Example 10.6 Landing Plane and Sandbag

Figure 10.9 Plane and sandbag

A light plane of mass 1000 kg makes an emergency landing on a short runway. With its engine off, it lands on the runway at a speed of $40\text{ m}\cdot\text{s}^{-1}$. A hook on the plane snags a cable attached to a 120 kg sandbag and drags the sandbag along. If the coefficient of friction between the sandbag and the runway is $\mu_k = 0.4$, and if the plane's brakes give an additional retarding force of magnitude 1400 N, how far does the plane go before it comes to a stop?

Solution: We shall assume that when the plane snags the sandbag, the collision is instantaneous so the momentum in the horizontal direction remains constant,

$$p_{x,i} = p_{x,1}\,. \tag{10.9.15}$$

We then know the speed of the plane and the sandbag immediately after the collision. After the collision, there are two external forces acting on the system of the plane and sandbag, the friction between the sandbag and the ground and the braking force of the runway on the plane. So we can use the Newton's Second Law to determine the acceleration and then one-dimensional kinematics to find the distance the plane traveled since we can determine the change in kinetic energy.

The momentum of the plane immediately before the collision is

$$\vec{\mathbf{p}}_i = m_p v_{p,i}\,\hat{\mathbf{i}} \tag{10.9.16}$$

The momentum of the plane and sandbag immediately after the collision is

$$\vec{\mathbf{p}}_1 = (m_p + m_s) v_{p,1}\,\hat{\mathbf{i}} \tag{10.9.17}$$

Because the x- component of the momentum is constant, we can substitute Eqs. (10.9.16) and (10.9.17) into Eq. (10.9.15) yielding

$$m_p v_{p,i} = (m_p + m_s) v_{p,1} . \tag{10.9.18}$$

The speed of the plane and sandbag immediately after the collision is

$$v_{p,1} = \frac{m_p v_{p,i}}{m_p + m_s} \tag{10.9.19}$$

The forces acting on the system consisting of the plane and the sandbag are the normal force on the sandbag,

$$\vec{\mathbf{N}}_{g,s} = N_{g,s} \hat{\mathbf{j}} , \tag{10.9.20}$$

the frictional force between the sandbag and the ground

$$\vec{\mathbf{f}}_k = -f_k \hat{\mathbf{i}} = -\mu_k N_{g,s} \hat{\mathbf{i}} , \tag{10.9.21}$$

the braking force on the plane

$$\vec{\mathbf{F}}_{g,p} = -F_{g,p} \hat{\mathbf{i}} , \tag{10.9.22}$$

and the gravitational force on the system,

$$(m_p + m_s) \vec{\mathbf{g}} = -(m_p + m_s) g \hat{\mathbf{j}} . \tag{10.9.23}$$

Newton's Second Law in the $\hat{\mathbf{i}}$-direction becomes

$$-F_{g,p} - f_k = (m_p + m_s) a_x . \tag{10.9.24}$$

If we just look at the vertical forces on the sandbag alone then Newton's Second Law in the $\hat{\mathbf{j}}$-direction becomes

$$N - m_s g = 0 .$$

The frictional force on the sandbag is then

$$\vec{\mathbf{f}}_k = -\mu_k N_{g,s} \hat{\mathbf{i}} = -\mu_k m_s g \hat{\mathbf{i}} . \tag{10.9.25}$$

Newton's Second Law in the $\hat{\mathbf{i}}$-direction becomes

$$-F_{g,p} - \mu_k m_s g = (m_p + m_s)a_x .$$

The x-component of the acceleration of the plane and the sand bag is then

$$a_x = \frac{-F_{g,p} - \mu_k m_s g}{m_p + m_s} \tag{10.9.26}$$

We choose our origin at the location of the plane immediately after the collision, $x_p(0) = 0$. Set $t = 0$ immediately after the collision. The x-component of the velocity of the plane immediately after the collision is $v_{x,0} = v_{p,1}$. Set $t = t_f$ when the plane just comes to a stop. Because the acceleration is constant, the kinematic equations for the change in velocity is

$$v_{x,f}(t_f) - v_{p,1} = a_x t_f .$$

We can solve this equation for $t = t_f$, where $v_{x,f}(t_f) = 0$

$$t_f = -v_{p,1} / a_x t .$$

Then the position of the plane when it first comes to rest is

$$x_p(t_f) - x_p(0) = v_{p,1} t_f + \frac{1}{2} a_x t_f^2 = -\frac{1}{2} \frac{v_{p,1}^2}{a_x} . \tag{10.9.27}$$

Then using $x_p(0) = 0$ and substituting Eq. (10.9.26) into Eq. (10.9.27) yields

$$x_p(t_f) = \frac{1}{2} \frac{(m_p + m_s)v_{p,1}^2}{(F_{g,p} + \mu_k m_s g)} . \tag{10.9.28}$$

We now use the condition from conservation of the momentum law during the collision, Eq. (10.9.19) in Eq. (10.9.28) yielding

$$x_p(t_f) = \frac{m_p^2 v_{p,i}^2}{2(m_p + m_s)(F_{g,p} + \mu_k m_s g)} . \tag{10.9.29}$$

Substituting the given values into Eq. (10.9.28) yields

$$x_p(t_f) = \frac{(1000\text{ kg})^2 (40\text{ m}\cdot\text{s}^{-1})^2}{2(1000\text{ kg} + 120\text{ kg})(1400\text{ N} + (0.4)(120\text{ kg})(9.8\text{m}\cdot\text{s}^{-2}))} = 3.8\times 10^2\text{ m} . \tag{10.9.30}$$