Let's now consider how to find the velocity of the center of mass of a wheel that's rolling without slipping down an inclined plane.

So let's draw a picture.

Here is the wheel that starts at t equals 0.

And this is an angle phi.

And at some later time t final, the wheel has dropped a distance h along the incline plane.

And now let's figure out what is the velocity of the center of mass of the wheel when it gets to the bottom.

And we're now going to apply our energy arguments.

Now the first thing we have to realize is a very subtle point here is that there can be friction.

And we saw that when a wheel is rolling without slipping, there can be static friction at the contact point between the wheel and the ground.

There is a gravitational force.

And a normal force.

Now static friction does no work because the point is always instantaneously at rest and f dot ds is 0 because the object is at rest.

And so the ds is 0.

So our static friction does no work.

And therefore, from our energy principle that the external work equals E final minus E initial, we have no external work and so our energy is constant.

Now how do we analyze our energy?

Well, I've set things up.

So I'm going to choose my potential energy to be 0 when it's at the bottom.

And that our initial energy is only equal to potential energy.

If the wheel has a mass m, gravitational force down, the potential energy initially is mgh.

The final energy is only kinetic energy.

The wheel is rolling with omega final.

Its center of mass is moving with v center of mass final.

And we just saw that the kinetic energy has two contributions; the translational kinetic energy, 1/2 m-- we'll just call this final squared.

And we'll make that the final just to make it easier.

And it has the kinetic energy of rotation.

Now the rolling without slipping condition is that the translational center of mass speed is equal to the radius of the wheel, r, times the angular speed, omega final.

So I can write E final as 1/2 of m plus I cm over r squared times the final squared, where I'm just replacing omega final equals v final over r.

And therefore, that the kinetic energy by the energy principle tells us that E initial equals E final.

So mgh equals 1/2 times m plus I cn over r squared v final square.

So we can find that v final is equal to the square root of 2 mgh divided by m plus 1/2 cm r square.

And that is the final speed of the center of mass translational's velocity of the wheel as it dropped a height h.