

Experiment CF Centripetal Force

Note: You will need both of your LVPS for this experiment. So build your second LVPS if you have not done so already.

Introduction

This experiment is about centripetal force, the force that keeps objects moving in a circular path at a constant velocity. This force is always at right angles to the motion and directed towards the center of the circular orbit. By Newton's Second Law when a centripetal force acts on a mass, the force continually changes the direction of the velocity without changing its magnitude. This change in the direction of the velocity is always towards the center and accounts for the centripetal acceleration of the mass.

Examples of centripetal forces are tension in a string connected to a rotating mass; the gravitation force between a planet and a rotating satellite; and magnetic forces on moving charged particles in a magnetic field.

Experiment

In this experiment, the centripetal force is the restoring force of a stretched rubber band with one end attached to the shaft of a dc permanent-magnet motor powered by your LVPS and the other end attached to a # 6-32 nut. The nut has a mass $0.92 \pm 0.01\text{g}$ and the rubber band has a mass of $0.22 \pm 0.02\text{g}$. The motor whirls the nut in a circular path at a rotational frequency f that you will set using a stroboscopic method. You will measure the radius of the circular path, r , for various rotation speeds. This will allow you to calculate the centripetal acceleration

$$|a_r| = r 4\pi^2 f^2 .$$

You will set the rotational frequency by using a light-emitting diode (LED) that flashes 60 times per second when connected to the wall transformer. (The wall transformer supplies a nominal 12 Volts AC at 60 Hz). You can adjust the motor speed by adjusting the output voltage of the LVPS. You can then set your motor speed by a stroboscopic method. You will place a pattern of lines on the shaft of the motor. While the LED is directed downward onto the motor shaft, you adjust the LVPS until you see a stationary pattern of lines. This will correspond to a frequency that is some fraction of 60 Hz.

The rubber band stretches until the restoring force of the rubber band exerts the necessary centripetal force on the nut to keep the nut spinning in a circular orbit. You calibrate the rubber band by hanging pennies in a cup that stretch the rubber band and measure the stretched length of the rubber band as a function of the hanging weight. Thus by measuring the stretched length of the rotating rubber band (the radius of the circular orbit) you can use your calibration to

determine the centripetal force acting on the spinning nut. You can then verify that Newton's Second Law is indeed correct, namely that

$$|\vec{F}_{radial}| = m|\vec{a}_{radial}| = m r 4\pi^2 f^2$$

Theory

When an object is moving in circular motion the direction of velocity is always tangent to the circle. Therefore the direction of the velocity is always changing towards the center of the circle. This means that there is a non-zero component of the acceleration directed radially inward. This acceleration is called the *centripetal acceleration*. If our object is increasing its speed or slowing down there is also a non-zero *tangential acceleration*. But when the object is moving at a constant speed in a circle then only the centripetal acceleration is non-zero. Therefore there must be a '*centripetal force*', a radial force pointing inward, that produces this acceleration. Since Newton's Second Law is a vector equality, $\vec{F} = m\vec{a}$, can be applied to the radial direction yielding (in magnitude)

$$\vec{F}_{radial} = m\vec{a}_{radial}$$

Kinematics of Circular Motion

We choose the origin of our coordinate system as the fixed central point. We shall choose coordinates for a point P in the plane as follows (Figure 1). One coordinate, r , measures the radial distance from the origin to the point P. The coordinate r ranges in value from $0 \leq r < \infty$. Our second coordinate θ measures an angular distance along the circle. We need to choose some reference point to define the angle coordinate.

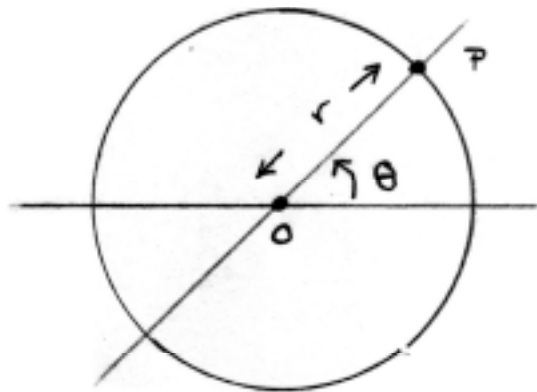


Figure 1: Polar Coordinate System

We choose a 'reference ray', a horizontal ray starting from the origin and extending to $+\infty$ along the horizontal direction to the right. (In a typical Cartesian coordinate system, our reference ray is the positive x-direction). We define the angle coordinate for the point P as follows. We draw a ray from the origin to the point P. We define the angle θ as the angle in the counterclockwise direction between our horizontal reference ray and the ray from the origin to the point P. All the other points that lie on a ray from the origin to infinity passing through P have the same value as θ . For any arbitrary point, our angle coordinate θ can take on values from $0 \leq \theta < 2\pi$.

Velocity and Angular Velocity

The rate of change of angle with respect to time is called the angular velocity and is denoted by the Greek letter ω ,

$$\omega \equiv \frac{d\theta}{dt}.$$

Angular velocity has units [rad/sec]. The component of the velocity v for circular motion is given by

$$v = r \frac{d\theta}{dt} = r\omega.$$

Uniform Circular Motion

When the magnitude of the velocity (speed) remains constant, the object moves in *uniform circular motion*. Since the speed $v = r\omega$ is constant, the amount of time that the mass takes to complete one circular orbit of radius r is a constant. This time interval is called the period and is denoted by T . In one period the object travels a distance s equal to the circumference, $s = 2\pi r$. Since the distance traveled in one period is the product of speed and period, we have

$$s = 2\pi r = vT.$$

Thus the period T is given by

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{r\omega} = \frac{2\pi}{\omega}.$$

The frequency f is defined to be the inverse of the period,

$$f = \frac{1}{T} = \frac{\omega}{2\pi},$$

The units of frequency are $[\text{sec}^{-1}] = [\text{hertz}] = [\text{Hz}]$.

Radial Acceleration

The magnitude of the component of radial acceleration (*centripetal acceleration*) is given by

$$|a_r| = |v\omega|.$$

The direction of the radial acceleration is towards the center of the circle.

The centripetal acceleration can be expressed in several equivalent forms since the velocity and the angular velocity are related by $v = r\omega$. Thus we have for the magnitude of the centripetal acceleration two alternative forms:

$$|a_r| = r\omega^2 \tag{6.1.1}$$

and

$$a_r = \frac{v^2}{r} \tag{6.1.2}$$

Recall that the angular velocity is related to the frequency by $\omega = 2\pi f$. So we have a third expression for the magnitude of the centripetal acceleration

$$a_r = r4\pi^2 f^2 \tag{6.1.3}$$

A fourth form commonly encountered uses the fact that the frequency is defined to be the inverse of the period $f = 1/T$. Thus we have the fourth expression for the centripetal acceleration

$$a_r = r4\pi^2 f^2 \tag{6.1.4}$$

Experiment: Building the Apparatus

Note: Do not plug in the wall transformer until you have completed the apparatus.

Clamping the motor

Take the 2-inch corner brace in the experiment kit and use the hose clamp to attach the motor to it as shown. Handle the motor carefully to avoid damage to the terminals of the motor. The clamp

should be tight enough to keep the motor from wiggling around, but not so tight as to keep it from turning freely. Use the 'C-clamp' to attach the assembly rigidly to your desk.

Attaching the nut

Attach the 6-32 nut to the rubber band and the rubber band to the motor shaft by looping the rubber band over itself as shown. Loop it in the direction such that the rotation of the shaft tends to tighten the loop. Try to keep the two strands the same length and not twisted up.

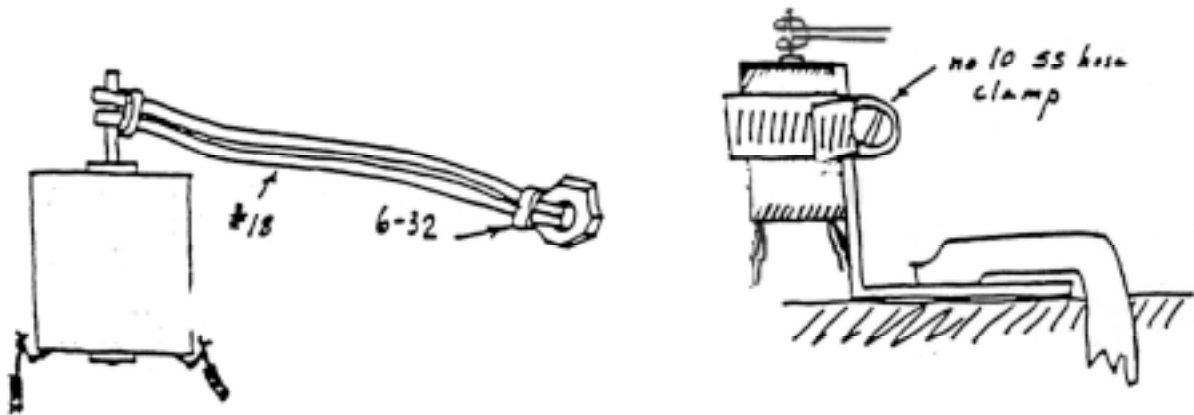


Figure 2: Clamping the motor and attaching the nut

Attaching the stroboscopic pattern

Cut out the four-line pattern (you can find it on the last page of this write-up), make a hole in the center with a pushpin and press it onto the motor shaft above the rubber band.

Connecting the LED

The LED needs an alternating voltage in order to flash so you will connect it to the ac input terminals of the rectifier in one of your partnerships' LVPS (call this one LVPS-1). Split the 'speaker wire' (two conductor stranded wire in transparent plastic insulation) an inch or so at each end of the 4 foot length provided in the experiment kit. Remove about 1/4 inch insulation from each of the ends and tin the exposed wire. Solder the pair at one end to the LED without twisting the wires. Use 1 1/2 inches of black tape folded lengthwise to protect the LED leads from shorting each other. Solder the pair at the other end to the ac input terminals of the rectifier in the LVPS-1. Again don't twist the wires.

Connecting the two LVPS

Your LVPS only goes down to 1.5 volts, which is still too high to obtain the low rotational speeds you will need for this experiment. You will use your other LVPS (call this one LVPS-2) with the voltage reversed to adjust the voltage down to zero voltage.

You only need one wall transformer to drive both LVPS. You will do this by connecting the output of LVPS-1 rectifier to the $1000\mu F$ capacitor of LVPS-2. Use a red clip lead to connect the positive sides of the $1000\mu F$ capacitors of LVPS-1 and LVPS-2 together. Use a black clip lead to connect the negative sides of the $1000\mu F$ capacitors of LVPS-1 and LVPS-2 together.

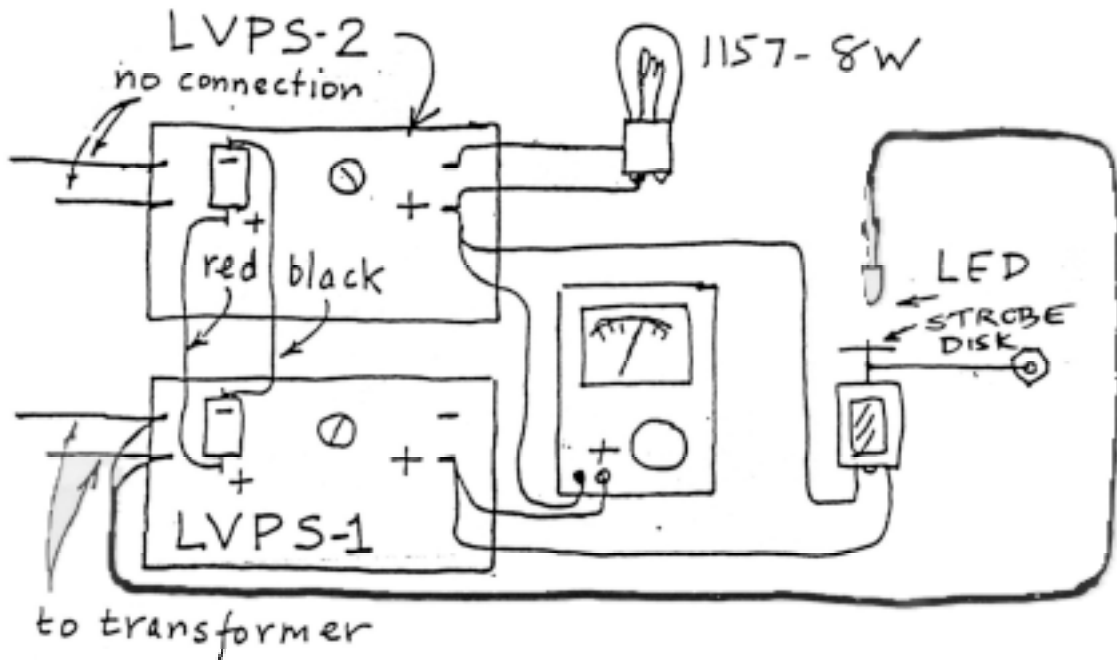


Figure 3: Connecting the two LVPS

Connecting the 1157 lamp

You will use the 8-watt filament of your 1157 lamp for a current regulating effect that makes settings less critical and more stable. As the current increases in the filament the resistance goes up. This increased resistance tends to limit the increase in the current. Solder the leads of the 8 W filament of the 1157 lamp to the output terminals of the LVPS-2.

Connecting the motor

Connect the motor leads to the plus terminals of the two LVPS with clip leads. By connecting the motor to the two positive terminals you are effectively subtracting the differences between the two voltage settings to get a net voltage across the motor between 0 and 1.5 volts. Connect

the input leads of the multimeter (MMM) at the 5 volt DC setting to the two positive output terminals of the LVPS so that you can measure the voltage across the motor.

Doing the Spinning Nut Experiment

You will first measure the radius of the circular orbit of the spinning nut for various rotational frequencies, 10 Hz (six stationary lines), 12 Hz (five stationary lines), 15 Hz (four stationary lines), and 20 Hz (three stationary lines). Then you will calibrate the rubber band by stretching the rubber band with pennies in a hanging cup.

Procedure:

- **Unclip one motor lead and plug in the wall transformer.** Adjust the voltage of LVPS-2 to about 7.5 volts. This is enough to make the 8 W filament of the 1157 lamp glow brightly. Turn down LVPS -1 all the way and then reconnect the motor.
- Hold the nut so the rubber band is slightly stretched; the band should not rub on the motor housing. Turn up LVPS-1 so that there is 0.5 to 0.75 volts across the motor. You will see the shaft start to turn; immediately launch the nut into orbit. Start all over if the rubber band gets twisted up around the shaft.
- In order to see the stroboscopic pattern you will need to make the area immediately around the experiment dark. One partner can hold the LED about 1 inch above the motor and continually adjust the LVPS-1 until the 4-line pattern appears to be stationary. This means that (the simplest case) the band is advancing 90 degrees = 1/4 of a revolution each 1/60 of a second, and therefore the shaft is turning at a rotational frequency $f = 15$ Hz.
- The other partner can measure the radius of the circular path of the spinning nut from the center of the shaft to the center of the nut.
- **Repeat these measurements for the various patterns corresponding to 10 Hz, 12 Hz, 15 Hz, and 20 Hz rotational frequencies.**

Calibrating the Rubber Band

Clamp the corner brace on its edge with the motor shaft extended well out over the edge of your table or desk so that the rubber band and nut hang freely. Bend a paper clip into an S-shaped hook. Press one end through the paper cup just below its rim and hook the other end to the nut.

Now add pennies to the cup and measure the length, L , of the rubber band. Keep adding pennies to cover the range of lengths you measured when spinning the nut. The mass of the paper cup and the paper clip together is $5.8. \pm 0.01$ g .

The weight required to stretch the rubber band to a length equal to the radius of the circular path of the nut is approximately equal to the centripetal force needed to accelerate the nut inward in its motion. We say "approximately", because the rubber band has mass and is stretched by the rotation even when there's no nut at the end. This isn't too serious, as you can see by spinning the rubber band by itself; it doesn't stretch much, and if it did, you could take data to make an appropriate correction. You should also note that excessive stretching of the rubber band could weaken it permanently; this could invalidate your results.

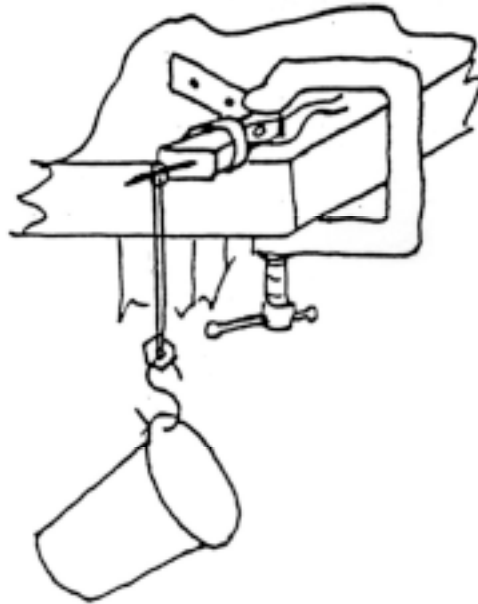


Figure 4: Calibrating the rubber band

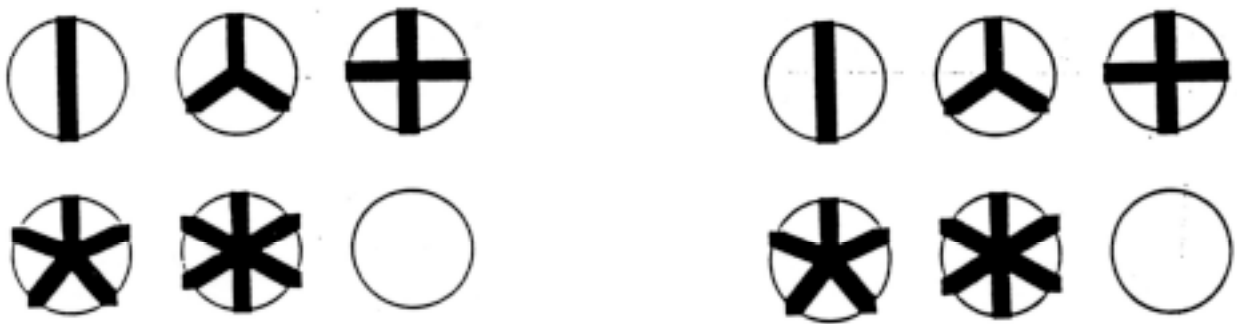


Figure 5: Stroboscopic line patterns

PARTS LIST

- 1 permanent magnet DC motor**
- 1 2" corner brace**
- 1 hose clamp #12 SS**
- 1 LED, wt. internal resistor**
- 4' #22 speaker wire**
- 2 nuts, 6-32 (5/16) steel**
- 2 rubber bands, #18**
- 1 paper clip, #1**
- 1 pushpin**
- 1 paper cup**
- 100 pennies**