Massachusetts Institute of Technology
Department of Physics
8.022 Fall 2004
Assignment 10: Displacement Current; Electromagnetic Waves
Due date: Monday, Dec 6th

1. Purcell 9.5: Electromagnetic Waves.
2. Purcell 9.8: Wave in a box.
3. Purcell 9.10: Magnetic field in a capacitor.
4. Purcell 9.13: Relativistic transformation of wave's fields.
5. Displacement Current.

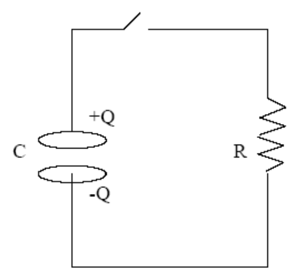

Figure 1: A RC circuit

A capacitor C with circular plates of radius b is charged to a voltage V_0. The space between the two plates is small compared to b so that we can safely ignore any fringing effects. At t = 0 the switch is closed and the capacitor discharges through the resistor R. In all the questions below give your answers in terms of C, b, V_0,R, t and any universal constants.

(a) Give an expression for the charge Q(t) as a function of time of the positively charged plate (upper one in the above figure) of the capacitor.
(b) Find the electric field $\vec{E}(t)$ between the two capacitor plates.
(c) Find the displacement current density $\vec{J}(t)$ between the two capacitor plates.
(d) Find the magnetic field $\vec{B}(t)$ anywhere in between the capacitor plates.

6. Electric and magnetic fields pair.

   A pair of electric and magnetic fields is given by $\vec{E} = E_0\hat{x}cos(\alpha y - \gamma z + \delta t)$, and $\vec{B} = B_0(\hat{y} + \hat{z})cos(\alpha y - \gamma z + \delta t)$, By substituting into all of Maxwell's equations in charge-free and current-free vacuum space derive the conditions that (constants) $\alpha, \gamma, \delta, B_0$ and E_0 have to obey in order the fields to satisfy them. Is this a legitimate electromagnetic wave? Why?

7. An infinite fat wire.

An infinite fat wire, with radius a, carries a constant current I, uniformly distributed over its cross section. A narrow gap in the wire, of width $\omega \ll a$, forms a parallel-plate capacitor, as shown in the figure. Find the magnetic field in the gap, at a distance $s < a$ from the axis.

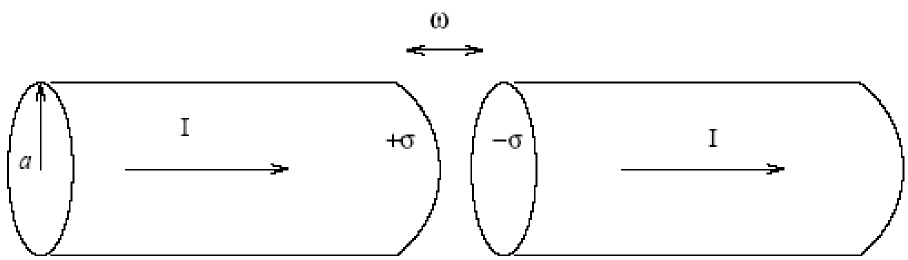

Figure 2: An infinite fatwire

(last problem on next page!!!)

Problem 8 (review problem in preparation for final)

You come across a spherically symmetric electric field with the following form:

$$\begin{aligned}
\vec{E}(r) &= E_0\left(\frac{r}{R}\right)^2\hat{r} \qquad 0 \le r \le R \\
&= 0 \qquad R < r \le 2R \\
&= E_0\left(\frac{r}{R} - 2\right)\hat{r} \qquad 2R < r \le 3R \\
&= E_0\left(\frac{3R}{r}\right)^2\hat{r} \qquad 3R < r \le 4R \\
&= 0 \qquad r > 4R
\end{aligned}$$

$\hat{r}$ is the radial unit vector in spherical coordinates.

(a) For all r, what is the charge $Q(r)$ contained within a radius r?

(b) Calculate the charge density $\rho(r)$ everywhere.

(c) Are there any surface charges in this charge distribution? If so, identify their location and give the magnitude of the surface charge density σ at each such location.

(d) The charge distribution is modified in some way. The new electric field is

$$\begin{aligned}
\vec{E}(r) &= E_0\left(\frac{r}{R}\right)^2\hat{r} \qquad 0 \le r \le R \\
&= 0 \qquad R < r \le 2R \\
&= E_0\left(\frac{r}{R} - 2\right)\hat{r} \qquad 2R < r \le 3R \\
&= E_0\left(\frac{3R}{r}\right)^2\hat{r} \qquad 3R < r \le 7R/2 \\
&= 0 \qquad r > 7R/2
\end{aligned}$$

Compute the difference in energy between this and the old configuration, $U_{\text{new}} - U_{\text{old}}$. Was work done on the system or did the system do work?