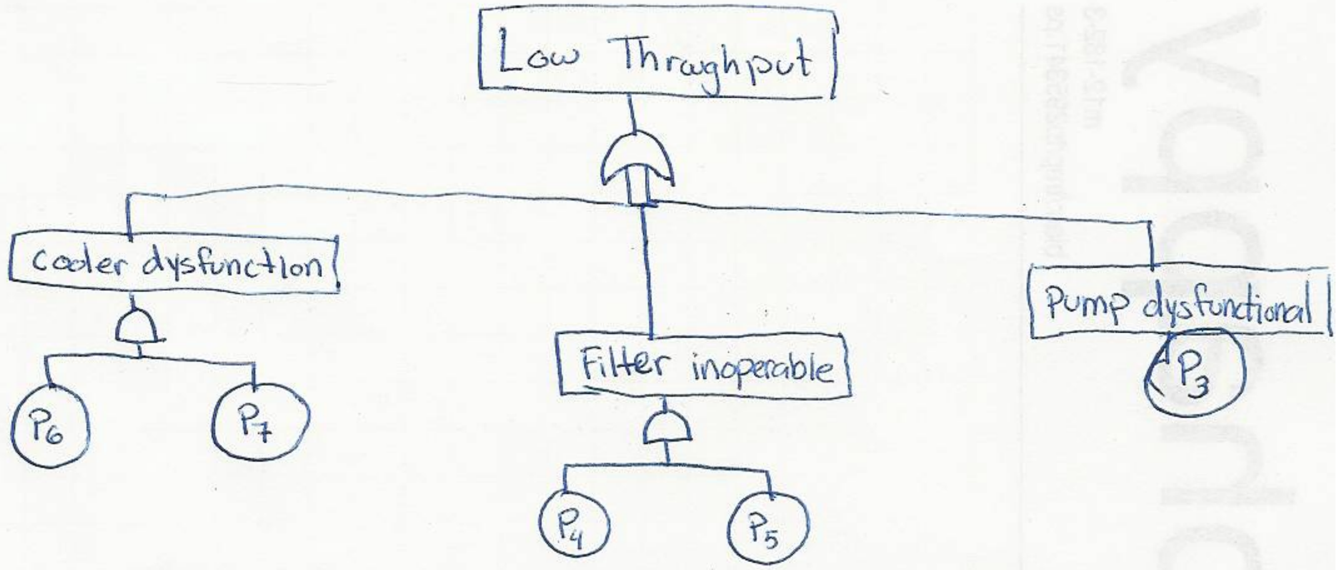
$= (1-.4)(1-.2)(1-.04) = 0.4608$

$P(\text{no free parking}) = P(\overline{AB}) = P(\bar{A}) P(\bar{B}/\bar{A}) = (1-.2)(1-.04) = .768$

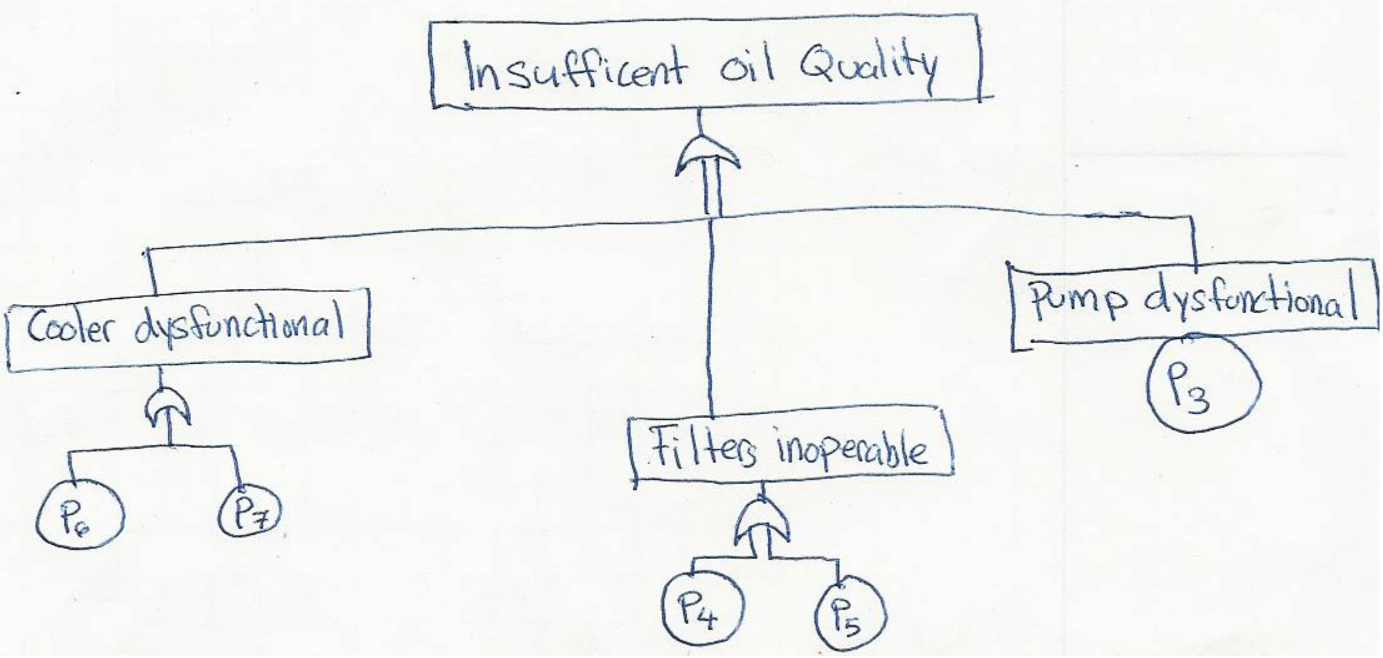
b) $P(\text{park}) = 1 - P(\text{no parking}) = 1 - .4608 = .5392$

c) $P(\text{Free} \mid \text{Parked}) = \dfrac{P(\text{free and parked})}{P(\text{parked})} = \dfrac{(1 - P(\overline{AB}))}{P(\text{park})} = \dfrac{(1-.768)}{.539} = .430$

a)



b)



Rausand and Hoyland 3.4

a) MPS:

~~1 AND (2 AND 3 AND~~

$$\left.\begin{array}{l} P_1 : 1\cdot 2\cdot 3\cdot 4 \\ P_2 : 1\cdot 2\cdot 5 \\ P_3 : 1\cdot 6 \end{array}\right\} P$$

Rausand and Hoyland 3.17

MCS:

$$\left.\begin{array}{l} K_1 : 1 \\ K_2 : 2 \text{ AND } 6 \\ K_3 : 3\cdot 5\cdot 6 \\ K_4 : 4\cdot 5\cdot 6 \end{array}\right\} K$$

b) X_T = system failure

$X = 1$ = failure
$= 0$ = success

$$X_T = 1-(1-X_1)(1-X_n)$$

$$X_n = (X_6)(1-(1-X_2)(1-X_5))(1-(1-X_2)(1-X_3)(1-X_4))$$

$$X_T = 1-(1-X_1)(1-((X_6)(1-(1-X_2)(1-X_5))(1-(1-X_2)(1-X_3)(1-X_4)))$$

(note $X_N^2 = X_N$)

$$= 1-(1-X_1)(X_6)[X_5X_3 + X_5X_4 + X_5X_2 - X_5X_3X_4 - X_5X_2X_4 - X_5X_2X_3 + X_5X_2X_3X_4 + X_2X_3 + X_2X_4 + X_2 - X_2X_3X_4 - X_2X_4 - X_2X_3 + X_2X_3X_4 - X_2X_5X_3 - X_2X_5X_4 - X_2X_5 + X_2X_3X_4X_5 + X_2X_4X_5 + X_2X_3X_5 - X_2X_3X_4X_5]$$

$$= 1-(1-X_1)(X_6)[X_2X_4 + X_2 + X_5X_3 + X_5X_4 - X_5X_3X_4 - X_5X_2X_4 - X_5X_2X_3 + X_2X_3X_4X_5]$$

$$= 1 - X_6X_2X_4 - X_2X_6 - X_6X_5X_3 - X_6X_5X_4 + X_6X_5X_3X_4 + X_6X_5X_2X_4 + X_6X_5X_2X_3 - X_6X_2X_3X_4X_5 + X_1X_2X_4X_6 + X_1X_2X_6 + X_1X_5X_3X_6 + X_1X_5X_4X_6 - X_1X_5X_3X_4X_6 - X_1X_5X_2X_4X_6 - X_1X_5X_2X_3X_6 + X_1X_2X_3X_4X_5X_6$$

if all have same failure probability: $P(X_i) = 1 - p^2 - 2p^3 + 6p^4 - 4p^5 + p^6$

For $Y_t = 1$ for success, $=0$ for failure, then:

$Y_t = Y_1[\ 1 - (1 - Y_1\ Y_2\ Y_3\ Y_4)\ (\ 1 – Y_2\ Y_5)\ (\ 1 – Y_6)]$