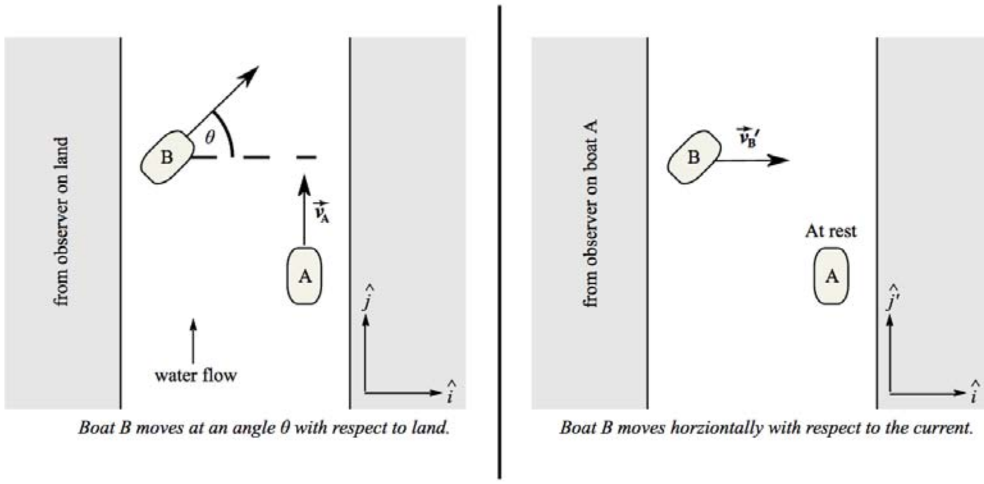
Problem Set 6

1. **Rowing Across the River**



Boat B moves at an angle θ with respect to land. *Boat B moves horziontally with respect to the current.*

An MIT student wants to row across the Charles River. Suppose the water is moving downstream at a constant rate of 1.0 m/s. A second boat is floating downstream with the current. From the second boat's viewpoint, the student is rowing perpendicular to the current at 0.5 m/s. Suppose the river is 800 m wide.

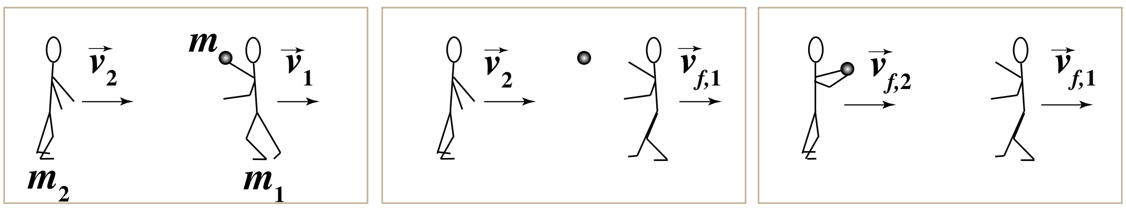
(a) What is the magnitude and direction of the velocity of the student as seen from an observer at rest along the bank of the river? The angle is measured as shown in the figure below.

(b) How far down river does the student land on the opposite bank?

(c) How long does the student take to reach the other side?

2. **Astronauts Playing Catch**

Two astronauts are playing catch in a zero gravitational field. Astronaut 1 of mass m_1 is initially moving to the right with speed v_1. Astronaut 2 of mass m_2 is initially moving to the right with speed $v_2 > v_1$. Astronaut 1 throws a ball of mass m with speed u **relative to herself** in a direction opposite to her motion. Astronaut 2 catches the ball. The final speed of astronaut 1 is $v_{f,1}$ and the final speed of astronaut 2 is $v_{f,2}$.



Before **During** **After**

(a) What is the speed $v_{f,1}$ of astronaut 1 after throwing the ball? Express you answer in terms of some or all of the following: m, m_1, m_2, u, and v_1.

(b) What is the required speed u of the ball (relative to astronaut 1) such that the final speed of both astronauts are equal $v_{f,1} = v_{f,2}$? Express you answer in terms of some or all of the following: m, m_1, m_2, v_1 and v_2.

3. **Multi-stage Rocket in Empty Space**

A rocket in zero gravitational field has a mass of $m_{r,i} = 2.81 \times 10^7$ kg, which is the sum of the mass of the fuel $m_{f,i} = 2.46 \times 10^7$ kg and the dry mass of the rocket (empty of fuel) $m_{r,d} \equiv m_{r,i} - m_{f,i} = 0.35 \times 10^7$ kg. The fuel is ejected at a speed $u = 3000$ m/s relative to the rocket. The total burn time is 510 s and the fuel is burned at a constant rate.

(a) What is the final speed v_f of the rocket in meters/second after all the fuel is burned assuming it starts from rest?

(b) Now suppose that the same rocket burns the fuel in two stages, expelling the fuel in each stage at the same relative speed $u = 3000$ m/s. In stage one, the available fuel to burn is $m_{f,1,i} = 2.03 \times 10^7$ kg with burn time 150 s. The total mass of the rocket after all the fuel in stage 1 is burned is $m_{r,1,d} = m_{r,i} - m_{f,1,i} = 0.78 \times 10^7$ kg. What is the change in speed after stage one is complete?

(c) Next, the empty fuel tank and accessories from stage one are disconnected from the rest of the rocket. These disconnected parts had a mass $m = 1.4 \times 10^6$kg, hence the remaining dry mass of the rocket is $m_{r,2,d} = 2.1 \times 10^6$ kg. All the remaining fuel with mass $m_{f,2,i} = 4.3 \times 10^6$ kg is burned during stage 2 with burn time of 360 s. What is the change in speed in meters/second after stage two is complete?

(d) What is the final speed in meters/second of the rocket after both stages are complete?

(e) How does this compare to your answer to Part A for the final speed if all the fuel were burned in one stage? Does your answer make sense?

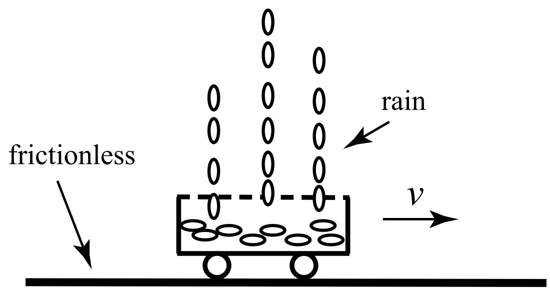
4. **Falling Drop**

A raindrop of mass m_0, starting from rest, falls under the influence of gravity. Assume that as the raindrop travels through the clouds, it gains mass at a rate proportional to the momentum of the raindrop, $\frac{dm_r}{dt} = km_r v_r$, where m_r is the instantaneous mass of the raindrop, v_r is the instantaneous velocity of the raindrop, and k is a constant with unit $[m^{-1}]$. You may neglect air resistance.

(a) Derive a differential equation for the raindrop's accelerations $\frac{dv_r}{dt}$ in terms of k, g, and the raindrop's instantaneous velocity v_r. Express your answer using some or all of the following variables: k, g for the gravitational acceleration and v_r, the raindrop's instantaneous velocity.

(b) What is the terminal speed, v_T, of the raindrop? Express your answer using some or all of the following variables: k and g for the gravitational acceleration.

5. **Moving Vehicle and Falling rain**



A vehicle of 'dry' mass m_0 , with no propulsion, is moving without friction on horizontal ground as shown in the figure above. Rain is falling vertically. While the rain is falling, for each time interval Δt, an amount of rain $\Delta m_r = b\Delta t$ hits the vehicle, sticks to it, and then moves along with zero relative velocity with respect to the vehicle. The vehicle is observed to be slowing down. In a reference system fixed to the ground, at time $t = 0$, the cart is initially moving with speed v_0.

(a) At time t, the vehicle is traveling with speed $v(t)$. For the time interval $[t, t + \Delta t]$, while the rain is falling, determine a differential relation for the change in speed of the vehicle Δv. Write your answer using some or all of the following: t, Δt, m_0 and b.

(b) Integrate your result from part a) to find the speed $v(t)$ of the cart at time t. Write your answer using some or all of the following: v_0, b, m_0 and t.

8.01 Classical Mechanics
Fall 2016