Class 05: Outline

Hour 1:

Gauss' Law

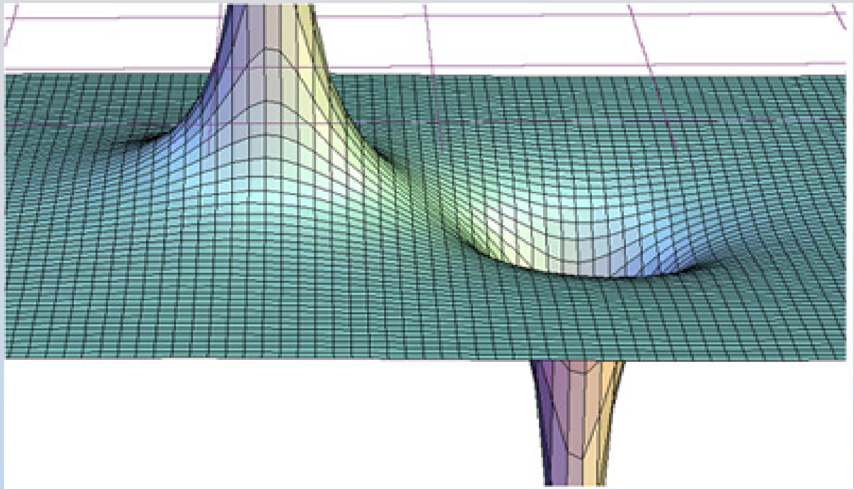
Hour 2:

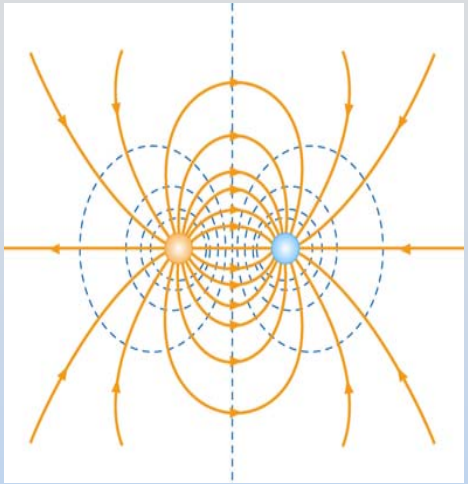
Gauss' Law

Six PRS Questions On Pace and Preparation

Last Time:
Potential and E Field

E Field and Potential: Creating





A point charge *q* creates a field and potential around it:

$$\vec{\mathbf{E}} = k_e \frac{q}{r^2}\hat{\mathbf{r}}; \;\; V = k_e \frac{q}{r}$$

Use superposition for systems of charges

They are related:

$$\vec{\mathbf{E}} = -\nabla V; \;\; \Delta V \equiv V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}}$$

E Field and Potential: Effects

If you put a charged particle, *q*, in a field:

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$$

To move a charged particle, *q*, in a field:

$$W = \Delta U = q\Delta V$$

Two PRS Questions: Potential & E Field

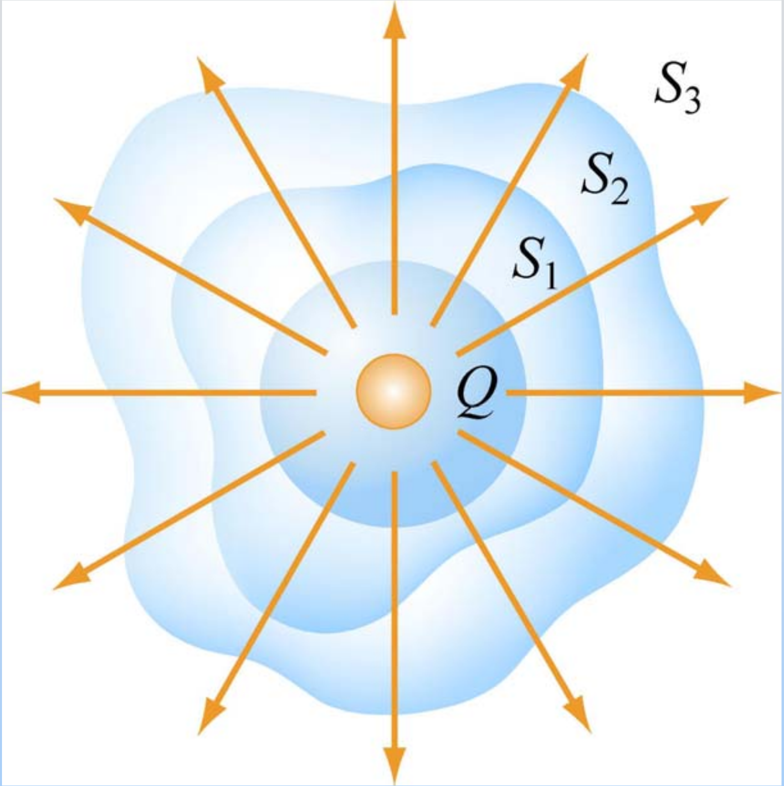
Gauss's Law

The first Maxwell Equation

A very useful computational technique

This is important!

Gauss's Law – The Idea



The total "flux" of field lines penetrating any of these surfaces is the same and depends only on the amount of charge inside

Gauss's Law – The Equation

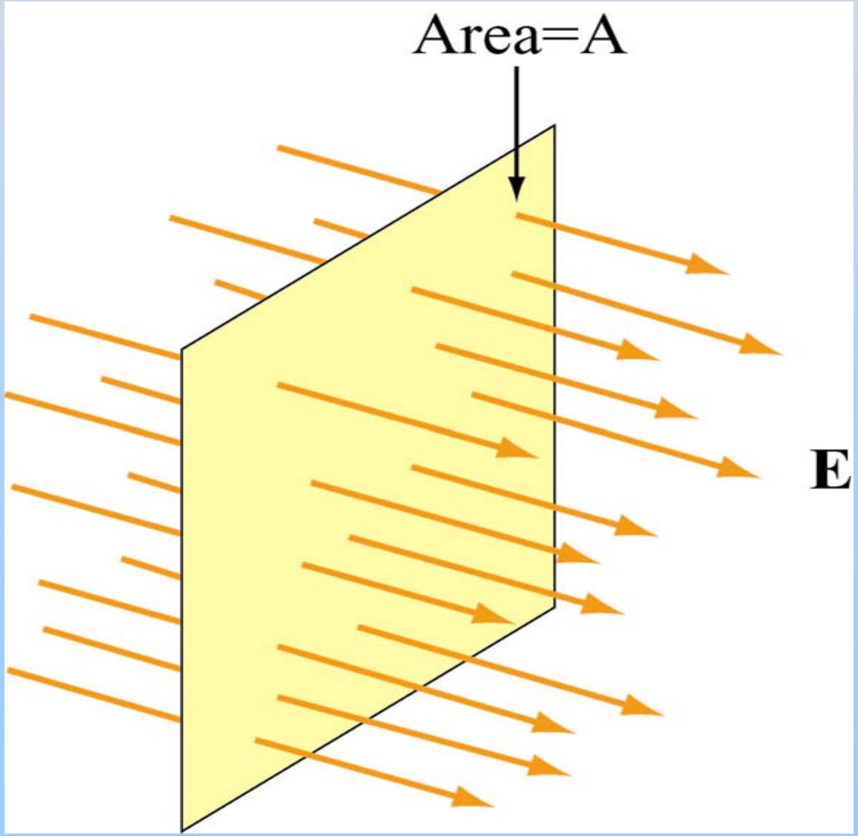
$$\boxed{\Phi_E = \underset{\substack{\text{closed} \\ \text{surface} S}}{\oint\!\!\oint} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_0}}$$

Electric flux Φ_E (the surface integral of E over closed surface *S)* is proportional to charge inside the volume enclosed by *S*

Now the Details

Electric Flux Φ_E

Case I: E is constant vector field perpendicular to planar surface S of area A



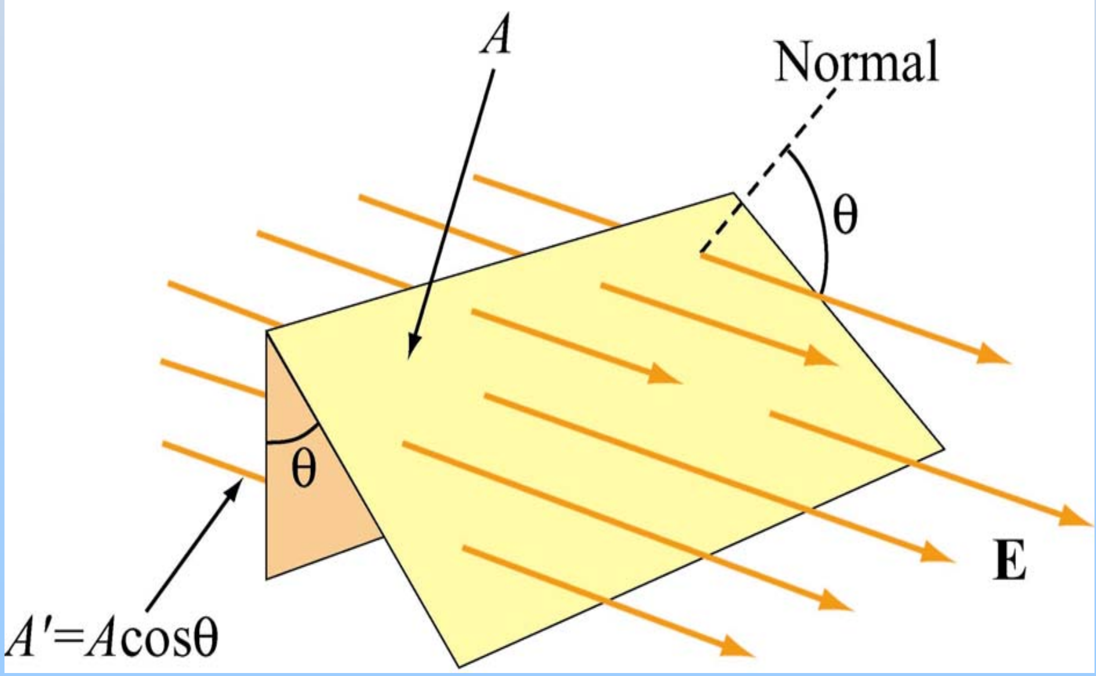
$$\Phi_E = \iint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

$$\Phi_E = +EA$$

Our Goal: Always reduce problem to this

Electric Flux Φ_E

Case II: E is constant vector field directed at angle θ to planar surface S of area A



$$\Phi_E = \iint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

$$\Phi_E = EA\cos\theta$$

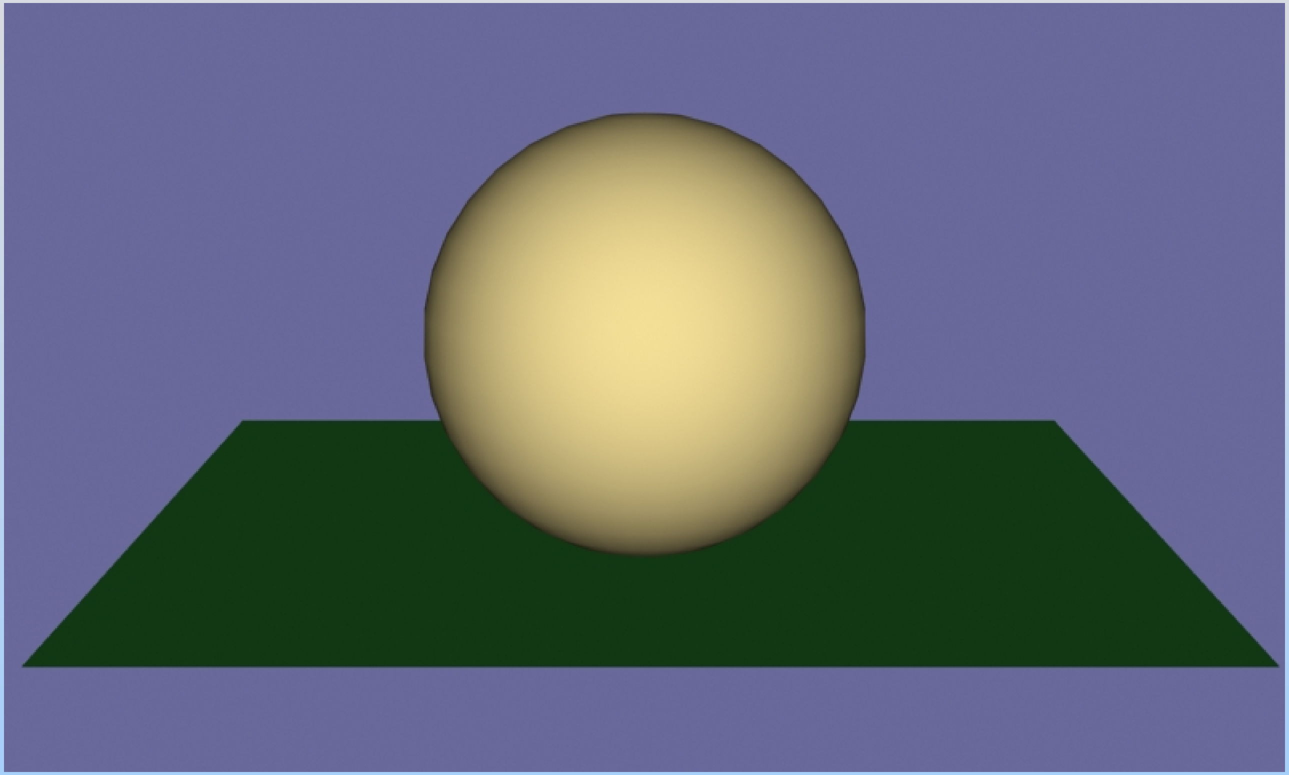
PRS Question: Flux Thru Sheet

Gauss's Law

$$\Phi_E = \underset{\substack{\text{closed} \\ \text{surfaceS}}}{\oiint} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_0}$$

Note: Integral must be over closed surface

Open and Closed Surfaces



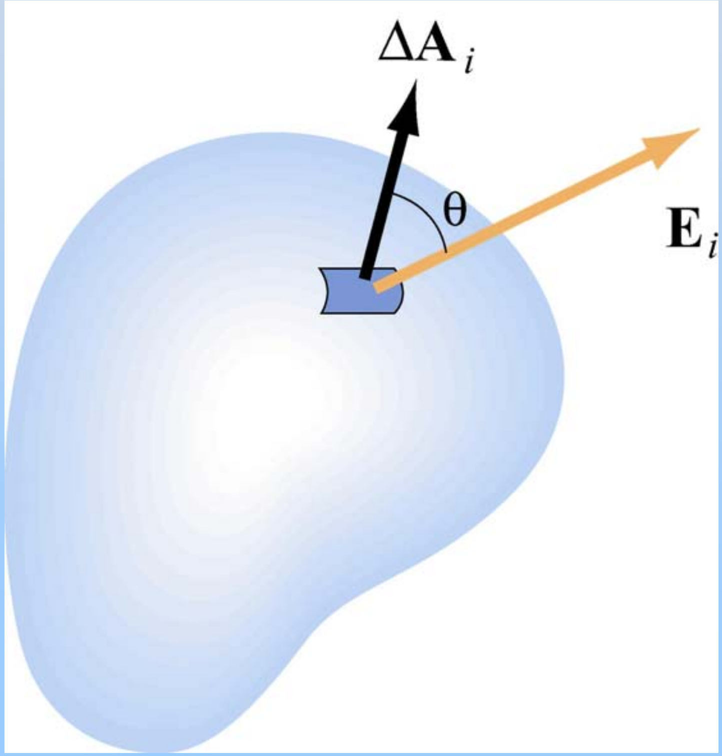
A rectangle is an open surface — it does NOT contain a volume

A sphere is a closed surface — it DOES contain a volume

Area Element dA: Closed Surface

For closed surface, d**A** is normal to surface and points outward

(from inside to outside)

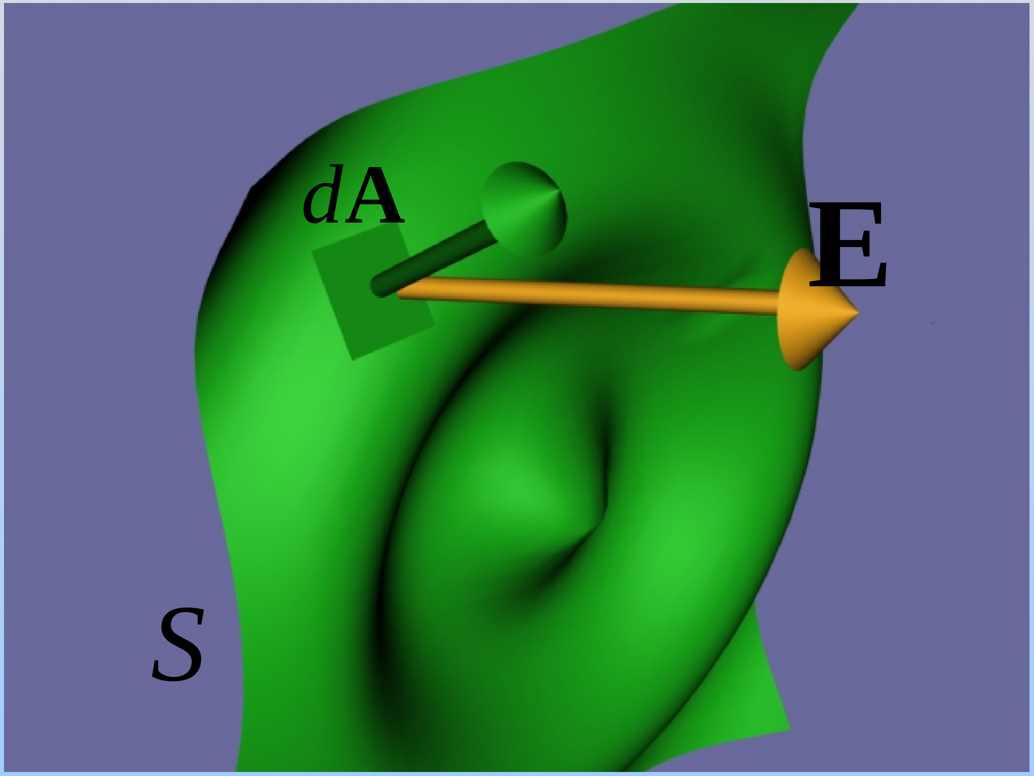


$\Phi_E > 0$ if E points out

$\Phi_E < 0$ if E points in

Electric Flux Φ_E

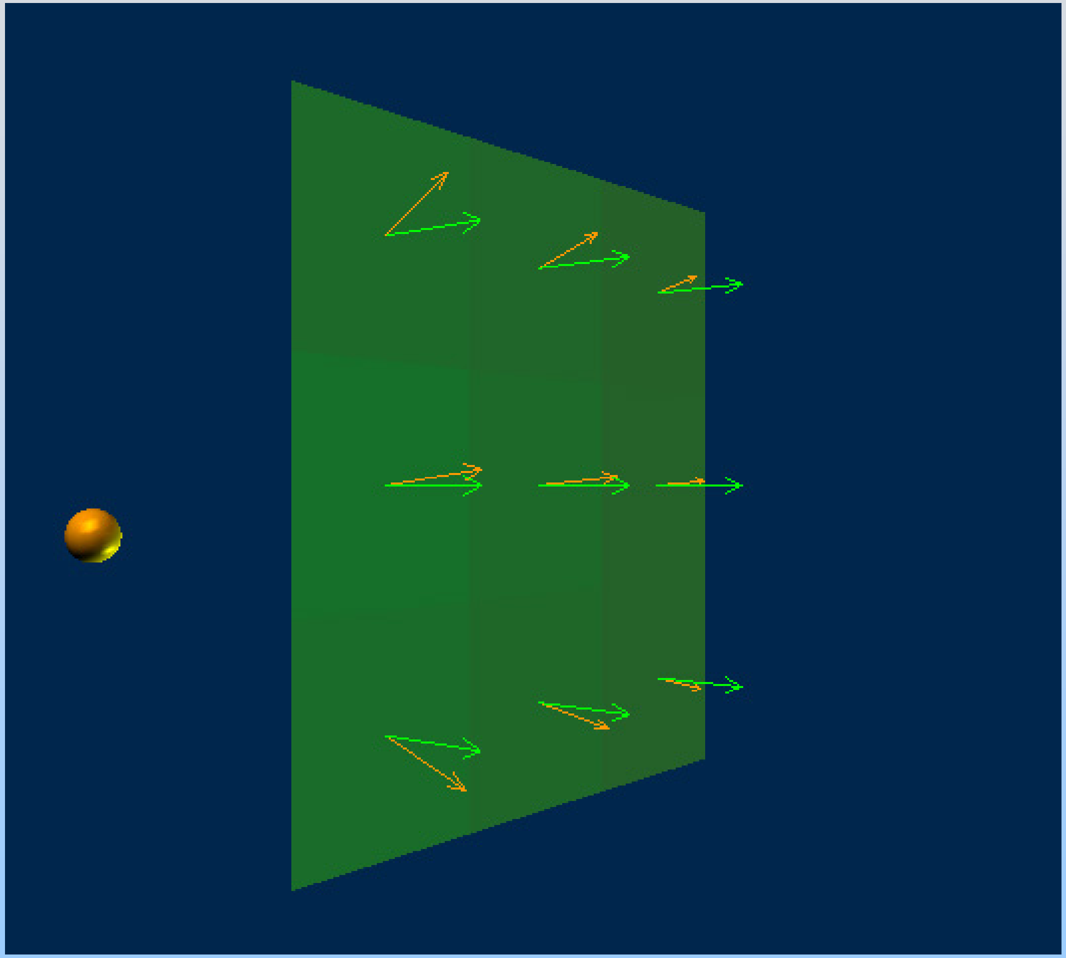
Case III: E not constant, surface curved



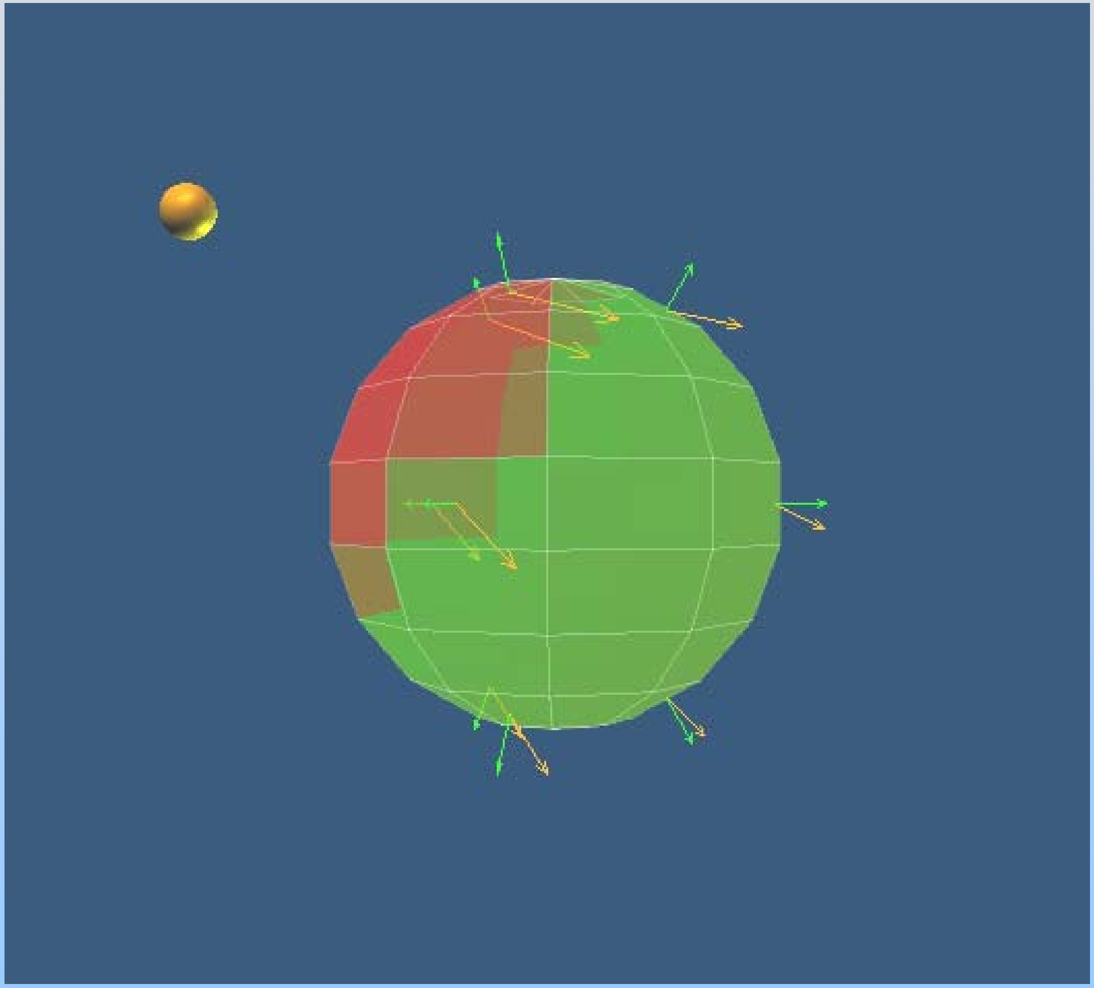
$$d\Phi_E = \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

$$\Phi_E = \iint d\Phi_E$$

Example: Point Charge Open Surface



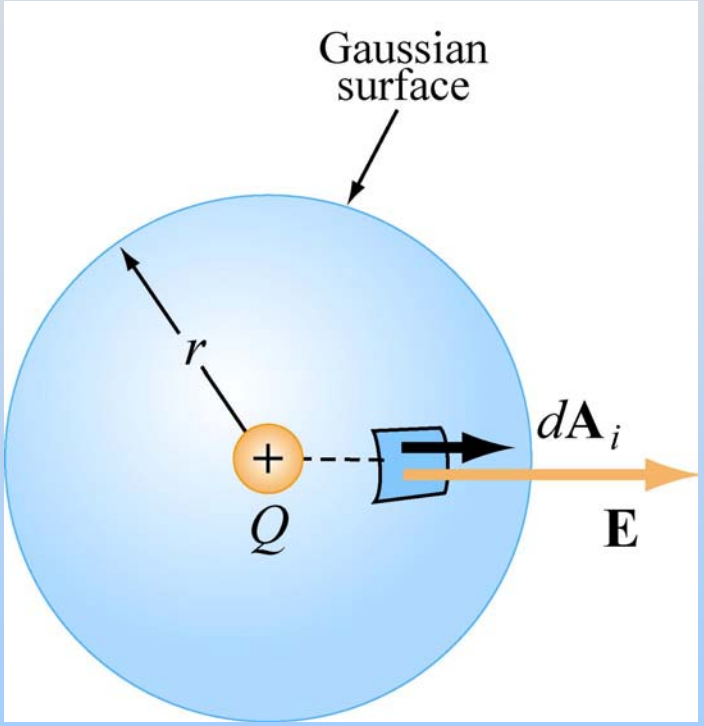
Example: Point Charge Closed Surface



PRS Question:
Flux Thru Sphere

Electric Flux: Sphere

Point charge $\boldsymbol{Q}$ at center of sphere, radius r



E field at surface:

$$\vec{\mathbf{E}} = \frac{Q}{4\pi\varepsilon_0 r^2}\,\hat{\mathbf{r}}$$

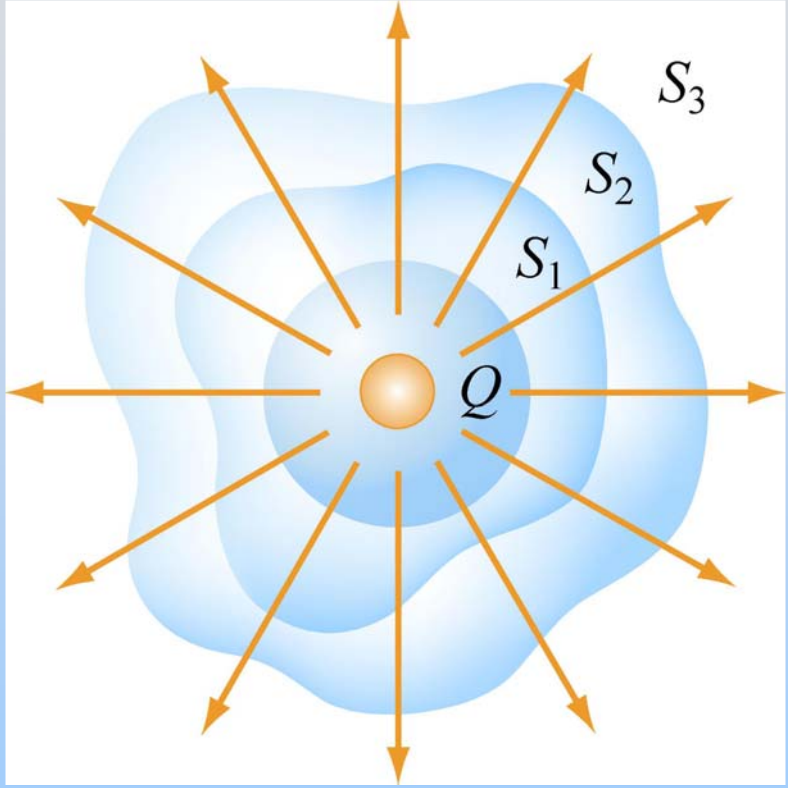
Electric flux through sphere:

$$\Phi_E = \oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oiint_S \frac{Q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}} \cdot dA\hat{\mathbf{r}}$$

$$= \frac{Q}{4\pi\varepsilon_0 r^2}\oiint_S dA = \frac{Q}{4\pi\varepsilon_0 r^2}4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\boxed{d\vec{\mathbf{A}} = dA\hat{\mathbf{r}}}$$

Arbitrary Gaussian Surfaces



$$\Phi_E = \underset{\substack{\text{closed}\\ \text{surface S}}}{\oiint} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q}{\varepsilon_0}$$

For all surfaces such as S_1, S_2 or S_3

Applying Gauss's Law

1. Identify regions in which to calculate E field.
2. Choose Gaussian surfaces S: Symmetry
3. Calculate $\Phi_E = \oint\!\!\oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$
4. Calculate q_{in}, charge enclosed by surface S
5. Apply Gauss's Law to calculate E:

$$\boxed{\Phi_E = \oint\!\!\oint_{\text{closed surface} S} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_0}}$$

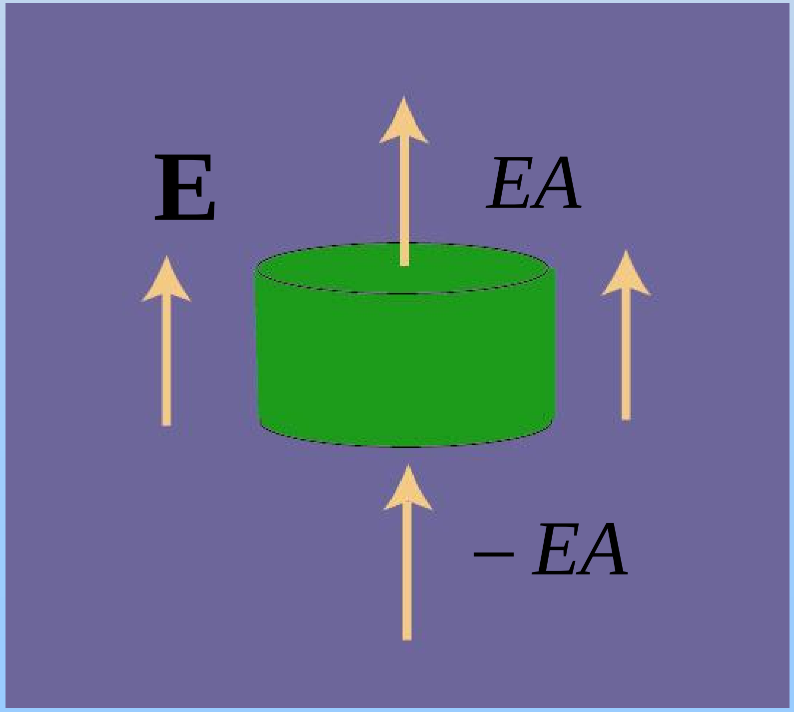
Choosing Gaussian Surface

Choose surfaces where **E** is perpendicular & constant.
Then flux is EA or -EA.

OR

Choose surfaces where **E** is parallel.

Then flux is zero



Example: Uniform Field

Flux is EA on top
Flux is –EA on bottom
Flux is zero on sides

Symmetry & Gaussian Surfaces

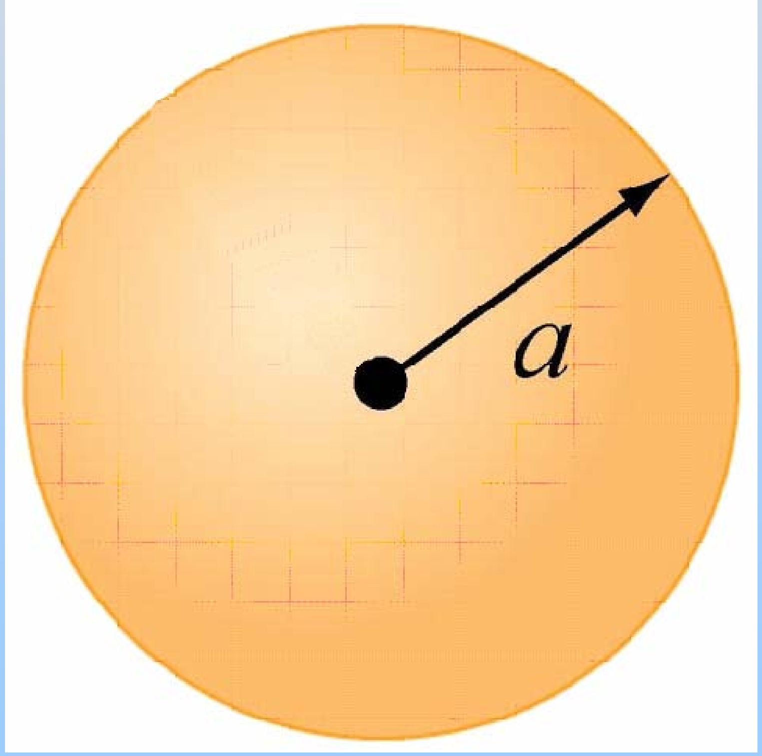
Use Gauss's Law to calculate E field from highly symmetric sources

Symmetry	**Gaussian Surface**
Spherical	Concentric Sphere
Cylindrical	Coaxial Cylinder
Planar	Gaussian "Pillbox"

PRS Question:
Should we use Gauss' Law?

Gauss: Spherical Symmetry

$+Q$ uniformly distributed throughout non-conducting solid sphere of radius a. Find **E** everywhere

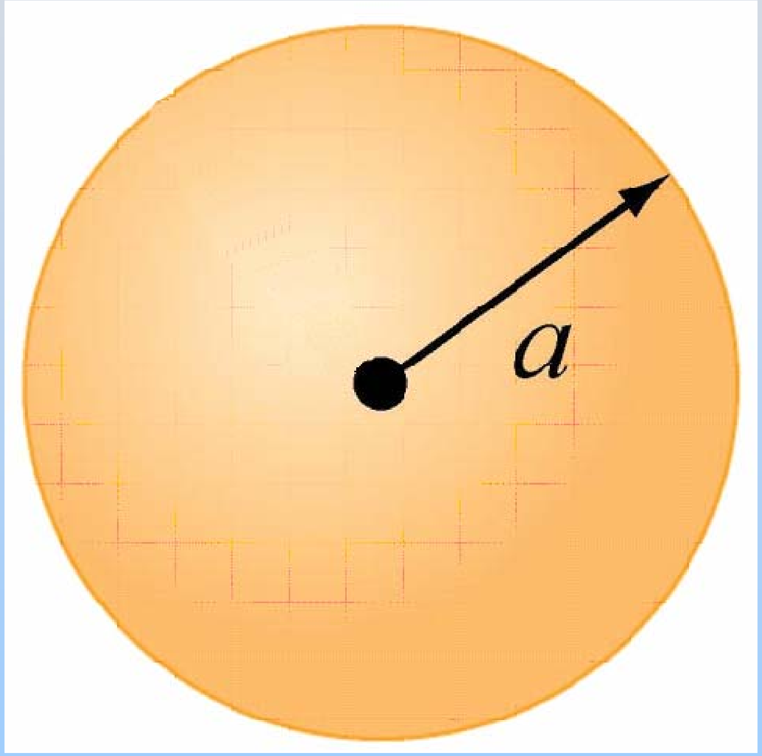


Gauss: Spherical Symmetry

Symmetry is Spherical

$$\vec{\mathbf{E}} = E\,\hat{\mathbf{r}}$$

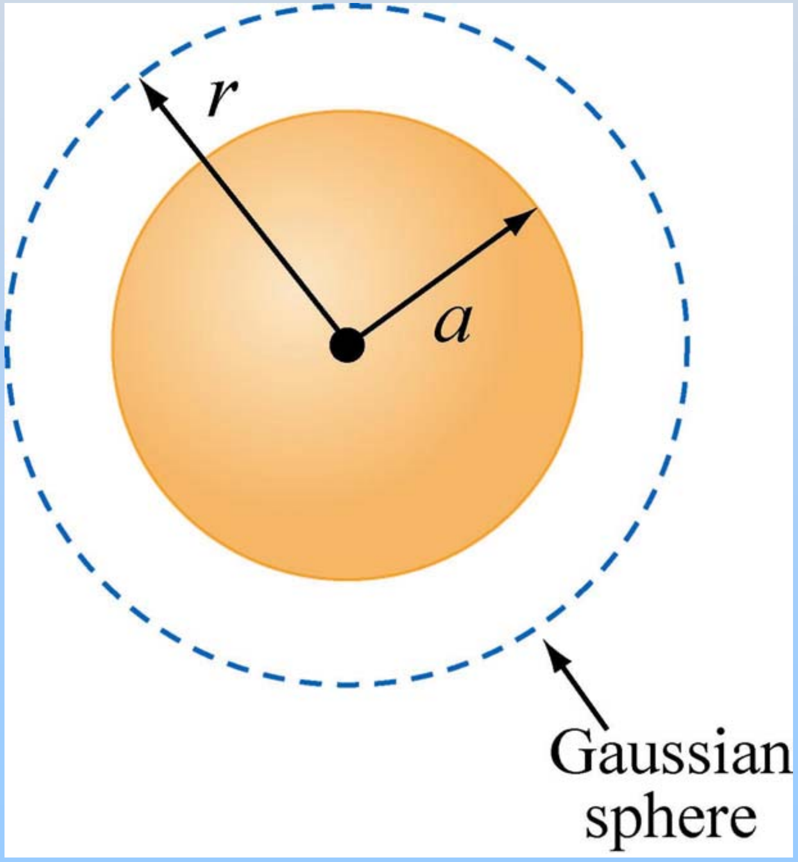
Use Gaussian Spheres



Gauss: Spherical Symmetry

Region 1: $r > a$

Draw Gaussian Sphere in Region 1 ($r > a$)



Note: r is arbitrary **but** is the radius for which you will calculate the E field!

Gauss: Spherical Symmetry

Region 1: $r > a$

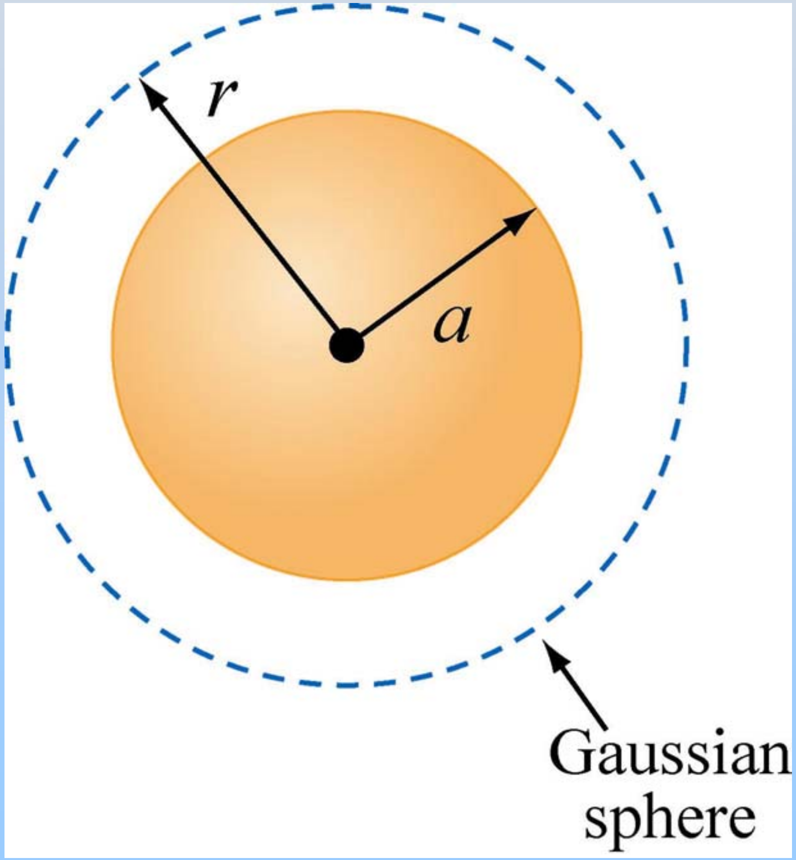
Total charge enclosed $q_{in} = +Q$

$$\Phi_E = \oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E \oiint_S dA = EA$$

$$= E\left(4\pi r^2\right)$$

$$\Phi_E = 4\pi r^2 E = \frac{q_{in}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \boxed{\Rightarrow \vec{\mathbf{E}} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}}$$

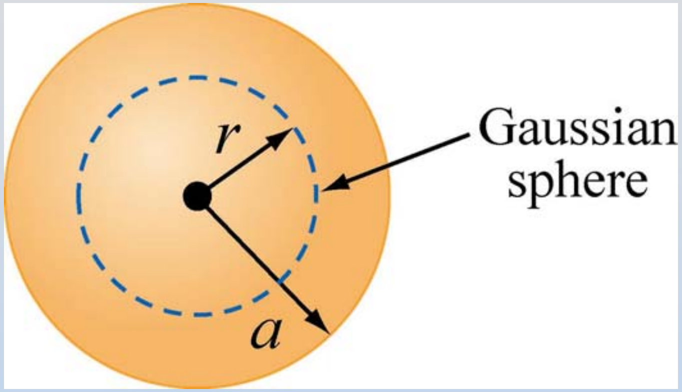


Gauss: Spherical Symmetry

Region 2: $r < a$

Total charge enclosed:

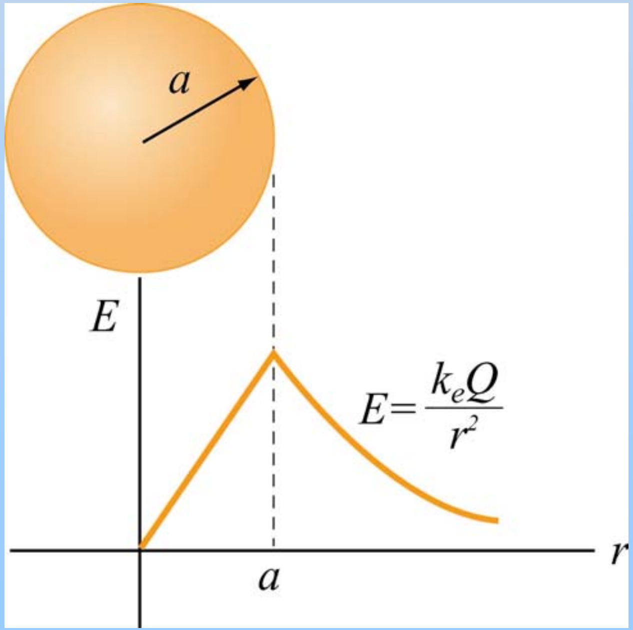
$$q_{in} = \left(\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi a^3} \right) Q = \left(\frac{r^3}{a^3} \right) Q \quad \text{OR} \quad q_{in} = \rho V$$



Gauss's law:

$$\Phi_E = E\left(4\pi r^2\right) = \frac{q_{in}}{\varepsilon_0} = \left(\frac{r^3}{a^3} \right) \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0} \frac{r}{a^3} \quad \boxed{\Rightarrow \vec{\mathbf{E}} = \frac{Q}{4\pi\varepsilon_0} \frac{r}{a^3} \hat{\mathbf{r}}}$$

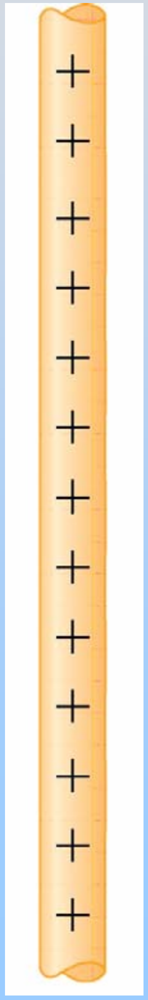


PRS Question:
Field Inside Spherical Shell

Gauss: Cylindrical Symmetry

Infinitely long rod with uniform charge density λ

Find **E** outside the rod.



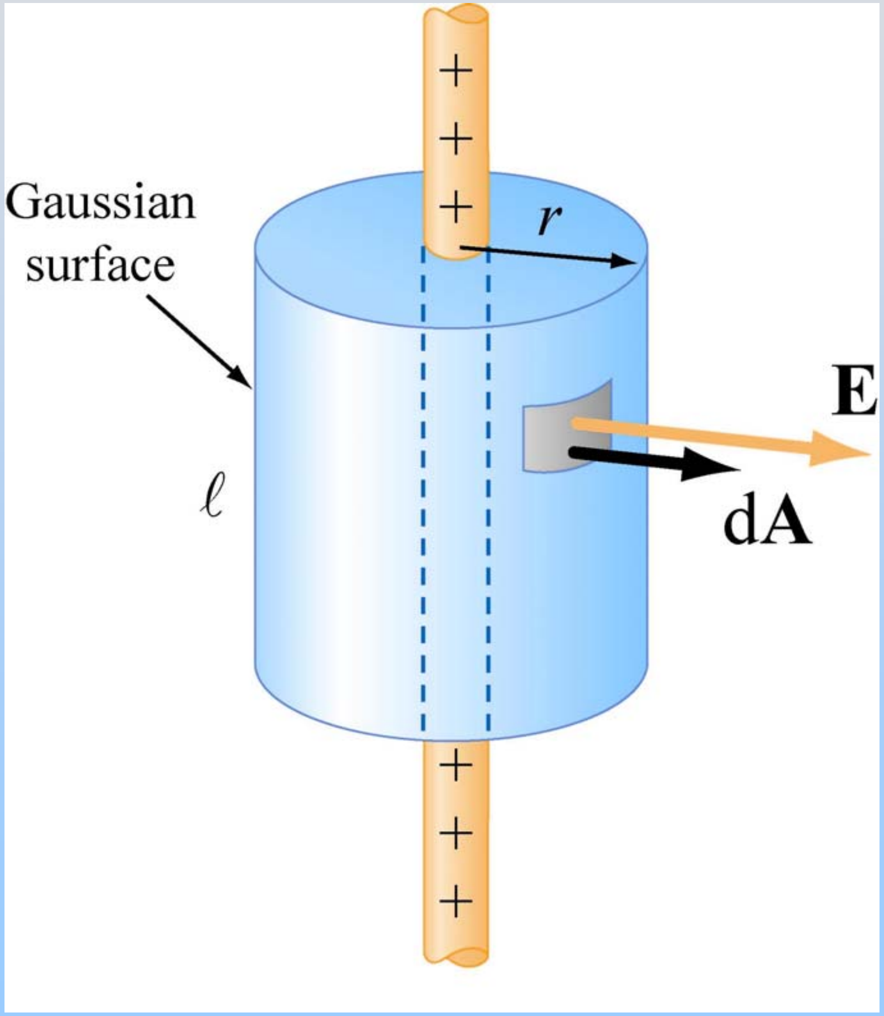
Gauss: Cylindrical Symmetry

Symmetry is Cylindrical

$$\vec{\mathbf{E}} = E\,\hat{\mathbf{r}}$$

Use Gaussian Cylinder

Note: *r* is arbitrary **but** is the radius for which you will calculate the E field! ℓ is arbitrary and should divide out



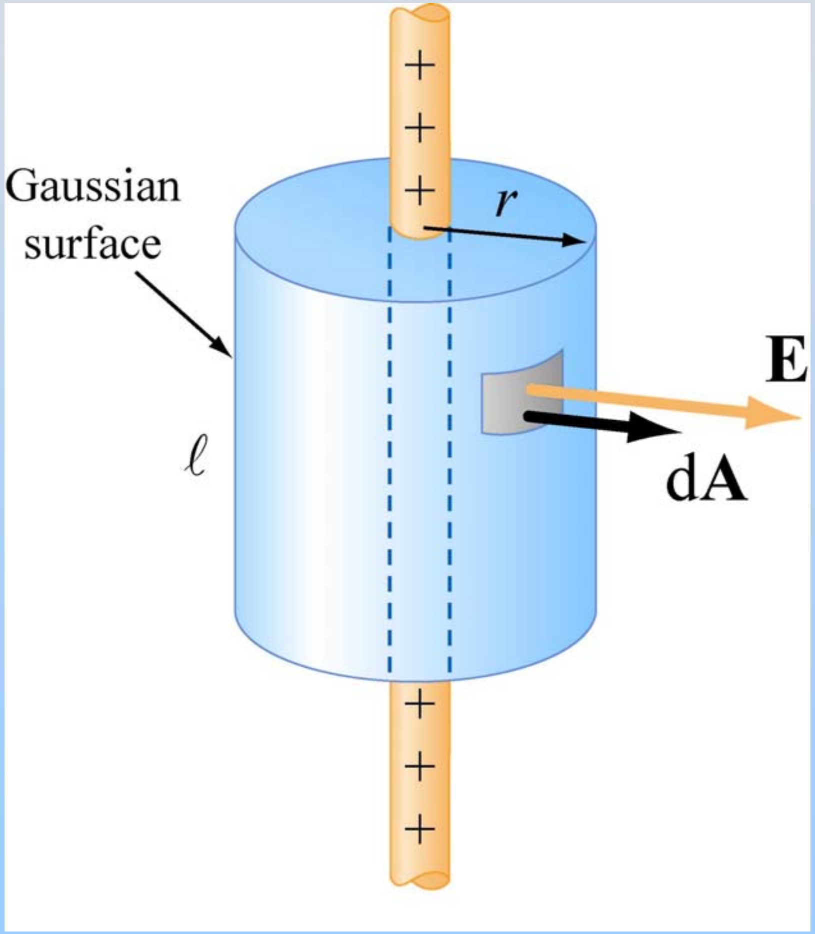
Gauss: Cylindrical Symmetry

Total charge enclosed: $q_{in} = \lambda \ell$

$$\Phi_E = \oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E \oiint_S dA = EA$$

$$= E\left(2\pi r \ell\right) = \frac{q_{in}}{\varepsilon_0} = \frac{\lambda \ell}{\varepsilon_0}$$

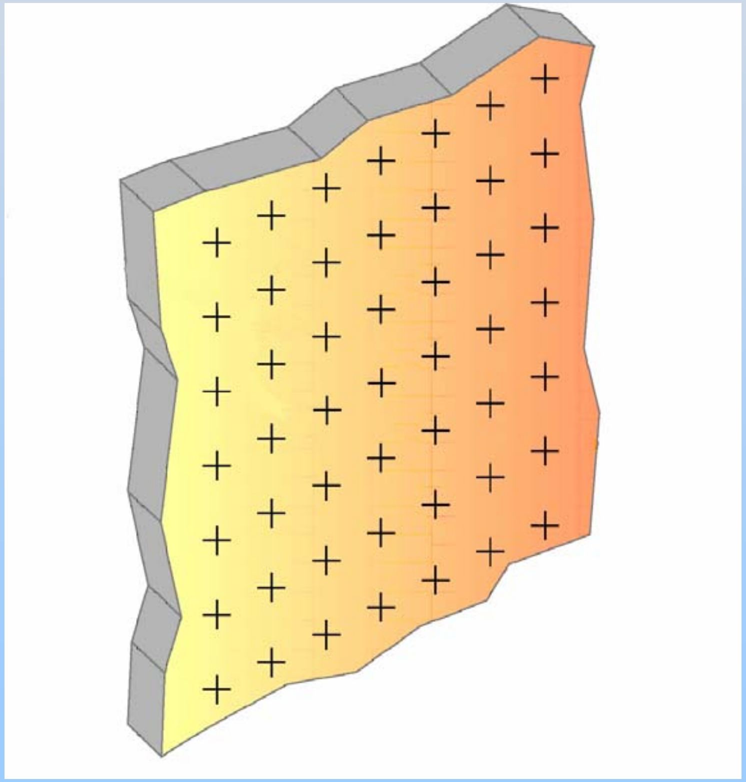
$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \boxed{\Rightarrow \vec{\mathbf{E}} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{\mathbf{r}}}$$



Gauss: Planar Symmetry

Infinite slab with uniform charge density σ

Find **E** outside the plane



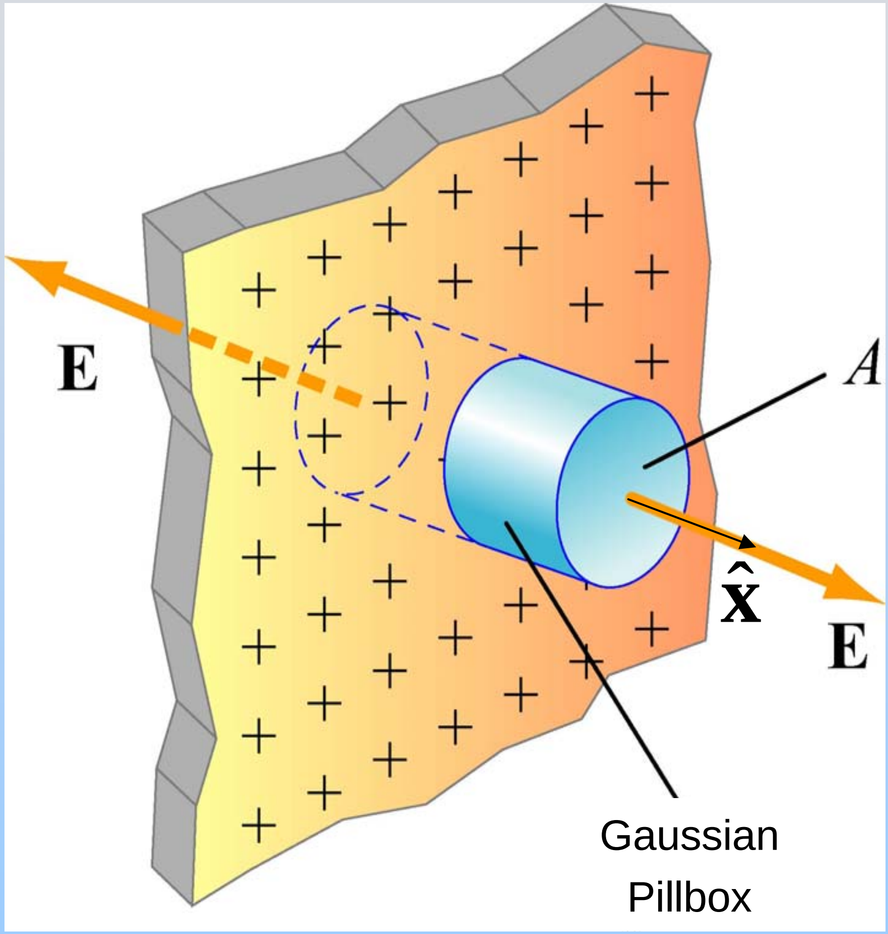
Gauss: Planar Symmetry

Symmetry is Planar

$$\vec{\mathbf{E}} = \pm E\,\hat{\mathbf{x}}$$

Use Gaussian Pillbox

Note: *A* is arbitrary (its size and shape) and should divide out



Gauss: Planar Symmetry

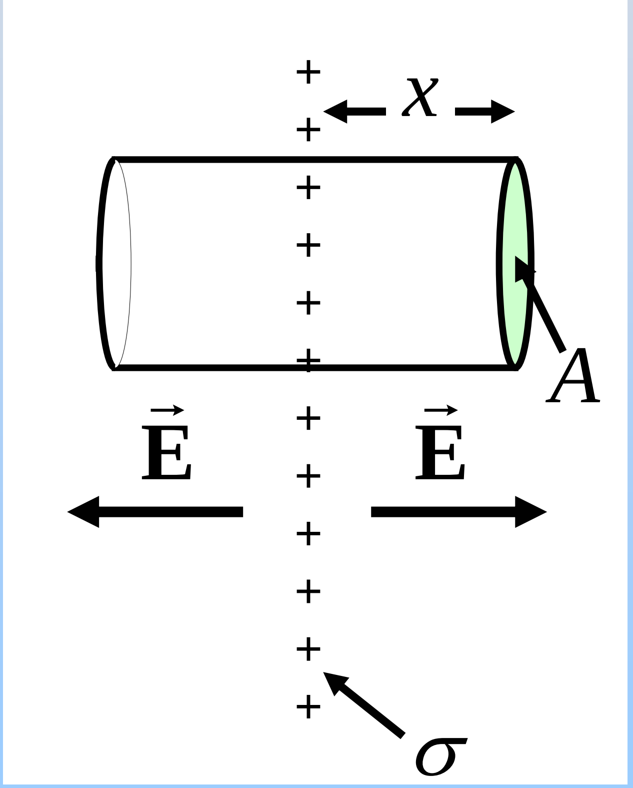
Total charge enclosed: $q_{in} = \sigma A$

NOTE: No flux through side of cylinder, only endcaps

$$\Phi_E = \underset{S}{\oiint} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E \underset{S}{\oiint} dA = EA_{Endcaps}$$

$$= E\left(2A\right) = \frac{q_{in}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0} \boxed{\Rightarrow \vec{\mathbf{E}} = \frac{\sigma}{2\varepsilon_0}\begin{Bmatrix} \hat{\mathbf{x}} & \text{to right} \\ -\hat{\mathbf{x}} & \text{to left} \end{Bmatrix}}$$



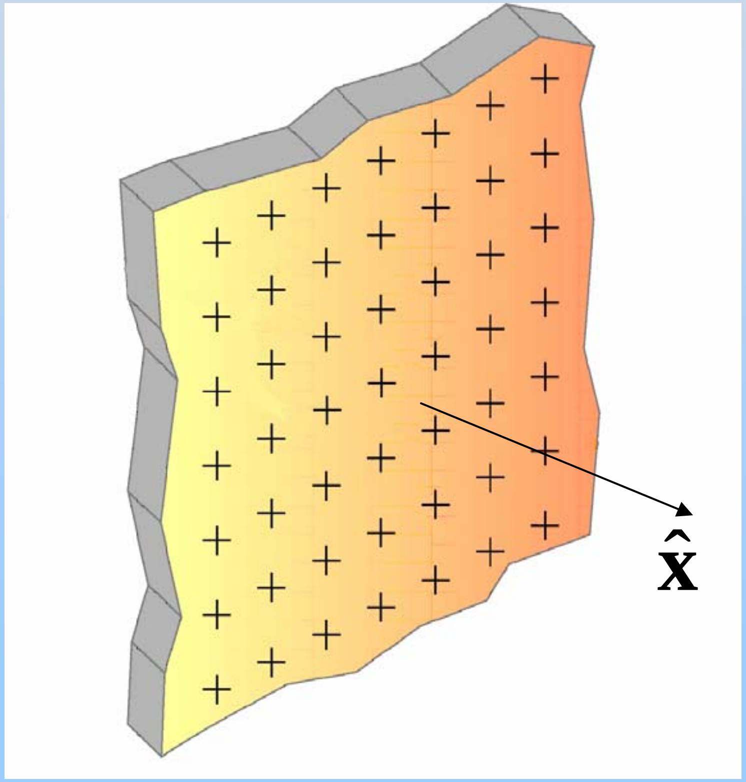
PRS Question:
Slab of Charge

Group Problem: Charge Slab

Infinite slab with uniform charge density ρ

Thickness is 2d (from x=-d to x=d).

Find **E** everywhere.



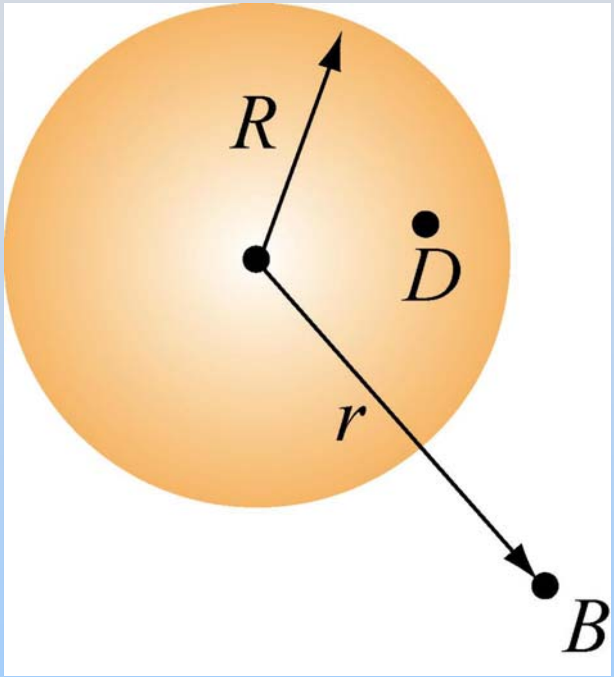
PRS Question:
Slab of Charge

Potential from E

Potential for Uniformly Charged Non-Conducting Solid Sphere

From Gauss's Law

$$\vec{\mathbf{E}} = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}, & r > R \\ \dfrac{Qr}{4\pi\varepsilon_0 R^3}\hat{\mathbf{r}}, & r < R \end{cases}$$



Use $\quad V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}}$

Point Charge!

Region 1: $r > a$

$$V_B - \underbrace{V(\infty)}_{=0} = -\int_{\infty}^{r} \frac{Q}{4\pi\varepsilon_0 r^2} dr = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Potential for Uniformly Charged Non-Conducting Solid Sphere

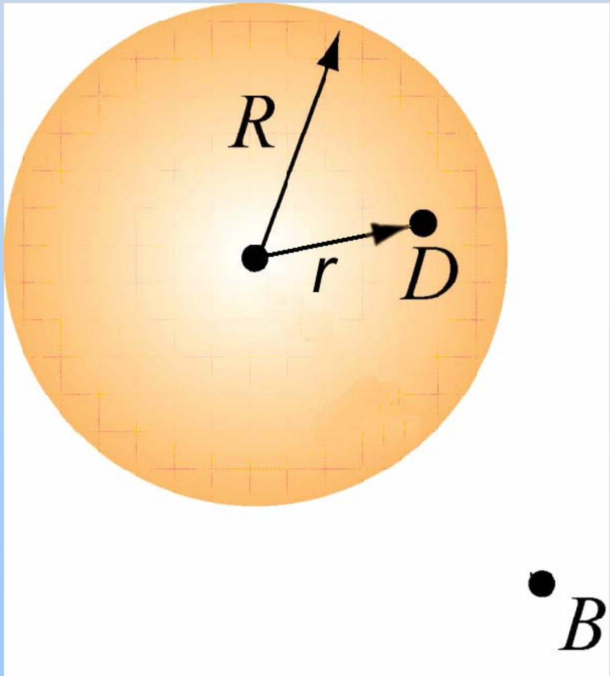
Region 2: $r < a$

$$V_D - \underbrace{V(\infty)}_{=0} = -\int_\infty^R dr E(r > R) - \int_R^r dr E(r < R)$$

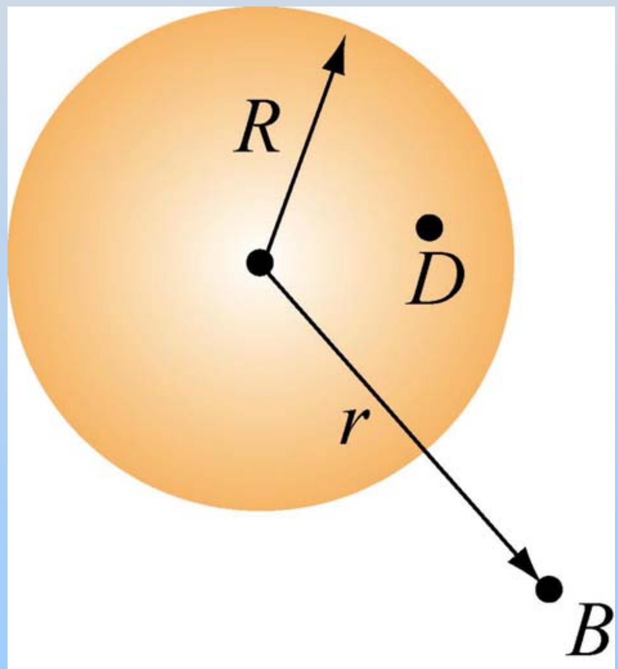
$$= -\int_\infty^R dr \frac{Q}{4\pi\varepsilon_0 r^2} - \int_R^r dr \frac{Qr}{4\pi\varepsilon_0 R^3}$$

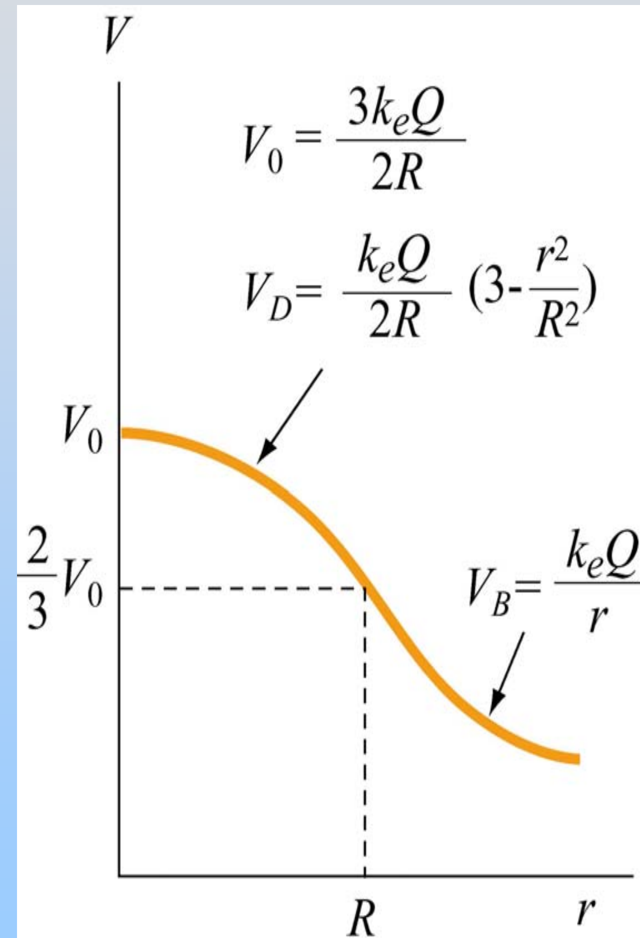
$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}\frac{1}{2}\left(r^2 - R^2\right)$$

$$= \frac{1}{8\pi\varepsilon_0}\frac{Q}{R}\left(3 - \frac{r^2}{R^2}\right)$$



Potential for Uniformly Charged Non-Conducting Solid Sphere



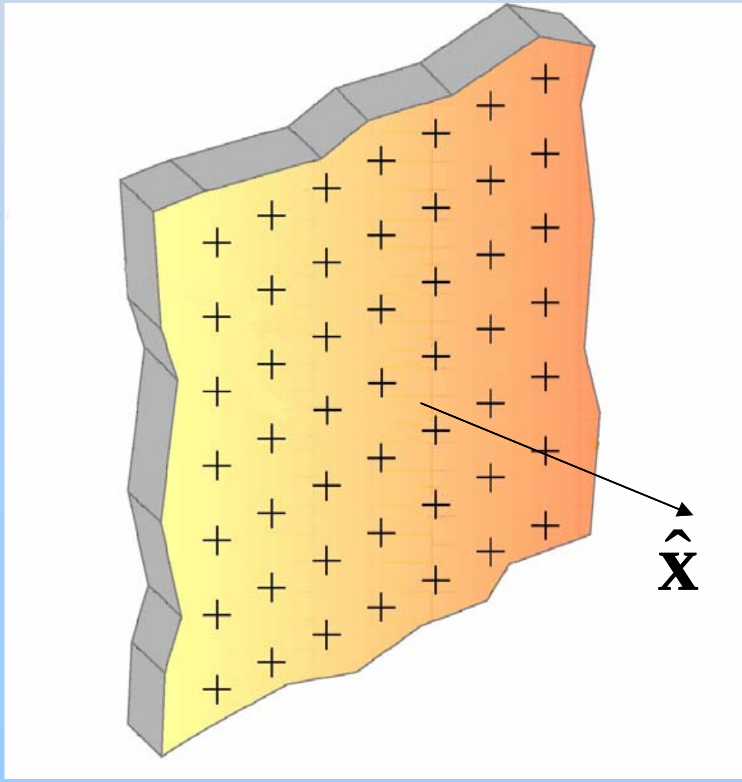


Group Problem: Charge Slab

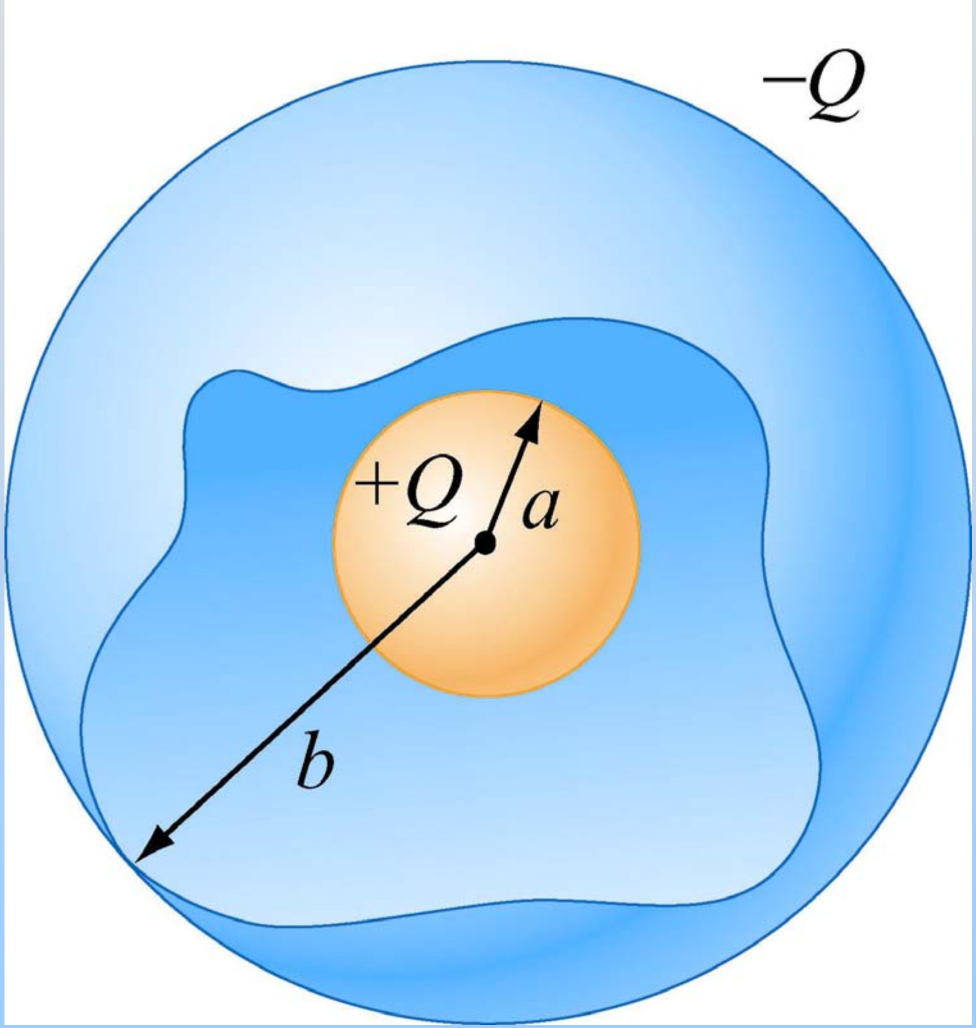
Infinite slab with uniform charge density ρ

Thickness is 2d (from x=-d to x=d).

If V=0 at x=0 (definition) then what is V(x) for x>0?



Group Problem: Spherical Shells



These two spherical shells have equal but opposite charge.

Find E everywhere

Find V everywhere (assume $V(\infty) = 0$)