8.03 Lecture 19

*Today: 1. EM waves in matter and 2. Brewster's Angle.
*Review of Gauss's Theorem:

$$Q = \int_Q (\vec{\nabla} \cdot \vec{A}) d\tau = \oint_S \vec{A} \cdot \vec{da}$$

*Review of Stokes' Theorem:

$$\vec{V} = \int_S (\vec{\nabla} \times \vec{A}) \vec{da} = \oint_L \vec{A} \times \vec{dl}$$

*Review of Polarization (dipole moment):

$$P(r) = \oint_V \rho(r_0)(r_0 - r) d^3 r_0$$

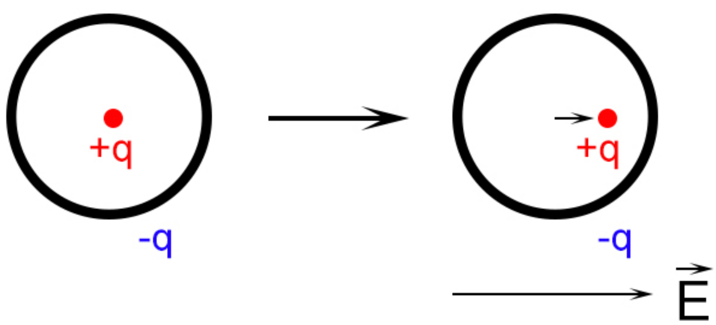
*Review of magnetic moment:

$$M = \frac{1}{2} \int_V r \times J \, dV$$

We have talked about EM waves in the vacuum. We know how to generate EM waves. Now: we are interested in EM waves in dielectrics.

1. In perfect conductors: We have unlimited supply of charges. It costs nothing to move them around.
2. In dielectrics: All charges are attached to specific atoms or molecules. They are bounded. They can only move a bit within an atom or molecule.

In the presence of an electric field: there is an induced dielectric polarization:



In the presence of the matter, there are bound charges and free charges: $\rho = \rho_f + \rho_b$. Take a look at the effect of free charge, we define the electric displacement field, $\vec{D}$:

$$\vec{D} \equiv \epsilon_0 \vec{E} + \vec{P}$$

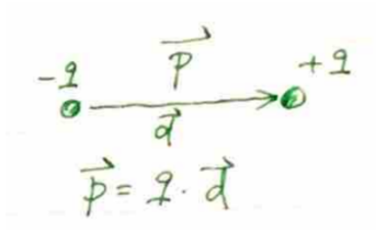
Where

$$-\nabla \cdot \vec{P} \equiv \rho_b$$

and $\vec{P}$ is the electric dipole moment.
Gauss' Law:

$$\begin{aligned}
&\epsilon_0 \nabla \cdot \vec{E} = \rho_f + \rho_b = \rho_f - \nabla \cdot P \\
&\Rightarrow \nabla \cdot \vec{D} = \nabla \cdot (\epsilon_0 \vec{E} + \vec{P}) = \rho_f \\
&\Rightarrow \nabla \cdot \vec{D} = \rho_f
\end{aligned}$$

$\vec{D}$ field is related to the effect of free charge.



Similiary in the presense of the matter, there are bound currents and free currents.

$$\vec{J} = \vec{J}_f + \vec{J}_b + \vec{J}_p$$

Where the last term is the polarization current.

$$\vec{H} \equiv \frac{\vec{B}}{\mu_0} - \vec{M} \qquad \text{where} \qquad \vec{\nabla} \times \vec{M} = \vec{J}_b$$

Where $\vec{H}$ is the demagnetizing field and $\vec{M}$ is the magnetic dipole moment.
Ampere's Law:

$$\nabla \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

$$\Rightarrow \frac{1}{\mu_0} (\nabla \times \vec{B}) = \vec{J}_f + \underbrace{\vec{\nabla} \times \vec{M}}_{\text{Band Cur.}} + \overbrace{\frac{\partial \vec{P}}{\partial t}}^{\text{Polarization cur.}} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\begin{aligned}
\Rightarrow \nabla \times \left(\frac{1}{\mu_0} \vec{B} - \vec{M} \right) &= \vec{J}_f + \frac{\partial \vec{D}}{\partial t} \\
\nabla \times \vec{H} &= \vec{J}_f + \frac{\partial \vec{D}}{\partial t}
\end{aligned}$$

Maxwell's Equation in matter where there is no free charge ($\rho_f = 0$, $\vec{J}_f$=0)

$$\begin{cases} \vec{\nabla} \cdot \vec{D} = 0 \\ \vec{\nabla} \cdot \vec{B} = 0 \\ \vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \vec{\nabla} \times \vec{H} = \quad \dfrac{\partial \vec{D}}{\partial t} \end{cases}$$

$$\text{If} \quad \vec{P} \propto \vec{E} \Rightarrow \vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon \vec{E}$$

$$\text{If} \quad \vec{M} \propto \vec{B} \Rightarrow \vec{H} = \frac{1}{\mu_0} \vec{B} - \vec{M} = \frac{\vec{B}}{\mu}$$

Where $\epsilon \equiv k_e \epsilon_0$ is the permittivity which goes up. Roughly it can be thought of as the resistance of forming an electric field. Usually μ, the permeability, is approximately μ_0.
Happens when $\vec{E}$ and $\vec{B}$ are small and linear, homogeneous, isotropic material.

$$\begin{cases} \vec{\nabla} \cdot \vec{E} = 0 \\ \vec{\nabla} \cdot \vec{B} = 0 \\ \vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \vec{\nabla} \times \vec{B} = \mu\epsilon \dfrac{\partial \vec{B}}{\partial t} \end{cases}$$

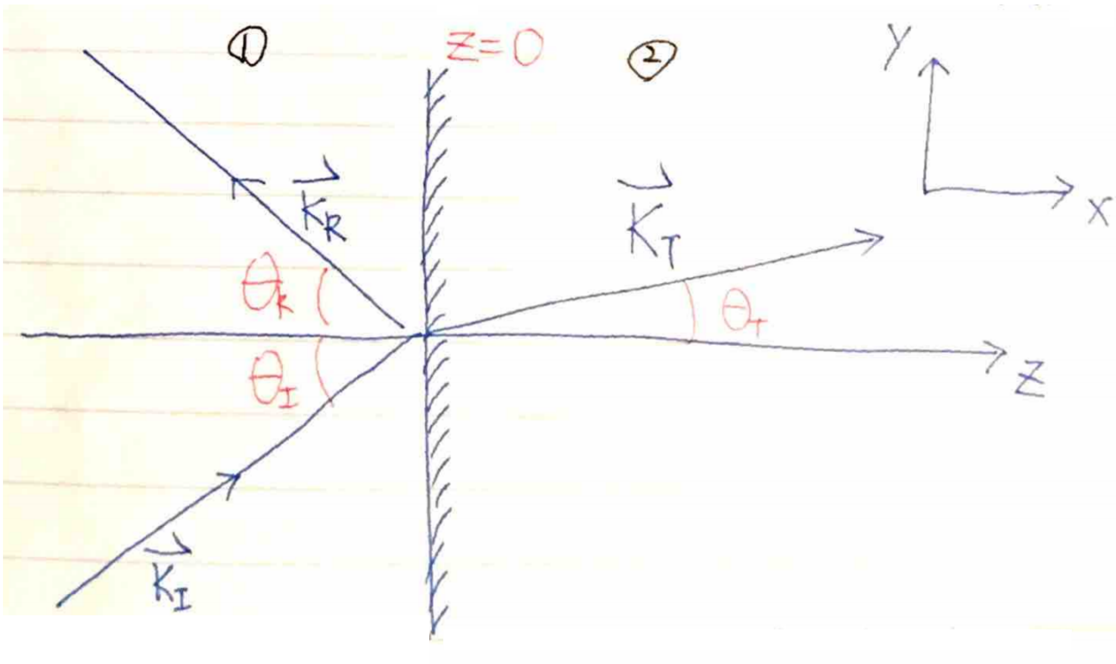
Where the velocity is $\dfrac{1}{\sqrt{\mu\epsilon}} = \dfrac{c}{n}$ and $n = \dfrac{\sqrt{\mu\epsilon}}{\sqrt{\mu_0\epsilon_0}}$
Usually $\mu \approx \mu_0$ if $\epsilon > \epsilon_0 \rightarrow n > 1$ The phase velocity of light in matter is SLOWER
Poynting vector:

$$\frac{1}{\mu} \vec{E} \times \vec{B} \approx \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

The refraction index, n, may depend on the wave length (or frequency). $\Rightarrow n = n(\omega)$ which is usually decreasing versus wavelength. Example:

$$\begin{aligned} &\vec{E} = \vec{E}_0 \cos(kz - \omega t) \Rightarrow \vec{k} = k\hat{z} \\ &\vec{E}_0 \perp \vec{k} \qquad \Downarrow \text{usually } \mu \approx \mu_0 \\ &\frac{\omega}{k} = \frac{c}{n} = \frac{c\sqrt{\mu_0\epsilon_0}}{\sqrt{\mu\epsilon}} \approx \frac{c\sqrt{\epsilon_0}}{\sqrt{\epsilon}} = \frac{c}{\sqrt{k_e}} \end{aligned}$$

Question: What happens when an EM wave passes from one transparent medium to another?



Suppose we have an incident plane wave, with a given $\vec{E}_{0I}$

$$\vec{E}_I(\vec{r},t) = \vec{E}_{0I}\cos(\vec{k}_I \cdot \vec{r} - \omega t)$$
$$\vec{B}_I(\vec{r},t) = \frac{1}{v_1}(\hat{k}_I \times \hat{E}_I)$$

Reflected wave:

$$\vec{E}_R(\vec{r},t) = \vec{E}_{0R}\cos(\vec{k}_R \cdot \vec{r} - \omega t)$$
$$\vec{B}_R(\vec{r},t) = \frac{1}{v_1}(\hat{k}_R \times \hat{E}_R)$$

where $\vec{E}_{0R}$ is unknown.
Transmitted wave:

$$\vec{E}_T(\vec{r},t) = \vec{E}_{0T}\cos(\vec{k}_T \cdot \vec{r} - \omega t)$$
$$\vec{B}_T(\vec{r},t) = \frac{1}{v_2}(\hat{k}_T \times \hat{E}_T)$$

where $\vec{E}_{0T}$ is unknown. We showed before in previous lecture:
At the boundary $z = 0$

$$(a) \qquad \vec{k}_I \cdot \vec{r} = \vec{k}_R \cdot \vec{r} = \vec{k}_T \cdot \vec{r}$$
$$(b) \qquad \theta_I = \theta_R$$

$$(c) \qquad n_1 \sin\theta_I = n_2 \sin\theta_T$$

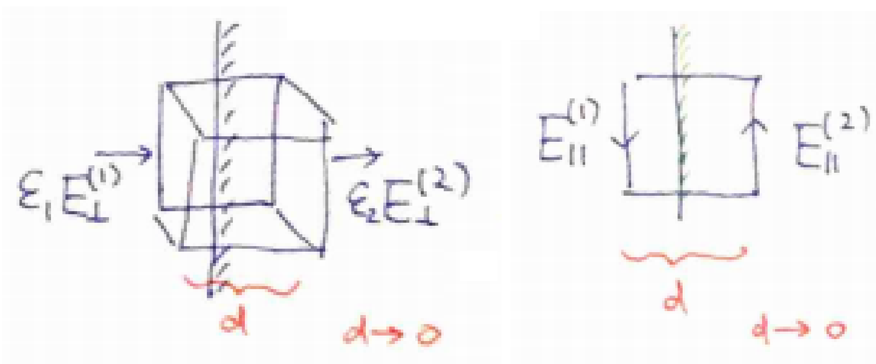
Using these criteria: $\vec{E}_I + \vec{E}_R$ are in medium 1 and $\vec{E}_T$ is in medium 2. At $z = 0$:

$$\vec{E}^{(1)} = \vec{E}_{0I} + \vec{E}_{0R} \qquad \text{and} \qquad \vec{E}^{(2)} = \vec{E}_{0T}$$

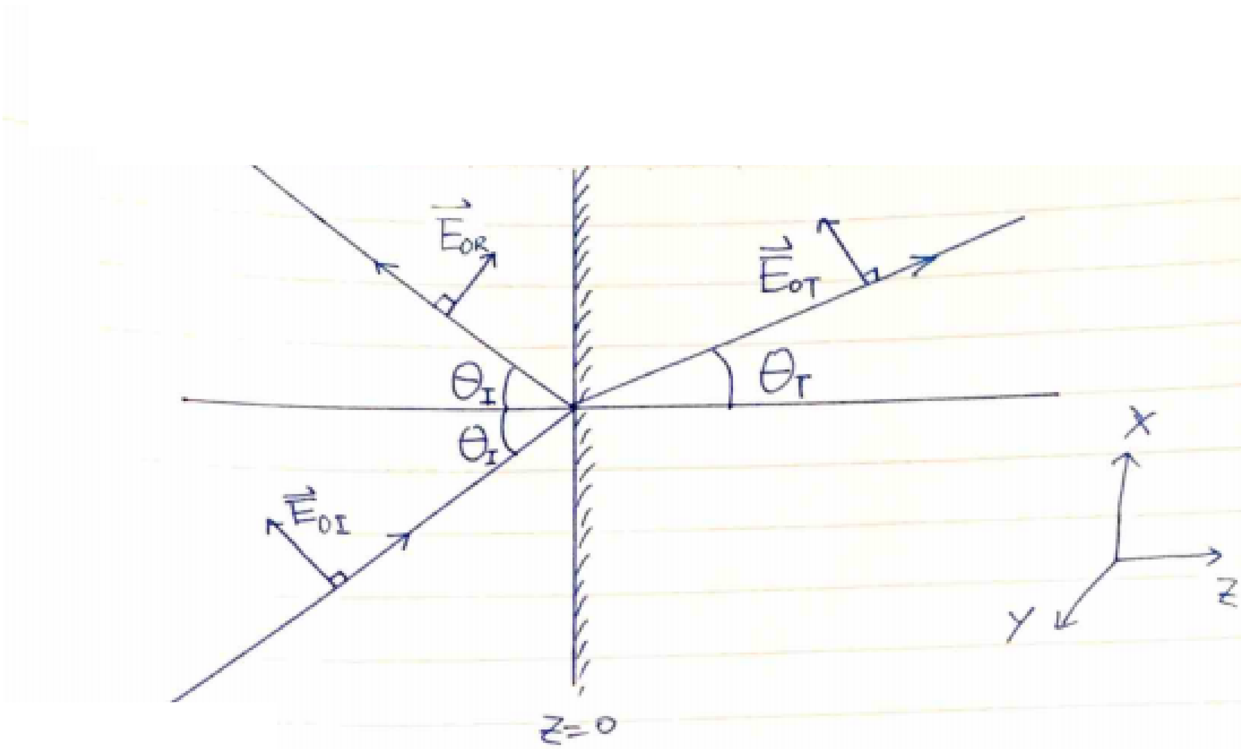
EM wave specific boundary conditions:

1. $\perp$ Direction:
$$\oint D \cdot da = 0, \qquad \vec{\nabla} \cdot \vec{D} = 0 \quad \Rightarrow \quad \epsilon_1 E_{\perp}^{(1)} = \epsilon_2 E_{\perp}^{(2)}$$

2. $\|$ Direction:
$$\oint E \cdot dl = -\frac{\mathrm{d}}{\mathrm{dt}} \int B \, da, \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial \mathrm{t}}$$



If we <u>assume</u> that the polariztion of the incident wave is <u>parallel</u> to the plane (x, z):



Perpendicular direction:

$$(i) \quad \epsilon_1(-E_{0I}\sin\theta_I + E_{0R}\sin\theta_I) = -\epsilon_2 E_{0T}\sin\theta_T$$

Parallel direction:

$$(iii) \quad E_{0I}\cos\theta_I + E_{0R}\cos\theta_I = E_{0T}\cos\theta_T$$

$$\begin{aligned}
(i) \Rightarrow (E_{0I} - E_{0R}) &= \frac{\epsilon_2}{\epsilon_1}\frac{\sin\theta_I}{\sin\theta_2}E_{0T} \\
&= \frac{\epsilon_2}{\epsilon_1}\frac{n_1}{n_2}E_{0T} = \beta E_{0T} \\
(iii) \Rightarrow (E_{0I} + E_{0R}) &= \frac{\cos\theta_T}{\cos\theta_I}E_{0T} = \alpha E_{0T}
\end{aligned}$$

$$\begin{aligned}
\Rightarrow E_{0R} &= \frac{\alpha-\beta}{\alpha+\beta}E_{0I} \Rightarrow R = \frac{\alpha-\beta}{\alpha+\beta} \\
E_{0T} &= \frac{2}{\alpha+\beta}E_{0I} \Rightarrow T = \frac{2}{\alpha+\beta}
\end{aligned}$$

What do we learn from this?

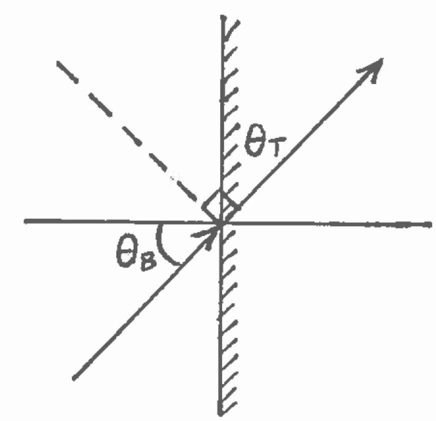
(1.) Normal incidence:

$$
\begin{aligned}
&\alpha = \frac{\cos\theta_T}{\cos\theta_I} = 1 \\
&\text{if } \mu_1 \approx \mu_2 \approx \mu_0 \\
&\Rightarrow \beta = \frac{\epsilon_2}{\epsilon_1}\frac{n_1}{n_2} = \frac{n_2}{n_1} \\
&\Rightarrow R = \frac{n_1 - n_2}{n_1 + n_2} \quad T = \frac{2n_1}{n_1 + n_2}
\end{aligned}
$$

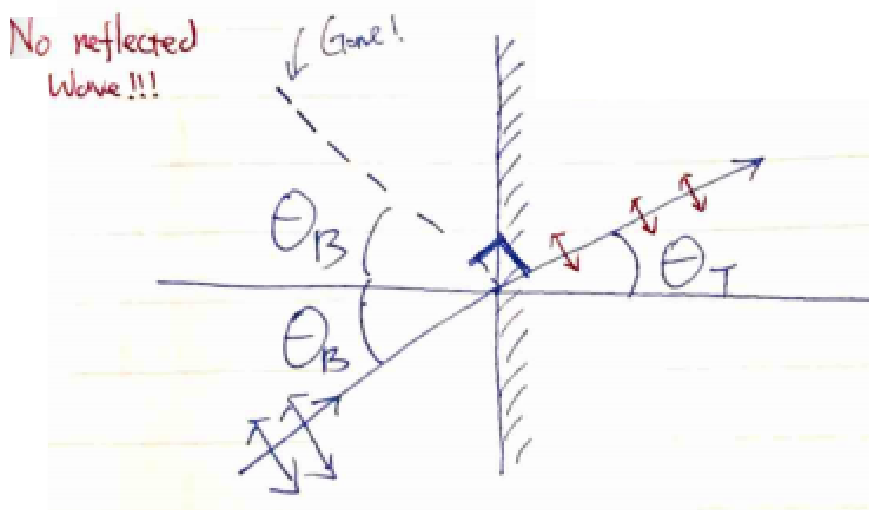
(2.) Grazing incidence:

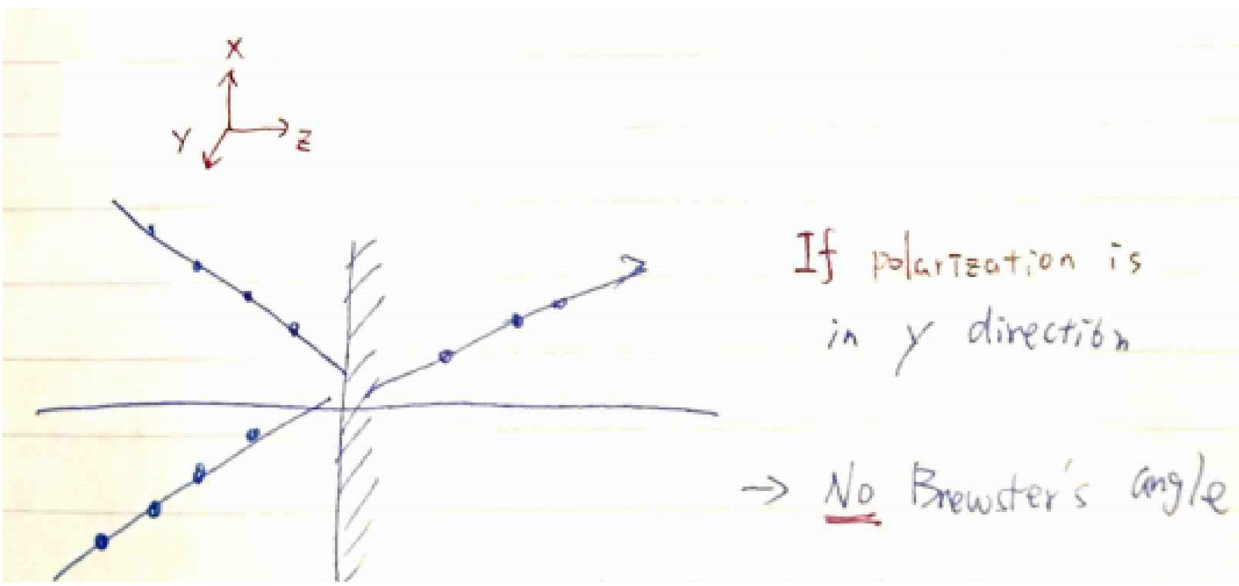
$$
\begin{aligned}
&\theta_I \approx 90^\circ \\
&\alpha \to \infty \\
&R \approx 1\ ,\ T \approx 0
\end{aligned}
$$

(3.) A very special angle: θ_B. Brewster's Angle! When $\alpha = \beta \Rightarrow R = 0$ and $T = 1$!



$$
\begin{aligned}
&\alpha = \beta \quad \Rightarrow \quad \frac{\cos\theta_T}{\cos\theta_B} = \frac{n_2}{n_1} = \frac{\sin\theta_B}{\sin\theta_T} \\
&\sin\theta_T\cos\theta_T = \sin\theta_B\cos\theta_B \\
&\Rightarrow \sin 2\theta_T = \sin 2\theta_B \\
&\Rightarrow 2\theta_B = 2\theta_T \therefore n_1\sin\theta_B = n_2\sin\theta_T \\
&\text{or } 2\theta_B = \pi - 2\theta_T \\
&\Rightarrow \theta_B + \theta_T = \frac{\pi}{2}
\end{aligned}
$$


No reflected Wave!!!
Gone!
θ_B
θ_B
θ_T


x
y
z
If polarization is in y direction
→ No Brewster's angle