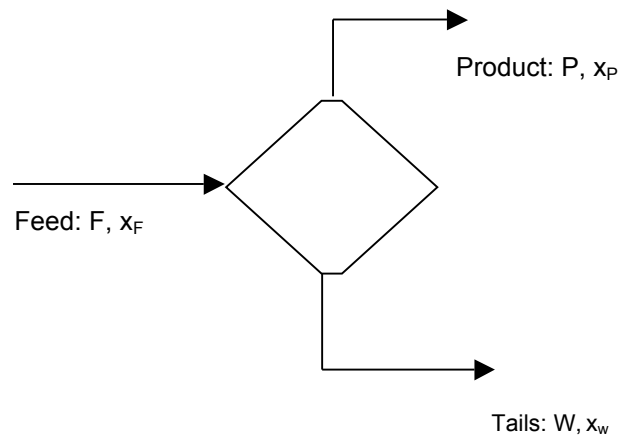


ClassnoteAdditional Note on Uranium Enrichment and Separative Work

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Enrichment is one of the biggest cost components in the nuclear fuel cycle. The basic task involves separating natural uranium (enriched to 0.711% in fissile U-235) into two streams, one enriched in the fissile isotope, and the other depleted in it.

The relationship between the three streams is given by the material balance equations for the enrichment plant.



where the  $F$ ,  $P$ , and  $W$  are expressed as the mass of U in the feed, product, and tails, respectively, and the  $x_i$  are the weight fractions of U-235 in the three streams.

These six variables are related by two material balance equations:

$$\text{On U: } F = P + W$$

$$\text{On U-235: } Fx_F = Px_P + Wx_W$$

In these equations, P and  $x_P$  are determined by the in-core fuel management scheme;  $x_F$  is given by the U-235 content of natural uranium; and  $x_W$  is set to optimize enrichment plant operations (see below). Thus we have two equations in two unknowns. Solving for F, we get that:

$$F = P \frac{(x_P - x_W)}{(x_F - x_W)}$$

and for  $x_P = 0.03$ ,  $x_F = 0.00711$ ,  $x_W = 0.002$ ,  $F/P = 5.48$

### What is the cost of a given uranium enrichment task?

The answer obviously depends on what enrichment process is being used. For each technology, the cost depends on the magnitude of the separation task. Intuitively we expect that the bigger the gap between  $x_P$  and  $x_W$  the bigger the separation task. Also, the size of the task will depend on the amount of material, F, that is processed.

It turns out that for many enrichment technologies, it is useful to express the magnitude of the separation task in terms of the separative work, S, where S is given by:

$$S = P(2x_P - 1) \ln \frac{x_P}{1 - x_P} + W(2x_W - 1) \ln \frac{x_W}{1 - x_W} - F(2x_F - 1) \ln \frac{x_F}{1 - x_F}$$

(It can be shown that S is proportional to the total volume of material flowing within an ideal cascade producing this amount of separation, and this, in turn, is proportional to the inputs of both energy and capital necessary to perform the separation task. Thus the total cost of the separation task is directly proportional to S.)

The units of this measure of work are mass units. Thus, for example, we speak of :

- “an enrichment task requiring 100kg of separative work”
- “an enrichment plant with a capacity of 1 million kg of separative work per year”

A kilogram of separative work is also known as a separative work unit.

The total cost of an enrichment task =  $C_S \times S$

Where:

$C_S$  = cost per kilogram of separative work  
 $S$  = total number of kilograms of separative work required

Example:

How much separative work is required to produce 1 kg of uranium enriched to 3% in U-235 ( $x_P=0.03$ ) from natural uranium ( $x_F=0.00711$ ) in a plant operating with a tails 'assay' of  $x_W=0.002$ ? How much would it cost to produce 20 MT of 3% enriched uranium in an enrichment plant for which the cost of separative work is \$100 per kilogram SW?

From the above equation we can write:

$$\frac{S}{P} = (2x_P - 1) \ln \frac{x_P}{1 - x_P} + \frac{W}{P} (2x_W - 1) \ln \frac{x_W}{1 - x_W} - \frac{F}{P} (2x_F - 1) \ln \frac{x_F}{1 - x_F}$$

and substituting for W/P and F/P we have

$$\frac{S}{P} = (2x_P - 1) \ln \frac{x_P}{1 - x_P} + \frac{x_P - x_F}{x_F - x_W} (2x_W - 1) \ln \frac{x_W}{1 - x_W} - \frac{x_P - x_W}{x_F - x_W} (2x_F - 1) \ln \frac{x_F}{1 - x_F}$$

And for  $x_P=0.03$ ,  $x_F=0.00711$ , and  $x_W=0.002$

S/P ~ 4.4 kg separative work/kg of product

The cost of producing 20 MT of 3% enriched uranium

= 20,000 (kg product U) x (S/P) (kg SW/kg product) x 100 (\$/kg SW)

= \$8.8 million.

Example

How much separative work is required to produce 1 kg of uranium enriched to 4.51% in U-235 ( $x_P=0.0451$ ) from natural uranium ( $x_F=0.00711$ ) in a plant operating with a tails 'assay' of  $x_W=0.003$ ?

From the previous equation we can write:

$$\frac{S}{P} = (2x_p - 1) \ln \frac{x_p}{1 - x_p} + \frac{W}{P} (2x_w - 1) \ln \frac{x_w}{1 - x_w} - \frac{F}{P} (2x_F - 1) \ln \frac{x_F}{1 - x_F}$$

and substituting for W/P and F/P we have

$$\frac{S}{P} = (2x_p - 1) \ln \frac{x_p}{1 - x_p} + \frac{x_p - x_F}{x_F - x_W} (2x_w - 1) \ln \frac{x_w}{1 - x_w} - \frac{x_p - x_W}{x_F - x_W} (2x_F - 1) \ln \frac{x_F}{1 - x_F}$$

$$= 2.777 + 53.326 - 49.857$$

$$= \underline{6.25 \text{ kg of separative work/per kg of product}}$$