Topics: Introduction to TEAL; Fields; Review of Gravity; Electric Field
Related Reading:

      Course Notes (Liao et al.):   Sections 1.1 – 1.6; 1.8; Chapter 2

Topic Introduction

The focus of this course is the study of electricity and magnetism. Basically, this is the study of how charges interact with each other. We study these interactions using the concept of "fields" which are both *created* by and *felt* by charges. Today we introduce fields in general as mathematical objects, and consider gravity as our first "field." We then discuss how electric charges create electric fields and how those electric fields can in turn exert forces on other charges. The electric field is completely analogous to the gravitational field, where mass is replaced by electric charge, with the small exceptions that (1) charges can be either positive or negative while mass is always positive, and (2) while masses always attract, charges of the same sign repel (opposites attract).

Scalar Fields

A *scalar field* is a function that gives us a single value of some variable for every point in space – for example, temperature as a function of position. We write a scalar field as a scalar function of position coordinates – e.g. $T(x, y, z)$, $T(r, \theta, \varphi)$, or, more generically, $T(\vec{\mathbf{r}})$. We can visualize a scalar field in several different ways:

(A)   (B)   (C)
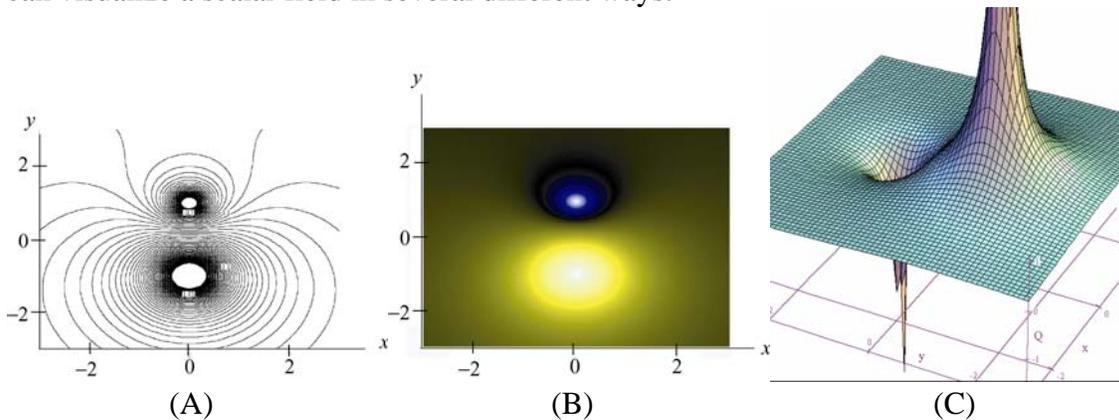
In these figures, the two dimensional function $\phi(x, y) = \dfrac{1}{\sqrt{x^2 + (y+d)^2}} - \dfrac{1/3}{\sqrt{x^2 + (y-d)^2}}$ has

been represented in a (A) contour map (where each contour corresponds to locations yielding the same function value), a (B) color-coded map (where the function value is indicated by the color) and a (C) relief map (where the function value is represented by "height"). We will typically only attempt to represent functions of one or two spatial dimensions (these are 2D) – functions of three spatial dimensions are very difficult to represent.

Vector Fields

A vector is a quantity which has both a magnitude and a direction in space (such as velocity or force). A *vector field* is a function that assigns a vector value to every point in space – for

example, wind speed as a function of position. We write a vector field as a vector function of position coordinates – e.g. $\vec{F}(x, y, z)$ – and can also visualize it in several ways:
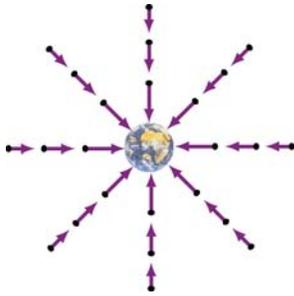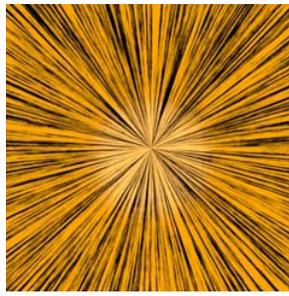
Image courtesy of the National Oceanic and Atmospheric Administration.

(A)   (B)   (C)

Here we show the force of gravity vector field in a 2D plane passing through the Earth, represented using a (A) vector diagram (where the field magnitude is indicated by the length of the vectors) and a (B) "grass seed" or "iron filing" texture. Although the texture representation does not indicate the absolute field direction (it could either be inward or outward) and doesn't show magnitude, it does an excellent job of showing directional details. We also will represent vector fields using (C) "field lines." A field line is a curve in space that is everywhere tangent to the vector field.

Gravitational Field
As a first example of a physical vector field, we recall the gravitational force between two masses. This force can be broken into two parts: the generation of a "gravitational field" **g** by the first mass, and the force that that field exerts on the second mass ($\vec{F}_g = m\vec{g}$). This way of thinking about forces – that objects create fields and that other objects then feel the effects of those fields – is a generic one that we will use throughout the course.

Electric Fields
Every charge creates around it an electric field, proportional to the size of the charge and decreasing as the inverse square of the distance from the charge. If another charge enters this electric field, it will feel a force $\left(\vec{F}_E = q\vec{E}\right)$.

Important Equations

Force of gravitational attraction between two masses: $\vec{F}_g = -G\dfrac{Mm}{r^2}\hat{r}$

Strength of gravitational field created by a mass M: $\vec{g} = \dfrac{\vec{F}_g}{m} = -G\dfrac{M}{r^2}\hat{r}$

Force on mass m sitting in gravitational field g: $\vec{F}_g = m\vec{g}$

Strength of electric field created by a charge Q: $\vec{E} = k_e\dfrac{Q}{r^2}\hat{r}$

Force on charge q sitting in electric field E: $\vec{F}_E = q\vec{E}$