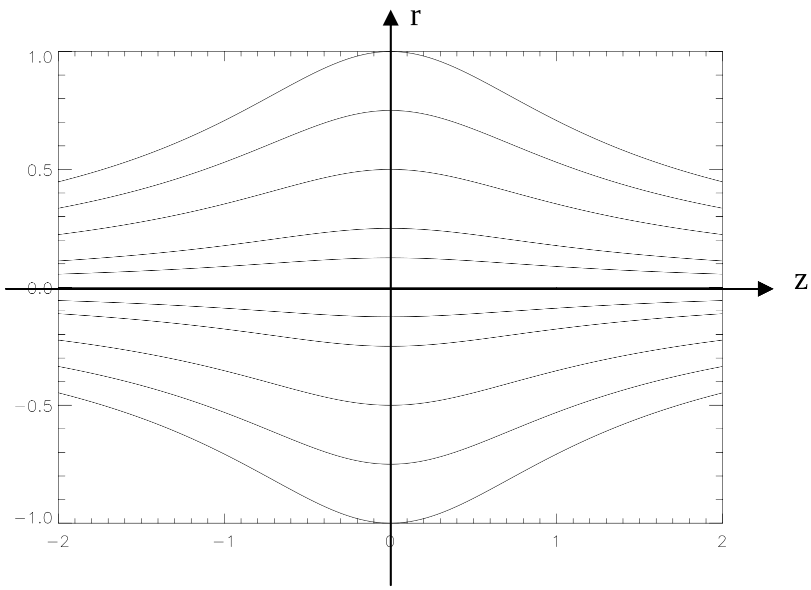
Massachusetts Institute of Technology

Subject 6.651J/8.613J/22.611J R. Parker
19 September 2006 Due: 26 September 2006

Problem Set 2

Problem 1.

The figure shows field lines in a cylindrically symmetric magnetic mirror plasma confinement device.



The field is given by

$$\vec{B} = B_0(1+\frac{z^2}{\ell^2})\hat{z} - B_0\frac{rz}{\ell^2}\hat{r}$$

a) A particle orbits in the mirror with its guiding center moving along the z-axis. At z=0, it has total energy $W_0 = W_{\perp 0} + W_{//0}$ where the subscripts $\perp$ and // refer to perpendicular and parallel to the field. Calculate $z(t)$, the motion of the guiding center.

b) Calculate the grad B and curvature components of the drift velocity of the particle when its guiding center is located at

$$z = 0, r = \ell/2$$

c) Find an equation for z_t, the turning point of a particle with a guiding center that passes through the point $z = 0, r = \ell/2$ with energy $W_0 = W_{\perp 0} + W_{//0}$. **Do not solve the equation.** (Hint: this will require calculation of the equation of the field lines.)

Problem 2.

The earth's dipole magnetic field is given by

$$\vec{B} = 3x10^{-5}\,\frac{R_e^3}{r^3}\left(-2\sin\theta\hat{r} + \cos\theta\hat{\theta}\right)$$

where R_e is the radius of the earth, approximately 6400 km. Here θ is the latitude and r is the distance from the earth's center. The actual field in the vicinity of the earth is more complicated due to interaction with the interplanetary magnetic field and solar wind, but this complication will be ignored for the purpose of this problem.

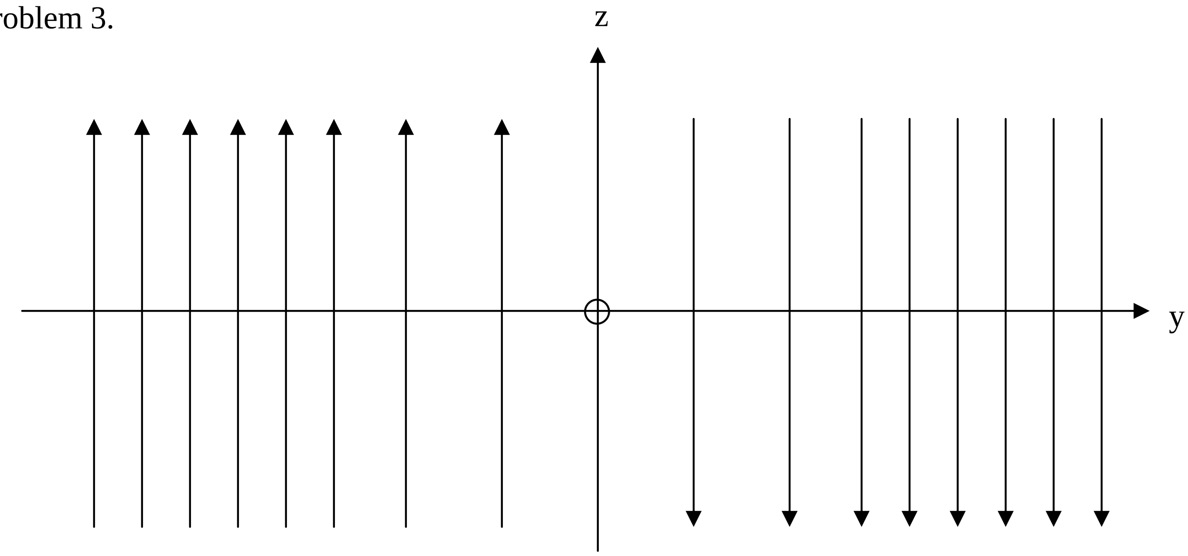
i) The magnetic field lines are found by solving the differential equation $\frac{dr}{rd\theta} = \frac{B_r}{B_\theta}$. Carry out the solution to this ODE to find the equation of the field lines in the form $r = r(\theta)$.

ii) Let B_e be the value of B on the equator ($\theta = 0$) at some r. Determine B in terms of B_e and θ, where B is the magnitude of $\vec{B}$ that one would measure as one followed a field line from the equator.

iii) A particle is injected into the earth's field at some radius R_0 at the equator. The particle has energy W_0, with $W_\perp = \frac{2}{3}W_0$ and $W_\| = \frac{1}{3}W_0$. Calculate the latitude θ_{ref} at which the particle is reflected. Assume θ_{ref} is small enough that small argument expansions to lowest non-zero order in θ can be used for all trigonometric terms.

iv) Using the particle's drift velocity at the equator as characteristic of the particle's drift velocity as it undergoes oscillation in latitude, estimate the time required for the particle to circle the earth. Evaluate numerically for a proton with W_0 = 1 Mev injected at r =2R_e. Is r_L small enough for our derivation of the ∇B and curvature drifts to be valid?

Problem 3.



The figure illustrates a sheared magnetic field given by

$$\vec{B} = \begin{Bmatrix} B_0 \hat{z} & y < -a \\ B_0 \dfrac{y}{a} \hat{z} & -a < y < a \\ -B_0 \hat{z} & y > a \end{Bmatrix},$$

where $\pm a$ defines the sheared region and B_0 is the constant field amplitude outside the sheared region. Such field configurations occur both in laboratory as well as in terrestrial and astrophysical plasmas.

i) Consider first a particle whose guiding center is located at $x = 0$, $y = a/2$ at $t = 0$. Describe its motion assuming $r_L << a$. What constraint does this condition imply for the particle's $W_\perp$?

ii) Consider now a particle whose position is located at $x = 0$, $y = 0$ with velocity $v_y = \text{V}$, $v_x = 0$, all at t=0. Derive the differential equations describing its motion.

iii) By solving these equations, determine the maximum extent of the particle's orbit in the y-direction. Assume that the parameters are such that the particle does not reach the region of uniform field. What condition assures that this will be a valid assumption?