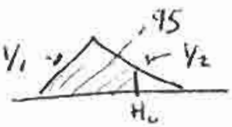
22.38 - PS#4 Solutions

3-31) Geometric distribution: $20 = \sum_{i=1}^{\infty} i p^{(i-1)} q = \frac{1}{q}$

a)

$\Rightarrow p = .95$



$Y_1 = X-5$, $Y_2 = -X+7$

$.95 = P(H<H_0) = \int_5^6 Y_1\,dx + \int_6^{x'} Y_2\,dx$

$\Rightarrow .95 = .5 + \frac{6^2}{2} - \frac{x^2}{2} + 7x - 42$

$\Rightarrow x = h_{20} = \underline{6.68 \text{ meters}}$

b) $P(\geq 1 \text{ flood}) = 1 - P(0)$ $\Rightarrow$ assume a Binomial Distribution:

$P(\geq 1) = 1 - (.95)^{20} = \underline{.6415}$

d) $P(\leq 2) = P(0) + P(1) + P(2)$

$$\left.\begin{aligned} P(1) &= \binom{5}{1}(.05)^1(.95)^4 = \underline{.204} \\ P(2) &= \binom{5}{2}(.05)^2(.95)^3 = \underline{.021} \\ P(0) &= (.95)^5 = \underline{.774} \end{aligned}\right\} \sum_{i=0}^{2} = \underline{.9994}$$

c) from d, $P(1) = .204$

3-34)

a) $P(F_A) = P(F_I) + P(F_{II}) - P(F_I F_{II}) = \frac{1}{20} + \frac{1}{10} - \frac{1}{200} = \underline{.145}$

b) $P(F_T) = \sum_i P(F_i) - \sum_i \sum_j P(F_j F_i) + \sum_i \sum_j \sum_k P(F_j F_i F_k)$

$= P(F_I) + P(F_{II}) + P(F_{III}) - [P(F_I F_{II}) + P(F_I F_{III}) + P(F_{II} F_{III})] + P(F_I F_{II} F_{III})$

$= .145 + .04 - (\frac{1}{25})(\frac{1}{20}) - (\frac{1}{25})(\frac{1}{10}) + (\frac{1}{10})(\frac{1}{20})(\frac{1}{25})$

$= \underline{.1792}$

c) $P(\text{no flood in 4 yrs}) = (1 - P(F_T))^4 = (1 - .1792)^4 = \underline{.4539}$

3-38) $\lambda = \frac{2}{5} = .4$ earthquakes / year average

a) $P(1 \text{ in } 3 \text{ yr}) = \frac{(.4(3))^1}{1!} e^{-.4(3)} = \underline{.361}$

b) $P(0 \text{ in } 3) = e^{-.4(3)} = \underline{.301}$

c) $P(\leq 2) = \sum_{x=0}^{2} \frac{(.4)^x}{x!} e^{-.4} = \underline{.9921}$

d) $P(\geq 1) = 1 - P(0 \text{ in } 5) = 1 - e^{-.4(5)} = \underline{.86466}$

3-41) $P(\text{strike}) = 1/3 = .33$ per year ; $\mu = 15$ days $\sigma = 5$ days

a) given a strike, $E(X_\$) = 15(10{,}000) = \$\underline{150{,}000}$

b) $P(X_\$ \geq \$20{,}000) = P(X_d \geq 2 \text{ days}) = 1 - \Phi(3)$

$S = 2-15/5 = -2.6 \implies \Phi(2.6) = 1 - \Phi(-2.6) = \underline{.995339}$

c) expected loss = cost/day · # days expected

days : $1/3\,(15) \cdot 2 = 10$ days $\implies$ Expected Loss $= 10(10{,}000) = \underline{\$100{,}000}$

3-48) $\nu = \frac{2}{50} = .04$ damaging (>5) earthquakes / year

a) $P(x<3 \text{ in } 20 \text{ yrs}) = P(0) + P(1) + P(2)$

$$= e^{-.04(20)}\left(1 + \frac{.04(20)}{1} + \frac{(.04(20))^2}{2}\right) = \underline{.952}$$

b) $P(\text{survival}) = P(n=0) + .8\,(P(n=1)) + .2\,(P(n=2))$

$$= e^{-.04(20)}\left(1 + .8\,(.04(20)) + .2\left(\frac{(.04(20))^2}{2}\right)\right) = \underline{.7650}$$

c) $P(\text{fail}) = P(\text{fail}_{EQ}) + P(\text{fail}_{\text{Tornado}}) - P(\text{fail}_{EQ})P(\text{fail}_{\text{Tornado}})$

$$= (1-.765) + (1-e^{-\frac{20}{200}}) - (1-.765)(1-e^{-20/200}) = \underline{.308}$$

3-50) $\mu = 500\text{k}$ gal/day ; $\sigma = 150\text{k}$ gpd

a) $P(\text{water shortage}) = P(\text{Consumption} > 600\text{k})\,P(600\text{k}) + P(\text{Consumption} > 750\text{k})\,P(750)$

$P(c<600\text{k})$: $s = 600-500/150 = .667 \Rightarrow \Phi(.667) = .748572$

$P(c<750\text{k})$: $s = 750-500/150 = 1.67 \Rightarrow \Phi(1.67) = .952540$

$P(c>600\text{k}) = 1 - P(c<600\text{k}) \simeq .25$

$P(c>750\text{k}) = 1 - P(c<750\text{k}) \simeq .0475$

$P(\text{shortage}) = .7(.25) + .3(.0475) = \underline{.1893}$

b) $P(\text{shortage in 1 week}) = 1 - P(\text{no shortage in 1 week})$

$= 1 - \binom{7}{0} p^0 (1-p)^7$ where $p = .1893$

$= \underline{.7698}$

c) $E(X) = \frac{1}{p} = (.189)^{-1} = 5.26$ days = MTTF

if Poisson; $\lambda = .1893$ shortage / day

$$P(\text{shortage}) = 1 - \frac{((.1893)(7))^0}{0!} e^{-.1893(7)} = \underline{.7342}$$

note, the outcome is very similar to the outcome in B using the binomial distribution, even though n is only 7.

3-50) continued

d) $\phi(s) = .99 \Rightarrow s = 2.329$

$2.325 = \frac{x - 500}{150} \Rightarrow x \simeq \underline{850,000 \text{ gpd}}$

3-59) 200 tendons $\longrightarrow$ 20 corroded tendons

- $p(0 \text{ corroded tendons}) = \binom{10}{0} p^0 (1-p)^{10} = .9^{10} = \underline{.349}$
- $p(\geq 1 \text{ corroded tendon}) = 1 - P(0) = 1 - .349 = \underline{.651}$