10.44J, 22.52J, 8.575J **Statistical Thermodynamics of Complex Liquids** (Spring 2004)
Quiz 1 (Part II, Prof. Chen)   Close book (2 hr)

1. This question is to ask you why hydrogen atom has an exceptionally large incoherent scattering cross section but relatively small coherent scattering cross section, while deuterium atom has similar magnitudes of incoherent and coherent scattering cross sections. You can answer this question by actually computing the cross sections according to the following definitions:

$$\sigma_{coh} = 4\pi \bar{b}^2$$
$$\sigma_{inc} = 4\pi\left(\overline{b^2} - \bar{b}^2\right) \qquad , \qquad (1)$$
$$\sigma_{sc} = 4\pi\bar{b}^2 + 4\pi\left(\overline{b^2} - \bar{b}^2\right) = 4\pi\overline{b^2}$$

given that

(A) (8%) for a H^1 atom, I=1/2, $b_+ = 1.03 \times 10^{-12}$ cm, and $b_- = $ -4.74×10^{-12} cm,

(B) (7%) but for a H^2 atom, I=1, $b_+ = 0.95$ x 10^{-12} cm and $b_- = 0.10$ x 10^{-12} cm.

2. Given that the particle structure factor for a randomly oriented cylinder with a cross-sectional radius R and length L is :

$$\bar{P}(Q) = \frac{1}{2}\int_{-1}^{1} d\mu \left[\frac{\sin(QL\mu/2)}{QL\mu/2}\right]^2 \left[\frac{2J_1(QR\sqrt{1-\mu^2})}{QR\sqrt{1-\mu^2}}\right]^2 , \qquad (2)$$

(A) (7%) make an argument (mathematical or physical), that when the particle is a long and thin rod, one has an asymptotic form,

$$\bar{P}(Q) \underset{QL>2\pi}{\to} \frac{\pi}{QL}\left[\frac{2J_1(QR)}{QR}\right]^2 \underset{QR<1}{\to} \frac{\pi}{QL}e^{-\frac{1}{4}Q^2R^2} . \qquad (3)$$

(B) (8%) Assuming that one has a dilute solution of mono-dispersed rod-like micelles, give a complete expression for the scattering intensity I(Q) (in unit of cm^{-1}). Show that a Guinier plot of $\ln[QI(Q)]$ vs Q^2 will yield a straight line, from the slope and the zero intercept of which one can extract respectively the radius and the aggregation number per unit length of the micelle.

3. It was shown in Chapter III that the scattering intensity from an isotropic porous material (a two-phase system) can be written succinctly as:

$$I(Q) = \langle \eta^2 \rangle \int_0^\infty dr \, 4\pi r^2 \frac{\sin Qr}{Qr} \Gamma(r), \tag{4}$$

where $\langle \eta^2 \rangle = (\Delta\rho)^2 \Phi_1 \Phi_2$, $\Delta\rho$ is the difference in the sld's of the two phases, Φ_1, Φ_2 their respective volume fractions and $\Gamma(r)$ the normalized Debye correlation. The Debye correlation function has the following three boundary conditions:

$$\Gamma(r = 0) = 1, \ \Gamma(r \to \infty) = 0. \tag{5}$$

$$\Gamma'(r = 0) = -\frac{1}{4\Phi_1(1 - \Phi_1)} \frac{S}{V}. \tag{6}$$

(A).(10%) Now if the microstructure (morphology) of the medium is characterized by a unique length scale ξ, then the Debye correlation function must be of the form $\Gamma\left(\dfrac{r}{\xi}\right)$. Show that under this condition the scaled

intensity distribution $\dfrac{I(Q)}{\langle \eta^2 \rangle \xi^3}$ $\qquad$ (7)

is a function of the dimensionless scaling parameter $y = Q\xi$ only. This implies that for a bunch of porous materials with different ξ, if we plot the scaled intensity distributions as a function of the scaling parameter y, they will all collapse into a single master curve.

(B).(5%)Calculate the intensity I(Q) explicitly for the case of the Debye model which assumes that $\Gamma(r) = e^{-\frac{r}{\xi}}$, and show the validity of the above theorem by calculating the scaled intensity.

You need the formula $\displaystyle\int_0^\infty 4\pi x^2 \frac{\sin xy}{xy} e^{-x} dx = \frac{8\pi}{\left(1 + y^2\right)^2}.$ $\qquad$ (8)

(C)(5%) Show that in this case the unique length scale ξ in the system is given by

$$\xi = 4\varphi_1\varphi_2\left(\frac{S}{V}\right)^{-1}. \tag{9}$$

where S/V is the surface to volume ratio of the porous material.