

Hints to Assignment #5 -- 8.022

Please note: in problem #1, σ , the conductivity of the material is involved in all the formulas and when the problem states "express all your answers in terms of V_a, V_b, a, b and L ", σ should be included too.

[1] Current Flow Between Cylindrical Plates

The two metal cylinders are equipotential surfaces at potentials fixed by the EmF.

- Imagine the space in between the cylindrical plates is broken into cylindrical shells of radial width dr and area $2\pi rL$. Could that be your infinitesimal resistor? Are these resistors in series or in parallel? We did lay out the solution to this problem in class...
 - In finding \mathbf{J} recall Ohm's law in the microscopic description. Its integral over area is a constant which you do not know yet. You will need to find \mathbf{E} from \mathbf{J} and then use the boundary conditions for the potential on the plates in order to determine this constant.
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[2] Snell's Law for Electric Currents

Assume that J_1 and J_2 are steady currents driven by corresponding electric fields E_1 and E_2 .

- Assume media are uniform and isotropic so that Ohm's law in the microscopic version applies (without invoking tensors etc...)
 - You need to find J_2 , which is a vector, i.e., it needs a magnitude and direction.
 - Define a cartesian coordinate system: Choose the x^{\wedge}, y^{\wedge} plane as the plane PERPENDICULAR to the interface onto which the J_1 vector lives.
 - Once you've identified $x^{\wedge}, y^{\wedge}, z^{\wedge}$, write J 's and E 's in terms of their projections.
 - Once at steady state, what "comes in" must "go out" in terms of the current through the interface...
 - One of the two E components must be conserved (why?) while the other E components' change must measure the surface density on the interface.
 - Watch out not to mess up the sigmas (conductivities) with sigmas (surface charge densities, use SIGMA, in capital if you wish).
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[3] Purcell's 4.21: Resistive Network

- For part (a), first find the current flowing in the circuit. You do not need any of Thevenin's or Kirchhoff's laws for that, just plain Ohm.
- Assume B is at V_b . Take a dq charge and move it from V_b to V_a . Plot its potential V versus point on the circuit (does it matter which way it goes? there are obviously two options ...). Specify on your answer if you've calculated $V_b - V_a$ or $V_a - V_b$. These are of course opposite, BUT, not all authors agree as to what they refer to when they talk about potential DIFFERENCE on a circuit. When in doubt, ASK, or write down the $V(1) - V(2)$ your answer refers to.
- For part (b), you probably want to use now Mr. Kirchhoff's laws. Identify the nodes and branches and write down a system of 3 eqns with 3 unknowns.
- For the last part, you will need some definitions at p.158, last two paragraphs.

[4] Purcell's 4.25: Discharging Capacitor

- Well, you shouldn't definitely break your friendship with your friends who do not believe capacitors really discharge... Let the math convince them. Open up together your old PC, read your transformer's DC outputs, should be $\pm 5V, \pm 12V$ (at least), a common voltage of operation for many integrated circuits and chips. Try to read some capacitor values (well, your PC must be really old if you manage to do that). Anyways, it must be somewhere in the pF to nF range. Assume that say 12V charged one of this capacitor. Find its charge and express it in terms of number of electrons.
- Assume now that the capacitor discharges, typically in your computer is over resistances of the order of $\times 100 - \times 1000$ Ohms, but having a hard to convince friend, offer him to discharge it over a more resistive element, like your body, so that to make the escaping of the charge even harder (i.e., slower). Use the exponential law with the RC you now have and find the time to reach a charge of $1e^-$ on the capacitor.

[5] Purcell's 4.32: Infinite Resistor Chain

That could have been an algebra (sequences) problem or an example of (mathematical) induction! I know some of you are already calculating the Nth term of the sequence... It might help you if you change notation for R_1 and R_2 to R_a and R_b in order to avoid confusion with the 1st and 2nd elements of the sequence.

- Your text gives the crucial hint, i.e., at $n \rightarrow \infty$, $R_{(n+1)} = R_n$. This should be enough to solve the problem. Optionally and for gaining confidence that the sequence indeed converges, you would like (NOT REQUIRED FOR YOUR HOMEWORK) to prove that the sequence CONVERGES. Use induction to show that $R_{(n+1)} \cdot R_n$ for all n (start with showing $R_2 \cdot R_1$ and then assume that $R_{(n)} \cdot R_{(n-1)}$ is true in order to show $R_{(n+1)} \cdot R_n$).

- Assume just two steps in this ladder where the final R_b is replaced by your "load" R_L . Try requesting $R_{\text{eff}}=R_L$ and you are ready to start your attenuator manufacturing company. (ps ".lt." is shortnotation for "less than" ... clearly do not know how to do it in html).

[6] Purcell's 4.33: Kirchhoff's Law and Minimum Power Requirement

Well, the way the problem is presented guides you through the solution. The only point you should make sure you understand is the power dissipation on a resistor R through which a current I flows.

- We can see how this comes along in the macroscopic picture of conductivity, i.e., in the picture where a net current flows between potential differences obeying Ohm's law. In order to see this, assume a charge dq (positive, BY DEFINITION in circuitry language) goes through the resistor in time dt (so that the current $I=dq/dt$ remains constant through the length of the circuit (but not necessarily through time...)). Its potential drops from V_i to V_f for a total energy loss of $(V_i-V_f)*dq$ We have already worked out (starting from the effect of the applied potential difference, i.e., electric field) on the charge carriers (assumed positive here) that $V_i-V_f=I*R$ Thus the energy lost on the resistor within dt will be $dE=I*R*dq$ If we divide by dt we find the dissipated power $P=dE/dt=I*R*I$ or I^2*R . This power is expended on thermal motion and thus heat of the material the current goes through.
- In the microscopic picture, i.e., individual charge carriers moving under the influence of electric fields (external) we will hopefully find the same answer. As we have seen in the microscopic description of conductivity, the effect of E on the q 's is described by an average drift velocity v . The power then of the applied force on one such carrier will be $dW/dt=Fv=qEv$ and for N such carriers then $dW/dt=NqEv=nALqEv$ where n is the number density of the carriers, A is the cross-section and L is the length of the conductor. You may identify $\Delta V=EL$ and $I=JA=nqvA$ reducing the above formula to $dW/dt=V*I=I^2*R$.
- Find I_1, I_2 by solving potential equations.
- Write down P in terms of one of the two (unknown) currents and use your familiar conditions for an extreme (but also minimum) of a function.