

**Topics:** Faraday's Law; Mutual Inductance & Transformers

**Related Reading:**

Course Notes (Liao et al.): Chapter 10; Section 11.1

Serway & Jewett: Chapter 31; Section 32.4

Giancoli: Chapter 29; Section 30.1

**Experiments:** (9) Faraday's Law of Induction

## Topic Introduction

Today we continue our discussion of induction (Faraday's Law), discussing another application – eddy current braking – and then continuing on to define mutual inductance and transformers.

### Faraday's Law & Lenz's Law

Remember that Faraday's Law tells us that a changing magnetic flux generates an EMF (electromotive force):

$$\mathcal{E} = -\frac{d\Phi_{B\Box}}{dt\Box}, \text{ where } \Phi_{B\Box} = \iint \vec{B} \cdot d\vec{A} \text{ is the magnetic flux, and } \mathcal{E} = \oint \vec{E}' \cdot d\vec{s} \text{ is the EMF}$$

In the formula above,  $\vec{E}'$  is the electric field measured in the rest frame of the circuit, if the circuit is moving. Lenz's Law tells us that the direction of that EMF is so as to oppose the change in magnetic flux. That is, if there were a physical loop of wire where you are trying to determine the direction of the EMF, a current would be induced in it that creates a flux to either supplement a decreasing flux or decrease an increasing flux.

### Applications

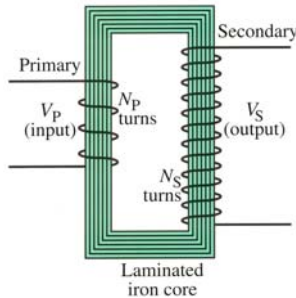
As we saw in the last class, a number of technologies rely on induction to work – generators, microphones, metal detectors, and electric guitars to name a few. Another common application is eddy current braking. A magnetic field penetrating a metal spinning disk (like a wheel) will induce eddy currents in the disk, currents which circle inside the disk and exert a torque on the disk, trying to stop it from rotating. This kind of braking system is commonly used in trains. Its major benefit (aside from eliminating costly service to maintain brake pads) is that the braking torque is proportional to angular velocity of the wheel, meaning that the ride smoothly comes to a halt.

### Mutual Inductance

As we saw last class, there are several ways of changing the flux through a loop – by changing the angle between the loop and the field (generators), the area of the loop (the sliding bar problem) or the strength of the field. In fact, this last method is the most common. Combining this idea with the idea that magnetic fields are typically generated by currents, we can see that changing currents generate EMFs. This is the idea of mutual inductance: given any two circuits, a changing current in one will induce an EMF in the other, or, mathematically,  $\mathcal{E}_2 = -M\frac{dI_1}{dt\Box}$ , where  $M$  is the mutual inductance of the two circuits. How does this work? The current in loop 1 produces a magnetic field (and hence flux) through loop 2. If that current changes in time, the flux through 2 changes in time, creating an EMF in loop 2. The mutual inductance,  $M$ , depends on geometry, both on how well the current in the first loop can create a magnetic field and on how much magnetic flux

through the second loop that magnetic field will create. Interestingly, mutual inductance is symmetric – if you flip the subscripts in the above equation, it remains true.

### Transformers



A major application of mutual inductance is the transformer, which allows the easy modification of the voltage of AC (alternating current) signals. At left is a picture/schematic of a step up transformer, which will take an input signal on the primary (of input voltage  $V_P$ ) and create an output signal on the secondary (of, in this case larger, output voltage  $V_S$ ). How does it work? The primary coil creates an oscillating magnetic field as an oscillating current is driven through it. This magnetic field is “steered” through the iron core – recall that ferromagnets like iron act like wires for magnetic fields, allowing the field to be bent around in a loop, as is done here. As the oscillating magnetic field punches through the many turns of the secondary coil, it generates an oscillating flux through them, which will induce an EMF in the secondary. If all else is equal and the core is perfect in its guiding of the field lines, the amount of flux generated and received is directly proportional to the number of turns in each coil. Hence the ratio of the output to input voltage is the same as the ratio of the number of turns in the secondary to the number of turns in the primary. As pictured we have more turns in the secondary, hence this is a “step up transformer,” with a larger output voltage than input.

The ease of creating transformers is a strong argument for using AC rather than DC power, and is one of the reasons that our main power system is AC. Why? Before sending power across transmission lines, the voltage is stepped up to a very high voltage (240,000 V), which leads to lower energy losses as the currents flow through the transmission lines (think about why this is the case). The voltage is then stepped down to 240 V before going into your home.

### Important Equations

Faraday’s Law (in a coil of  $N$  turns):

$$\mathcal{E} = -N \frac{d\Phi_B}{dt}$$

Magnetic Flux (through a single loop):

$$\Phi_{B\Box} = \iint \vec{B} \cdot d\vec{A}$$

EMF:

$$\mathcal{E} = \oint \vec{E}' \cdot d\vec{s} \quad \text{where } \vec{E}' \text{ is the electric field measured in the rest frame of the circuit, if the circuit is moving.}$$

Mutual Inductance:

$$\mathcal{E}_2 = -M \frac{dI_1}{dt}$$

### Experiment 9: Faraday’s Law of Induction

**Preparation:** Read lab write-up.