Hints to Assignment #4 -- 8.022

(20 points) [1] Energy of a conductor in a capacitor.

- You have two unknowns (sigma_W and sigma_N), so you will need to come up with two independent eqns for them. One is the total charge on each plate. What can be the other?
- Once you have sigma's, fields must be fairly easy to find.
- Use the definition. But what is V? Could you replace the system with two capacitors? Would they be in series or in parallel?
- There are plenty of ways to find the total U and hopefully all should yield the same answer (from E, from the capacitance and from the 'sum' of two capacitors).
- If you know the total energy, you can find its first derivative... But why will that be what you are looking for? Assume the conductor shifts by dx, why energy changes? Can you make a GENERAL STATEMENT on how conductor behave close to electric fields??

(15 points) [2] Line charge near a conducting plane (Purcell 3.6)

You may want to review the reasoning behind solving the problem of a point charge in front of a conducting plane before digging into this problem. Here are a few remarks that might assist you in solving it:

- Treat the wire as infinitely long and with a cross section that is insignificant respect to its distance from the earth.
- Treat the earth as a conducting plane (clearly, *grounded!*)
- Spell out the specifications of the electric field on the surface of the earth and ask yourself what "image" must be placed (i.e., of what "strenth"=charge density and position) in order to yield this field.
- We have done that in class and in previous HW for a point charge. As your book suggests, use superposition to construct the image of this line charge.
- With regards to the force, remember the force the wire feels must come from the charges in the *rest of the world* and in this case, *either* the induced charges on the ground *or* the image.

(15 points) [3] Points charges and conducting plane (Purcell 3.9)

This is a topological problem that you might find its reincarnations in many many fields! It's a good investment to understand what is going on here!

- Start by making a clean plot of the charges on the plane. If you've done the plotting right and you recall how a point charge "reflects" on a plane, then the answer on the first question is quite obvious.
- Now remove all charges but one and bring an "L"-shaped metal sheet to sit on top of the equipotential surfaces you've just identified. Follow the general rules for drawing field of lines as well as the fact that field of lines have to be perpendicular to a conductor.
- Now, let's clarify something that problem 3.9 might has biased you upon: does the picture change if the charge q is **NOT** symmetrically located between the two parts of the metal plate??
- To continue on this problem you will need to make a geometrical contruction to as good precision as you can in the following steps:
  - Draw a circle of an arbitrary radious R.
  - Draw an arbitrary angle of opening theta_0 with respect to the +x axis. Imagine these being the sides of a metal sheet that was bended accordingly.
  - Place a charge Q in an ARBITRARY position along the circular arc on the circle in between the opening of the angle theta_0. Let's call this angle (with respect to +x) theta.
  - Call the two sides of the angle (i.e., of the metal sheet) A and B. Draw the image of Q with respect to A and with respect to B and call them A' and B'.
  - Now, pick one of the two, say A' and find its image A" with respect to B, then its image A"' with respect to A and so on and so forth.
  - Can you see the condition that must be met in order the problem to be solvable with the method of images? After n reflections the image A""' (nth order) has to return to B (or equivalently, A itself).
  - Is that satisfied for an angle theta_0=120 deg? (Go back to your plot on the circle).
- There should be two things that you realize at the end of this problem:
  - The position of the charge Q in between the dihedral angle DOES NOT matter!
  - The number of images that are needed for an angle theta_0 are n=2(pi/theta_0)-1 when and only when pi/theta_0 is an integer.

(20 points) [4] Image on a sphere.

We have laid out the path to the solution of this problem in class.

- Use SYMMETRY to argue that there is a preferred direction for the position of the charge and use notion of "image" that it has to be "behind" the region of interest (our region of interest=space outside the sphere; thus image has to be inside the sphere...).

- Convince yourself that image can NOT be equal to Q. Assign a value Q' and seek its position x. Form a system of 2 eqns with 2 unknowns (Q' and x) by examining the potential of two 'preferred' points on the sphere.
- Write down the potential on ANY POINT on the sphere due to the Q and its image Q'. Use vector algebra to determine the length of a vector **C**=**A**+**B** from the lengths and angle of **A** and **B**.
- OPTIONAL, but rather interesting:
    - Find the (induced) charge distribution ON THE SPHERE
    - What is its integral??

(15 points) [5] Cylindrical capacitor (Purcell 3.23)

We have itemized the steps in finding capacitance:

- Find **E** in all space.
- If you know **E** you can find the potential.
- Use the definition of capacitance (from Q,phi) and you are done.
- The formal limiting behavior might require some Taylor expansion but you can probably get to right answer by some back of the envelope approximations.

(15 points) [6] Capacitors in Series and in Parallel

- Recall the definitions for connection in series and in parallel. You will need to justify why in the parallel connection the plates of adjacent capacitors are equally and oppositely charged. Moreover you will have to convinced yourself that the charges split among the capacitors when connected in parallel. The rest are trivialities.
- The second part is a bit tedious, but rather obvious. You are armed with the expressions for total capacitance when caps are in series/parallel and moreover you have established how voltages and charges behave in each case. You will find benefitial to REDRAW the circuit when the switch moves.
