Massachusetts Institute of Technology
Physics 8.03SC Fall 2016
Homework 8

Problems

Problem 8.1 (25 pts)

Consider the free transverse oscillations of the two-dimensional beaded string shown in Figure 1. The system is composed of 9 beads arranged in a 3x3 grid. All the horizontal strings have tension T_h, all the vertical strings have tension T_v, all the solid circles are beads with mass m. The ends of strings not attached to a bead are fixed. The square frame is fixed in the $z = 0$ plane.

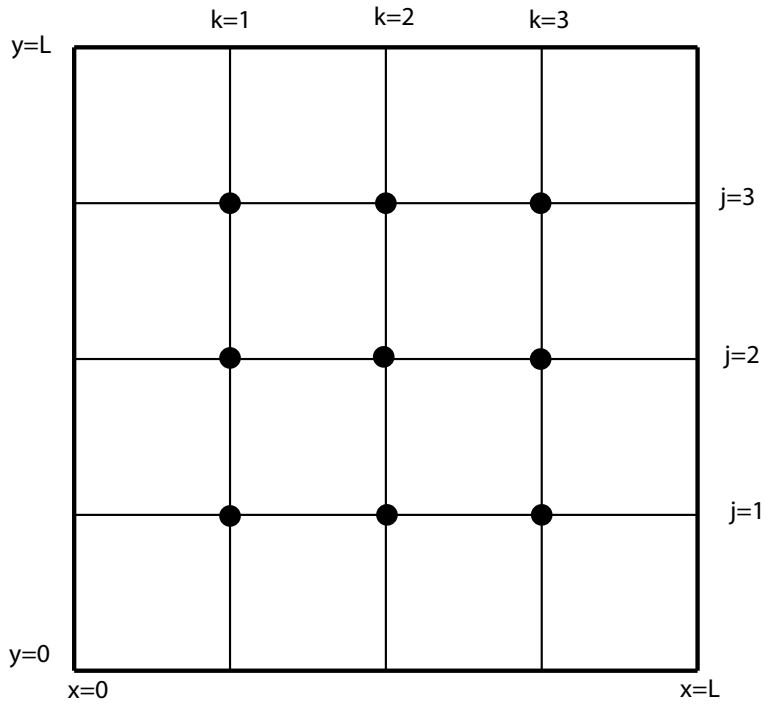

Figure 1: 2D mass lattice

a. Find the normal modes and the corresponding frequencies.

b. Suppose that $T_v = 1000T_h$. Draw nine diagrams, one for each normal mode, in order of increasing frequency, indicating which beads are moving up (by a + sign), which are moving down (by a - sign) and which are not moving (by a 0). You can interchange + and - and still have the right answer by changing the setting of your clock ir multiplying your normal mode vector by -1. For example the lowest frequency mode looks like:

+ + +
+ + +
+ + +

while the mode with the fifth highest looks like:

- 0 +
0 0 0
+ 0 -

Problem 8.2 (25 pts)

A light beam travels through vacuum ($n_1 = 1$) before reaching a transparent plate with index of refraction n_2, at an angle $\alpha = 60^o$. It traverses this plane and enters a new material with index of refraction n_3 at an angle $\beta = 30^o$. The configuration of this optical experiment is shown in Figure 2.

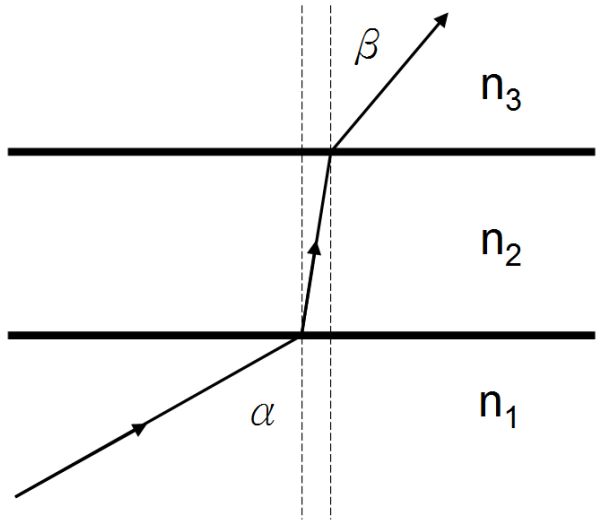

Figure 2: Light experiment

a. What is the possible range of value for n_2?

b. What is the value for n_3?

Problem 8.3 (25 pts)

Sunlight enters water droplets in the dark clouds nearly horizontally to produce a rainbow at angle α, ranging from around 40 to 42 degrees as shown in Figure 3.

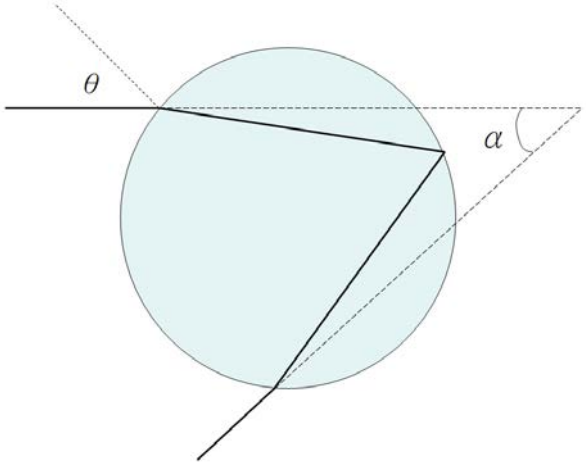

Figure 3: Water droplets

a. Find α in terms of the incident angle θ and n, the index of refraction of water. Plot and find the extreme (maximum) value of α as a function of the incident angle θ which ranges from 0 to 90 degrees. (Use $n = 1.33$ to find the maximum α and make plot with Mathematica or any plotting tools you like.)

b. Why does the rainbow appear at the extreme value of α?

c. Which color is at higher angle when you look up at the rainbow? (hint: the index of refraction for red light is slightly lower than that of the blue)

d. How do you explain the double rainbows? Which color is higher in the sky for the second rainbow? A qualitative discussion is sufficient for this part.

8.03SC Physics III: Vibrations and Waves

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