

## Hints to Assignment #6 -- 8.022

### [1] RC circuit: qualitative analysis.

The qualitative behaviour of an *uncharged* capacitor can be itemized as follows:

- at  $t=0$  it presents *zero* resistance in the circuit, thus it can be replaced with a *resistanceless* wire.  
**REDRAW THE CIRCUIT BY REPLACING THE CAPACITOR --||-- WITH A PLAIN WIRE ----**
- at  $t \rightarrow \infty$  it presents *infinite* resistance in the circuit, i.e., no current can flow through it and the potential it presents on its leads counterbalances whatever potential charges it.  
**REDRAW THE CIRCUIT BY REMOVING THE BRANCH THAT CONTAINS THE CAPACITOR.**

As you can appreciate, this does *not* mean that at  $t \rightarrow \infty$  there is no current flowing in a circuit that involves a capacitor; it **depends** on the circuit and how the capacitor is connected. For example, in the circuit under consideration in this problem, while  $I_{R3}$  goes to *zero* at  $t \rightarrow \infty$ ,  $I_{R1}$  and  $I_{R2}$  remain finite and non-zero at  $t \rightarrow \infty$ .

You are now ready to start filling out the following table where  $t=0$  is assumed to be the instance the switch is closed:

$t$	$I_{R1}$	$I_{R2}$	$I_{R3}$	$V_{R1}$	$V_{R2}$	$V_{R3}$	$V_C$
$\rightarrow 0$							
$\rightarrow \infty$							

The qualitative behaviour of a *charged* capacitor when it is the only source of energy in a circuit can be itemized as follows (assuming  $t=0$  is the instance we bring a charged capacitor in a circuit):

- at  $t > 0$ , the capacitor acts like an Emf of initial strength equal to the potential it was charged on.
- at  $t \rightarrow \infty$  the capacitor discharges ( $Q=0 \implies V_C=0$ ) and **all** currents in the circuit cease to exist (assuming the capacitor was the only source of energy in the circuit).

You are now ready to fill out a similar table like the one above.

Here are a few more things you should keep in mind:

- This is a **linear** circuit and all its state variables will be described by **superposition** of partial solutions. We know how a stand alone RC system

behaves, thus all points we have defined in the above two tables will be connected with **exponentials**. DO NOT WORK OUT the exact time constant for each of them. Simply draw exponentials connecting start and end of phenomenon.

- The transition from closed switch to open switch implies discontinuities in **ALL BUT ONE** of the seven variables we have been examining. In order to figure out which of the seven variables under discussion will be continuous, consider that **CHARGE ON THE CAPACITOR** is continuous and thus from the definition of capacitance, also ... is continuous. Convince yourself that the other 6 variables "jump" when we open the switch.
  - In setting up a system of equations for a detailed derivation of the quantities involved, we will assume the classical circuitry problem where EMF,  $R_1$ ,  $R_2$ ,  $R_3$  and  $C$  are given and we are asked to find  $I_1$ ,  $I_2$  and  $I_3$ . Write down Kirchhoff's rules #1 and #2 for these three quantities. Choose for  $I_3$  the direction that charges up the  $+Q$  plate of the capacitor AND KEEP THIS PROTOCOL THE SAME even for the discharge. If you end up with an EXTRA variable (maybe the  $Q$  on the capacitor?), recall how it is connected with  $I$  ...
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## [2] RC circuit: qualitative analysis.

- When the capacitor does not make part of the circuit (open switch), the current is rather straightforward to calculate. Notice that this is the *same* current flowing through all the elements ( $=V/(3R)$ ).
  - You should be able to set up a system of three equations with three unknowns ( $I_1, I_2, I_3$ ) or equivalently a system of four equations (the fourth one being  $I_c = dQ_c/dt$ ) with four unknowns ( $I_1, I_2, I_3$  and  $Q_c$ ). Clearly, you have to reduce them to a single one (differential) for  $Q_c$ . You should solve this equation for  $Q_c$ . Make some remarks on  $t \rightarrow 0$  and  $t \rightarrow \infty$  behaviour. Does it agree with our qualitative expectations discussed before?
  - You should find that after  $t \rightarrow \infty$ , the charge on the capacitor is  $Q_\infty = 4CV/3$  and thus you have the energy stored ready:  $Q^2/(2C)$ .
  - When we replace the two batteries with resistanceless wires, the capacitor "sees" two resistors in parallel: one of  $2R$  and another one of  $R$ , thus of total resistance  $2R/3$ . Can you see why?? If yes, then your answer in the final part is obvious.
  - **OPTIONAL:** This is a typical example where the *Thevenin* theorem can simplify our lives **a lot**. Find the Thevenin equivalent of what is presented on the leads of the capacitor. This should be a resistor of  $2R/3$  and an EMF of  $4V/3$ . Can you see this? If yes, you can rework the same problem by considering **just three elements in series:** an EMF of  $4V/3$ , a resistor of  $2R/3$  and a capacitor  $C$ . We have worked out the solution for this configuration in class.
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## [3] Invariance of intervals.

Assume the moving frame has a velocity  $v=vi^{\wedge}$ , i.e., along the x axis. Use Lorentz transformations and write down how event "1" coordinates are translated from the lab frame to the moving frame. Do the same for event "2" and derive how  $\Delta T=t_2-t_1$ ,  $\Delta X=x_2-x_1$ , etc transform under this scheme. Construct  $\Delta S'$  and substitute the primed  $\Delta$  variables from the above equations.

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#### [4] Lorentz transformations.

Fairly straightforward problem involving Lorentz transformations. You will have to find  $\gamma$  and from  $\gamma$  you can readily find  $\beta=v/c$ .

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#### [5] Low velocity Lorentz transformations.

Our strategy for solving this problem is identified in the spelling of the problem, i.e., Taylor expansion. Recall how  $\gamma=1/\sqrt{1-\beta^2}$  is approximated for small enough  $\beta$ 's. Keep first order in  $u/c$  terms only.

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#### [6] Transforming angles.

This problem has a lot of insight to offer us as it touches upon our laws of physics (Gauss' Law) as seen by two observers.

- Assume the stick is in the X-Y plane and forms an angle  $\phi$  with respect to X.
- Identify the X and Y projections of the stick in one of the frames, say the moving one.
- You should convince yourself that the Y projection is the **same** for both observers while the X projection is contracted (see Prof. Fisher handout and the Appendix of our textbook) thus  $X=X'/\gamma$
- Form the ratios  $Y/X$  and  $Y'/X'$  and find how they are related. This gives the transformation of the tangents of the angles  $\phi$  and  $\phi'$  between the two observers.
- Assume an observer is riding on a charge moving with velocity  $v$  with respect to the laboratory. For her, the field of lines are UNIFORM AND ISOTROPIC. If they are represented with sticks emanating from the charges, they will not be seen exactly this way by the laboratory observer. Use the result from the previous step and draw some representative sticks.