Welcome back to 8.033!


Image courtesy of Wikipedia.

Emmy Noether

1882-1935

MIT Course 8.033, Fall 2006, Lecture 2

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PRACTICAL STUFF:

- PS1 due Friday 4PM
- Symmetry notes posted

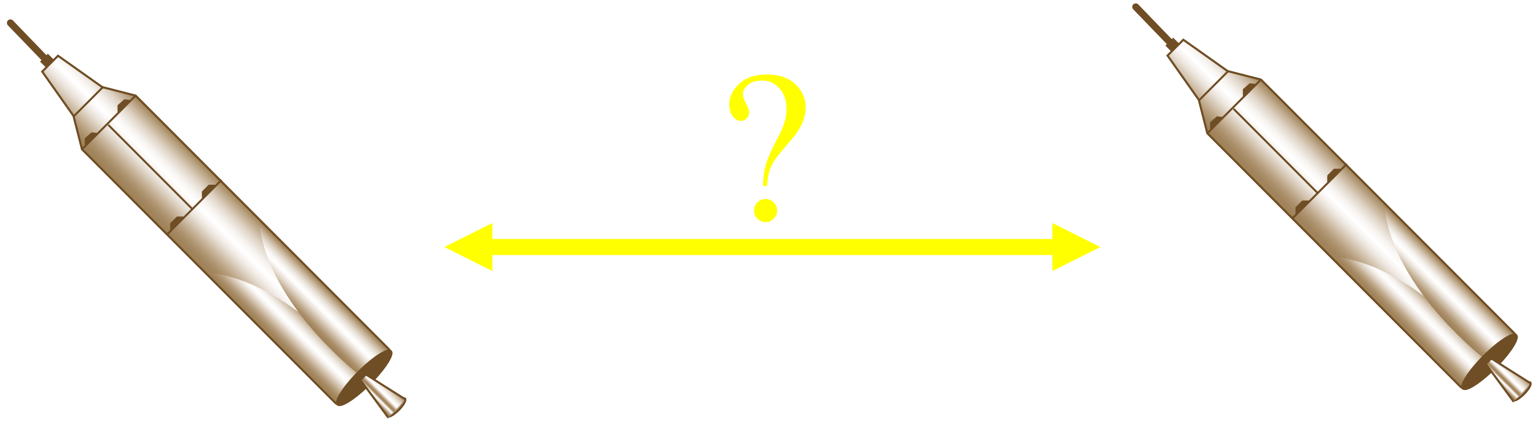
TODAY'S TOPIC: SYMMETRY IN PHYSICS

- **Key concepts:** frame, inertial frame, transformation, invariant, invariance, symmetry, relativity
- **Key people:** Galileo Galileo, Emmy Noether
- **Symmetry examples:** translation, rotation, parity, boost
- **Million Dollar question:** what are the symmetries of physics?

What do we mean by symmetry?

WHAT'S THE SYMMETRY OF THE UNIVERSE?

OF PHYSICS?



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Invariance under translation

- No experiment within your lab can determine whether it's been shifted
- Original frame: masses at $\mathbf{r}_1$ and $\mathbf{r}_2$.

$$F = \frac{GmM}{|\mathbf{r}_2 - \mathbf{r}_1|^2}$$

- Primed frame: masses at $\mathbf{r}_1' \equiv \mathbf{r}_1 + \mathbf{a}$ and $\mathbf{r}_2' \equiv \mathbf{r}_2 + \mathbf{a}$.

$$F' = \frac{GmM}{|\mathbf{r}_2' - \mathbf{r}_1'|^2} = \frac{GmM}{|(\mathbf{r}_2 + \mathbf{a}) - (\mathbf{r}_1 + \mathbf{a})|^2} = \frac{GmM}{|\mathbf{r}_2 - \mathbf{r}_1|^2} = F$$

Invariance under rotation

- No experiment within your spaceship can determine whether it's been rotated.
- Is everyone cool with 3×3 matrices?
- Primed frame: masses at $\mathbf{r}'_1 \equiv \mathbf{R}\mathbf{r}_1$ and $\mathbf{r}'_2 \equiv \mathbf{R}\mathbf{r}_2$

$$F' = \frac{GmM}{|\mathbf{R}\mathbf{r}_2 - \mathbf{R}\mathbf{r}_1|^2} = \frac{GmM}{|\mathbf{R}(\mathbf{r}_2 - \mathbf{r}_1)|^2} = \frac{GmM}{|\mathbf{r}_2 - \mathbf{r}_1|^2} = F$$

- Another example: Maxwell's equations in vacuum imply

$$\nabla^2 \mathbf{E} = \frac{1}{c^2}\ddot{\mathbf{E}}.$$

  Since only *differences* in position and time enter, it's translationally invariant. Here it's infinitesimal differences (derivatives), above it was a finite difference $|\mathbf{r}_2 - \mathbf{r}_1|^2$.

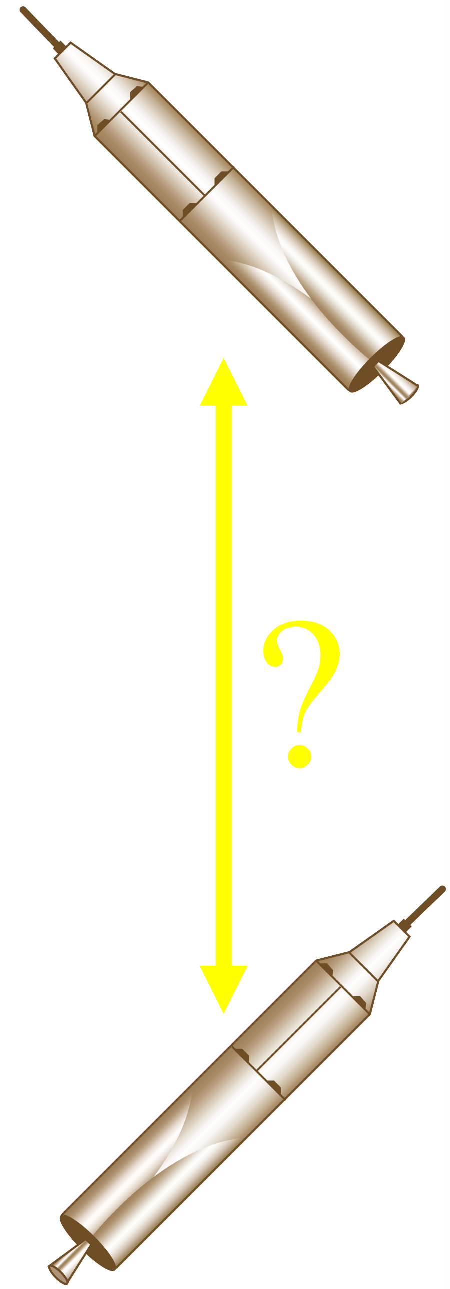
- ∇^2 is invariant under rotation (remember Gauss' theorem)
- At MIT:

$$\mathbf{E} = \frac{1}{c^2}\ddot{\mathbf{E}} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

- Near Australia:

$$\mathbf{E} = \frac{1}{c^2}\ddot{\mathbf{E}} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}.$$

- So both observer's agree that Maxwell was right, *i.e.*, the wave equation is translationally and rotationally invariant.



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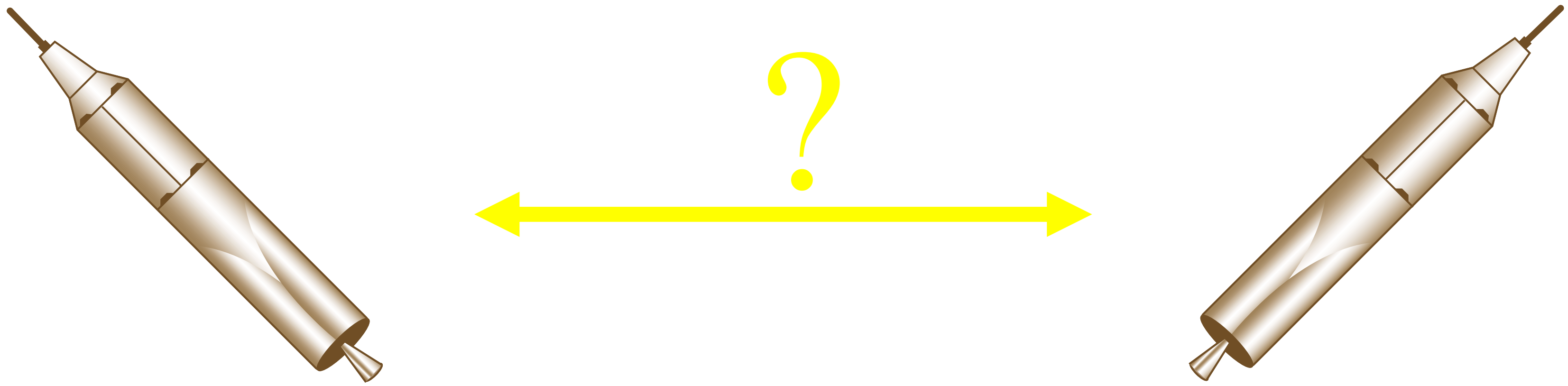

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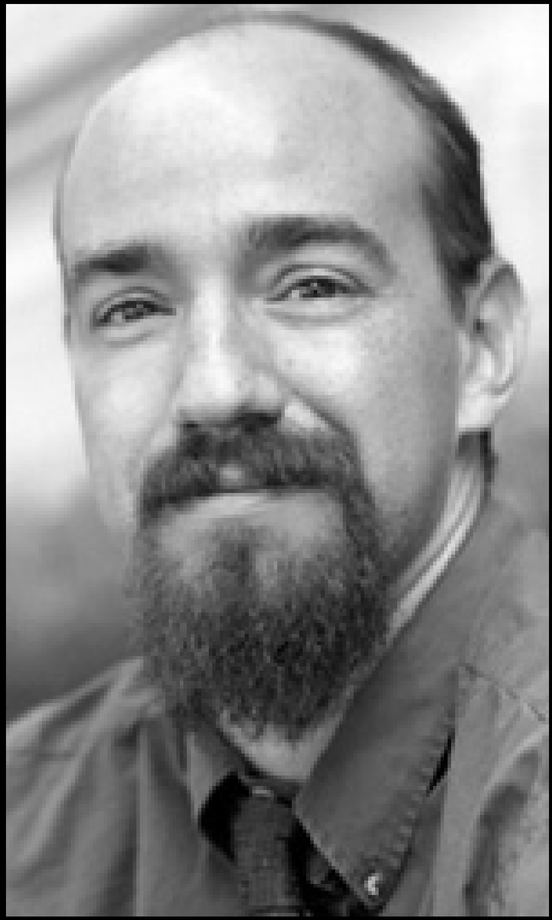
Invariance under reflection?

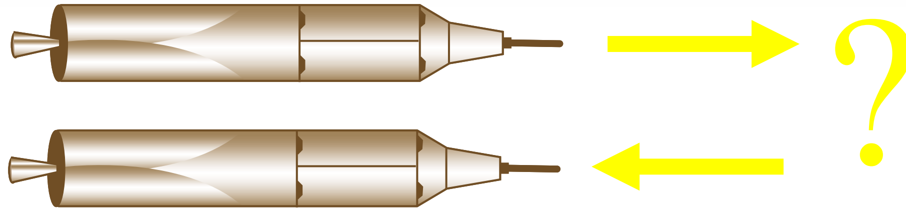
- Yes for all of classical physics
- Considered self-evident and obvious
- 1956: Chen Ning Yang & Tsung-Dao Lee propose that weak interactions violate parity; Chien-Shiung Wu demonstrates it with cobalt 60, Leon Lederman with accelerator. (Yang & Lee get 1957 Nobel prize.)

Symmetry is at the heart of modern physics

- Special relativity is all about so-called *Lorentz symmetry.*
- General relativity is about so-called *diffeomorphism symmetry.*
- Key topics in particle physics are C, P and T symmetry and combinations like CP and CPT symmetry.
- A cornerstone of particle physics is *gauge symmetry*
- In 2007, the Large Hadron Collider at CERN will search for *supersymmetry.*

WHAT'S THE SYMMETRY OF CLASSICAL MECHANICS?





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Invariance under Galilean transformation

- Demo with colliding carts, ball.
- So Newtonian mechanics appears to be invariant - let's understand exactly what the transformation is, and why this is so.
- Inertial frame definition ($\mathbf{a} = 0$ if $\mathbf{F} = 0$)
- Are we in an inertial frame? (PS1)
- Galilean transformation definition (between 2 inertial frames)
- Definition of *event*: a 4D point (x, y, z, t). Examples?
- $\mathbf{r}' = \mathbf{r} - \mathbf{v}t$
- Lengths invariant: $\Delta\mathbf{r}' \equiv \mathbf{r}'_2 - \mathbf{r}'_1 = (\mathbf{r}_2 - \mathbf{v}t) - (\mathbf{r}_1 - \mathbf{v}t) = \Delta\mathbf{r}$
- But we must measure $\mathbf{r}'_1$ and $\mathbf{r}'_2$ at the same time!
- Which we can, since time is invariant and unambiguous: $t' = t$

Spacetime transformation summary

- Translation:

$$\begin{cases} \mathbf{r}' = \mathbf{r} + \Delta r \\ t' = t + \Delta t \end{cases}$$

- Rotation:

$$\begin{cases} \mathbf{r}' = \mathbf{R}\mathbf{r} \\ t' = t \end{cases}$$

- Galilean:

$$\begin{cases} \mathbf{r}' = \mathbf{r} + \mathbf{v}t \\ t' = t \end{cases}$$

- Combined:

$$\begin{cases} \mathbf{r}' = \mathbf{R}\mathbf{r} + \Delta\mathbf{r} + \mathbf{v}t \\ t' = t + \Delta t \end{cases}$$

Transforming velocity

- How does $\mathbf{u}$ transform under a Galilean transformation?

$$\mathbf{u} \equiv \frac{d\mathbf{r}}{dt}$$

$$\mathbf{u}' \equiv \frac{d\mathbf{r}'}{dt'} = \frac{d}{dt}(\mathbf{r} - \mathbf{v}t) = \frac{d\mathbf{r}}{dt} - \mathbf{v} = \mathbf{u} - \mathbf{v}$$

So velocities add/subtract as you'd expect: $\mathbf{u}' = \mathbf{u} - \mathbf{v}$

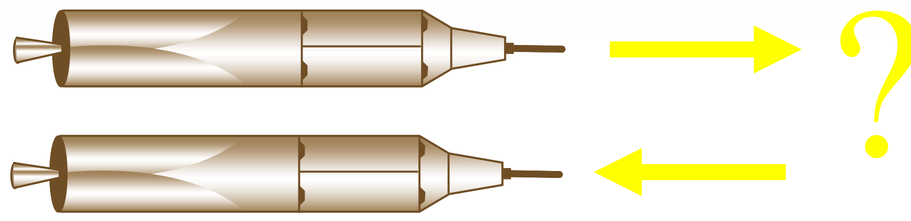
- But what about the flashlight on the train?

Transforming acceleration

$$\mathbf{a} \equiv \frac{d\mathbf{u}}{dt}$$

$$\mathbf{a}' \equiv \frac{d\mathbf{u}'}{dt'} = \frac{d}{dt}(\mathbf{u} + \mathbf{v}) = \frac{d\mathbf{u}}{dt} = \mathbf{a}$$

So acceleration is invariant.



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Transforming $\mathbf{F} = m\mathbf{a}$

- Consider forces that depend on *separation*:
  - Spring: $F = k(x_2 - x_1)$
  - Gravity: $F = \frac{GmM}{|\mathbf{r}_2 - \mathbf{r}_1|^2}$

  They are invariant, since lengths are.

- m is invariant
- Since $\mathbf{F}$, $\mathbf{a}$ and m are all invariant, so is the equation $\mathbf{F} = m\mathbf{a}$.
- So the physical law is invariant, but not the initial conditions!

Transforming energy & momentum

- Neither is invariant, since $\mathbf{v}$ isn't.
- But the conservation laws are invariant: E and $\mathbf{p}$ are conserved in any frame (PS1).
- Work-energy theorem:

$$W = \Delta KE,$$

  where work defined as

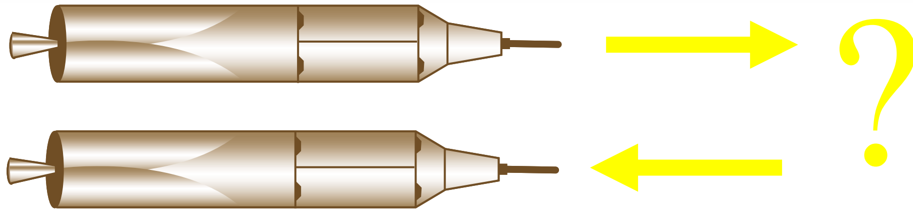
$$W = \int_{x_1}^{x_2} F dx.$$

- Proof:

$$\begin{aligned} W &= \int_{x_1}^{x_2} F dx = \int_{x_1}^{x_2} ma\, dx = m \int_{x_1}^{x_2} \frac{dv}{dt} dx \\ &= m \int_{v_1}^{v_2} \frac{dx}{dt} dv = m \int_{v_1}^{v_2} v\, dv = \frac{mv_2^2}{2} - \frac{mv_1^2}{2} = \Delta KE. \end{aligned}$$

  Only assumption here was $F = ma$, which is invariant, so the work-energy theorem is also invariant.

- W and KE alone are *not* invariant.



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Transforming trajectories

- Is the 3D shape of a trajectory *not* invariant?
- No! Basket ball example: line in frame A is parabola in frame B.

Key Galilean non-invariants

$$\begin{cases} \mathbf{r}' = \mathbf{r} + \mathbf{v}t \\ \mathbf{u}' = \mathbf{u} + \mathbf{v} \\ \mathbf{p}' = \mathbf{p} + m\mathbf{v} \end{cases}$$

SO WHICH DO YOU TRUST MORE:

Classical Mechanics, or E&M?