* Today: ① EM waves in matter

② Brewster's Angle.

* Reminder of radiation

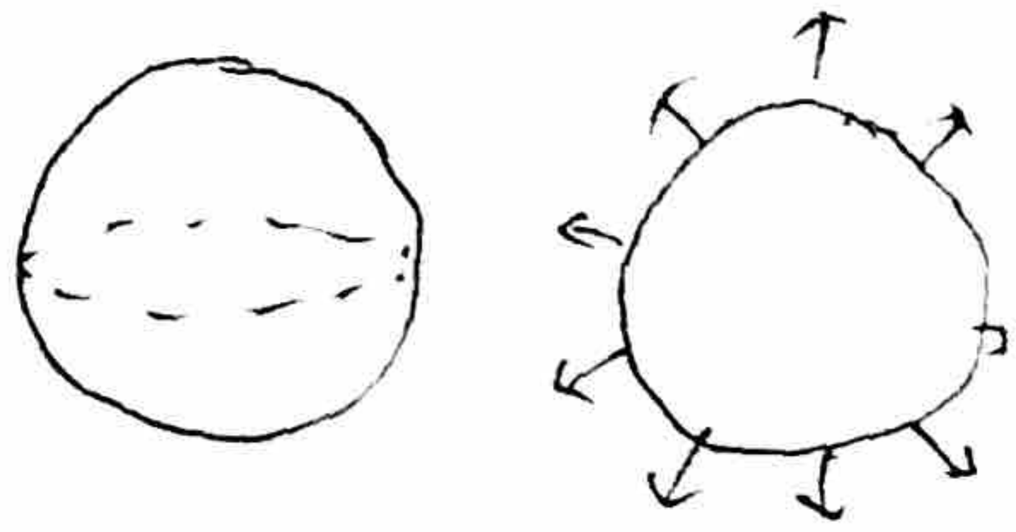
slides

* Polarizer and photography !

slides

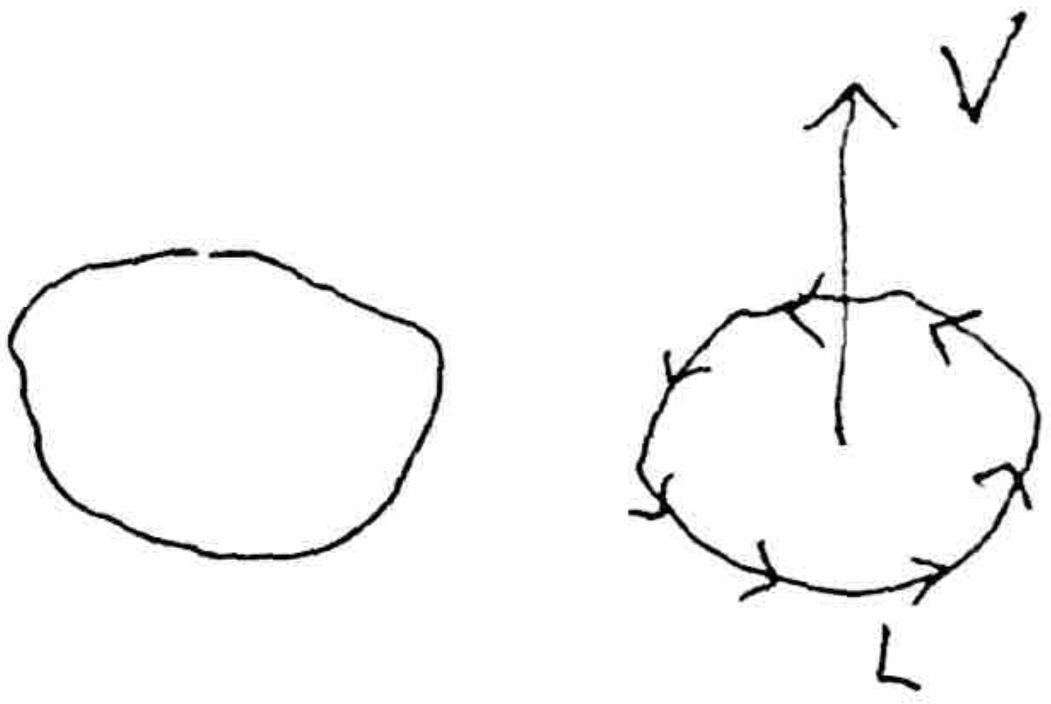
* Review of Gauss's Theorem

$$Q = \int_V (\vec{\nabla} \cdot \vec{A})\, d\tau = \oint_S \vec{A} \cdot d\vec{a}$$



* Review of Stoke's Theorem

$$\vec{V} = \int_S (\vec{\nabla} \times \vec{A})\, d\vec{a} = \oint_L \vec{A} \times d\vec{l}$$




* Review of Polarization (dipole moment) : $P(r) = \oint_V \rho(r_0)(r_0 - r)\, d^3 r_0$.

* Review of magnetic moment : $M = \frac{1}{2} \int_V r \times J\, dV$

We have talked about EM waves in vacuum.

We know how to generate EM waves.

Now: we are interested in the EM waves in dielectrics

(1) In perfect conductors:

We have unlimited supply of charges.

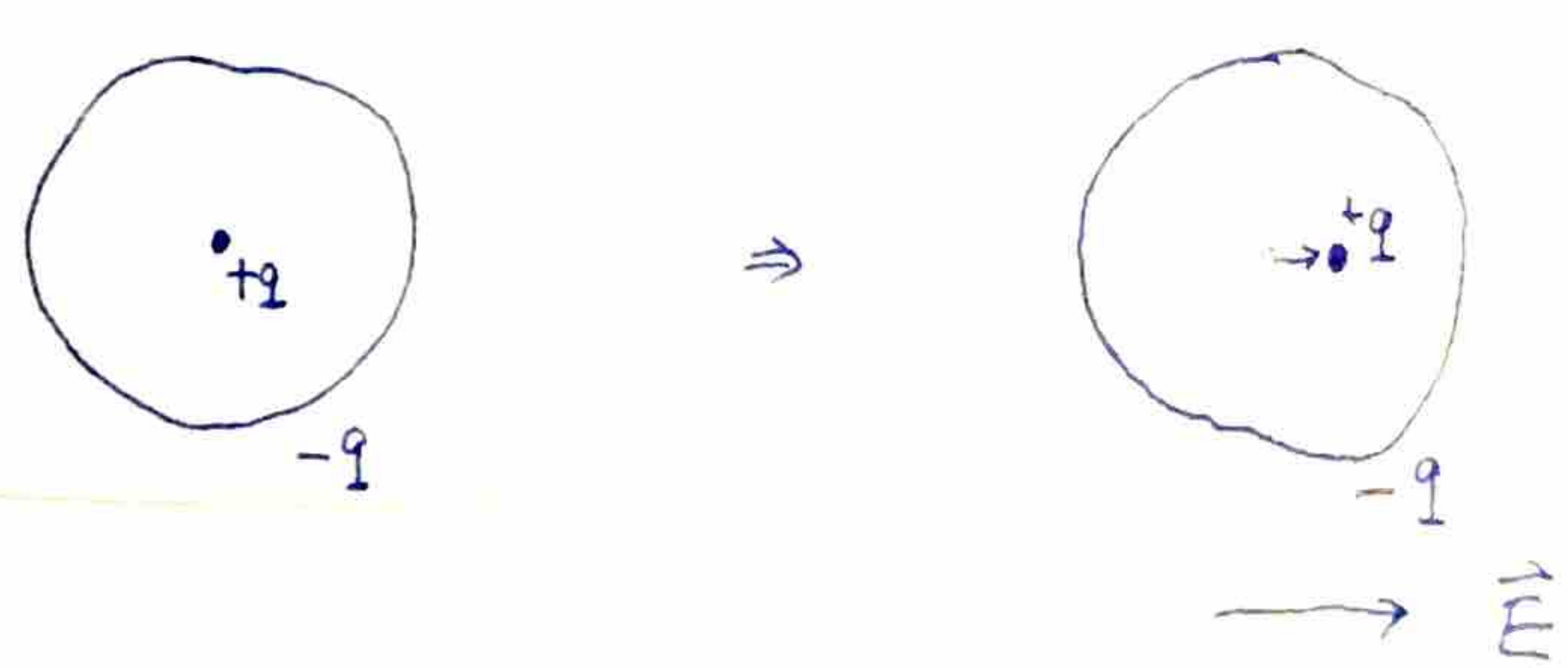
It costs nothing to move them around.

(2) In dielectrics :

All charges are attached to specific atom or molecules. They are bounded. They can only move a bit within atom or molecule.

The presence of electric field:

Induced dielectric polarization



In the presence of the matter, there are bound charges and free charges. $\rho = \rho_f + \rho_b$

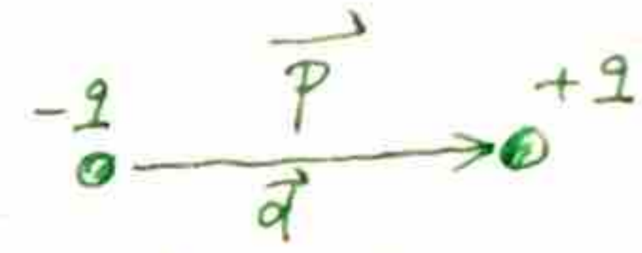
To look at the effect of free charge, we define

$$\vec{D} \equiv \varepsilon_0 \vec{E} + \vec{P} \qquad \text{where } -\nabla \cdot \vec{P} \equiv \rho_b$$

↳ electric displacement field ↳ electric dipole moment

Gauss's Law

$$\varepsilon_0 \nabla \cdot \vec{E} = \rho_f + \rho_b = \rho_f - \nabla \cdot P$$




$\vec{p} = q \cdot \vec{d}$

$$\Rightarrow \nabla \cdot \vec{D} = \nabla \cdot (\varepsilon_0 \vec{E} + \vec{P}) = \rho_f$$

$$\Rightarrow \nabla \cdot \vec{D} = \rho_f$$

$\vec{D}$ field is related to the effect of free charge

Similarly in the presence of the matter, there are bound currents and free current $\vec{J} = \vec{J}_f + \vec{J}_b + \vec{J}_p$

↳ polarization current

$$\vec{H} \equiv \frac{\vec{B}}{\mu_0} - \vec{M} \qquad \text{where } \vec{\nabla} \times \vec{M} \equiv \vec{J}_b$$

↳ demagnetizing field ↳ magnetic dipole moment

Ampere's Law

$$\nabla \times \vec{B} = \mu_0 \left( \vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

$$\Rightarrow \frac{1}{\mu_0} (\nabla \times \vec{B}) = \vec{J}_f + \underbrace{\vec{\nabla} \times \vec{M}}_{\text{Bound current}} + \underbrace{\underbrace{\frac{\partial \vec{P}}{\partial t}}_{\text{polarization current}} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}}_{\frac{\partial \vec{D}}{\partial t}}$$

$$\Rightarrow \nabla \times \left( \frac{1}{\mu_0} \vec{B} - \vec{M} \right) = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

Maxwell's Equation in matter where there is no free charge:

$(\rho_f = 0,\ \vec{J}_f = 0)$

$$\begin{cases} \vec{\nabla}\cdot\vec{D} = 0 \\ \vec{\nabla}\cdot\vec{B} = 0 \\ \vec{\nabla}\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t} \\ \vec{\nabla}\times\vec{H} = \dfrac{\partial\vec{D}}{\partial t} \end{cases}$$

"resistance of forming an electric field"

Permittivity changed (Goes up)

If $\vec{P} \propto \vec{E}$ $\Rightarrow$ $\vec{D} = \varepsilon_0\vec{E} + \vec{P} = \varepsilon\vec{E}$ $\quad \varepsilon = k_e\varepsilon_0$

If $\vec{M} \propto \vec{B}$ $\Rightarrow$ $\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M} = \frac{\vec{B}}{\mu}$

Happens when ① $\vec{E}$ and $\vec{B}$ small ② Linear, homogeneous isotropic material | usually $\mu \approx \mu_0$

Permeability

(i) $\vec{\nabla}\cdot\vec{E} = 0$

(ii) $\vec{\nabla}\cdot\vec{B} = 0$

(iii) $\vec{\nabla}\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t}$

(iv) $\vec{\nabla}\times\vec{B} = \mu\varepsilon\dfrac{\partial\vec{E}}{\partial t}$

$\Rightarrow$ velocity $= \dfrac{1}{\sqrt{\mu\varepsilon}}$

$= \dfrac{c}{n}$

$\Rightarrow n = \dfrac{\sqrt{\mu\varepsilon}}{\sqrt{\mu_0\varepsilon_0}}$

usually $\mu \approx \mu_0$

if $\varepsilon > \varepsilon_0 \Rightarrow n > 1$

$\Rightarrow$ the phase velocity of light in matter is SLOWER

Poynting vector: $\dfrac{1}{\mu}\vec{E}\times\vec{B} \approx \dfrac{1}{\mu_0}\vec{E}\times\vec{B}$

The retraction index $n$ may depends on the wave length (or frequency)

$\Rightarrow n = n(\omega)$

Usually decreasing v.s. wavelength.

Example:

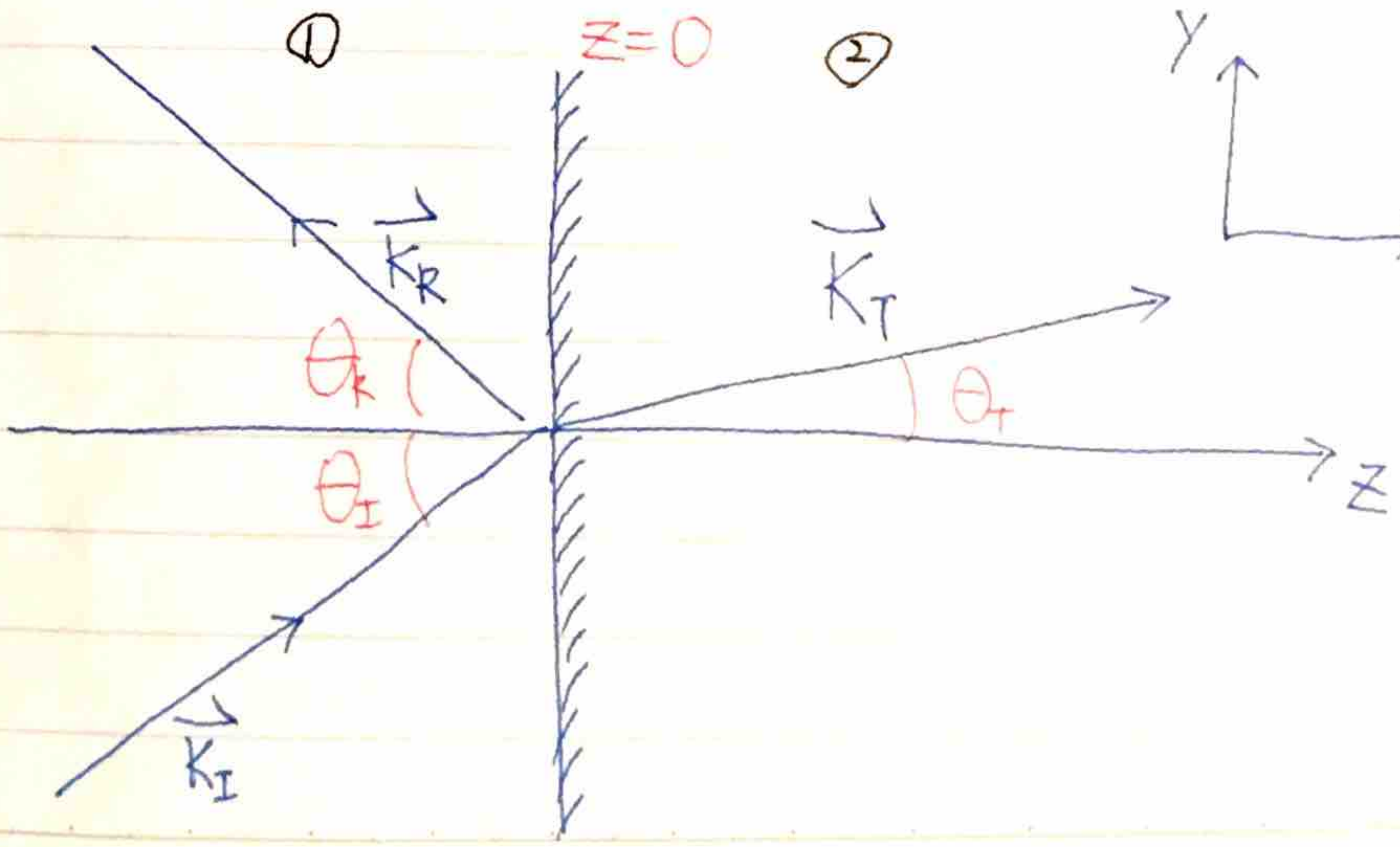
$$\vec{E} = \vec{E}_0 \cos(kz - \omega t) \Rightarrow \vec{k} = k\hat{z}$$

$$\vec{E}_0 \perp \vec{k}$$

usually $\mu \sim \mu_0$

$$\frac{\omega}{k} = \frac{c}{n} = \frac{c\sqrt{\mu_0 \varepsilon_0}}{\sqrt{\mu \varepsilon}} \approx \frac{c\sqrt{\varepsilon_0}}{\sqrt{\varepsilon}} = \frac{c}{\sqrt{K_e}}$$

Question: What happens when an EM wave passes from one transparent medium to another?



Suppose we have an incident plane wave

known, given

$$\begin{cases} \vec{E}_I(\vec{r},t) = \vec{E}_{0I} \cos(\vec{k}_I \cdot \vec{r} - \omega t) \\ \vec{B}_I(\vec{r},t) = \frac{1}{v_1}(\hat{k}_I \times \hat{E}_I) \end{cases}$$

Reflected Wave:

unknown

$$\begin{cases} \vec{E}_R(\vec{r},t) = \vec{E}_{0R} \cos(\vec{k}_R \cdot \vec{r} - \omega t) \\ \vec{B}_R(\vec{r},t) = \frac{1}{v_1}(\hat{k}_R \times \hat{E}_R) \end{cases}$$

Transmitted Wave:

unknown

$$\begin{cases} \vec{E}_T(\vec{r},t) = \vec{E}_{0T} \cos(\vec{k}_T \cdot \vec{r} - \omega t) \\ \vec{B}_T(\vec{r},t) = \frac{1}{v_2}(\hat{k}_T \times \hat{E}_T) \end{cases}$$

We showed before in Previous lecture:

(a) At $z=0$ the boundary

$$\Rightarrow \vec{k}_I \cdot \vec{r} = \vec{k}_R \cdot \vec{r} = \vec{k}_T \cdot \vec{r}$$

General properties for all waves.

(b) $\theta_I = \theta_R$ , (c) $n_1 \sin\theta_I = n_2 \sin\theta_T$

(or you can review it in the next page)

Using these criteria (a)-(c)

$\vec{E}_I + \vec{E}_R$ ... medium (1) at $z=0$

$\vec{E}_T$ ... medium (2) cos terms drop out.

$\vec{E}^{(1)} \equiv \vec{E}_{0I} + \vec{E}_{0R}$

$\vec{E}^{(2)} \equiv \vec{E}_{0T}$ at $z=0$

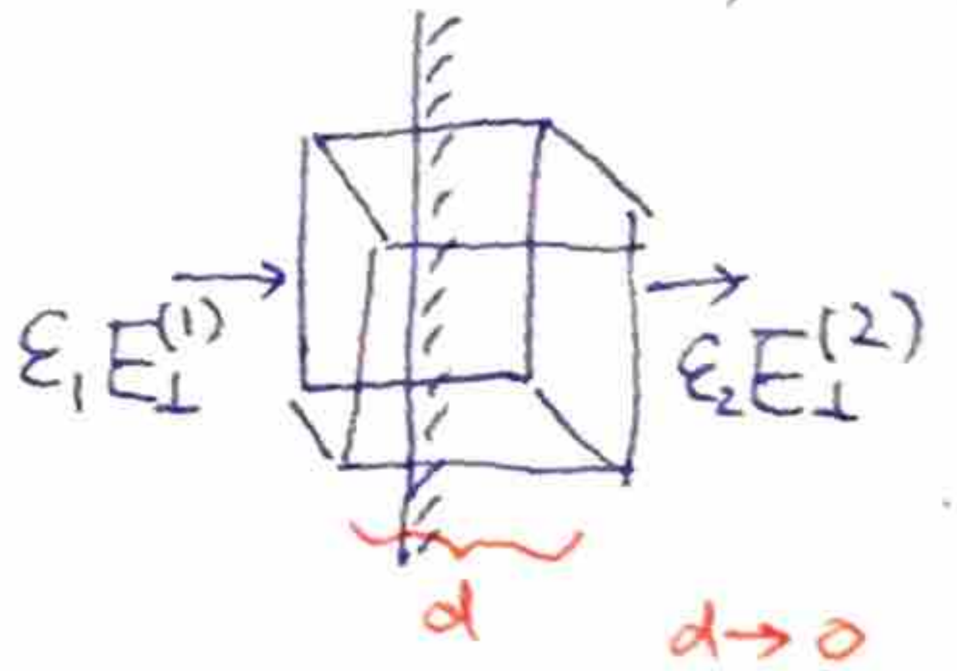
EM wave specific boundary conditions:

⊥ direction

(i), (ii)

(i) $\vec{\nabla}\cdot\vec{D}=0 \Rightarrow \varepsilon_1 E_\perp^{(1)} = \varepsilon_2 E_\perp^{(2)}$

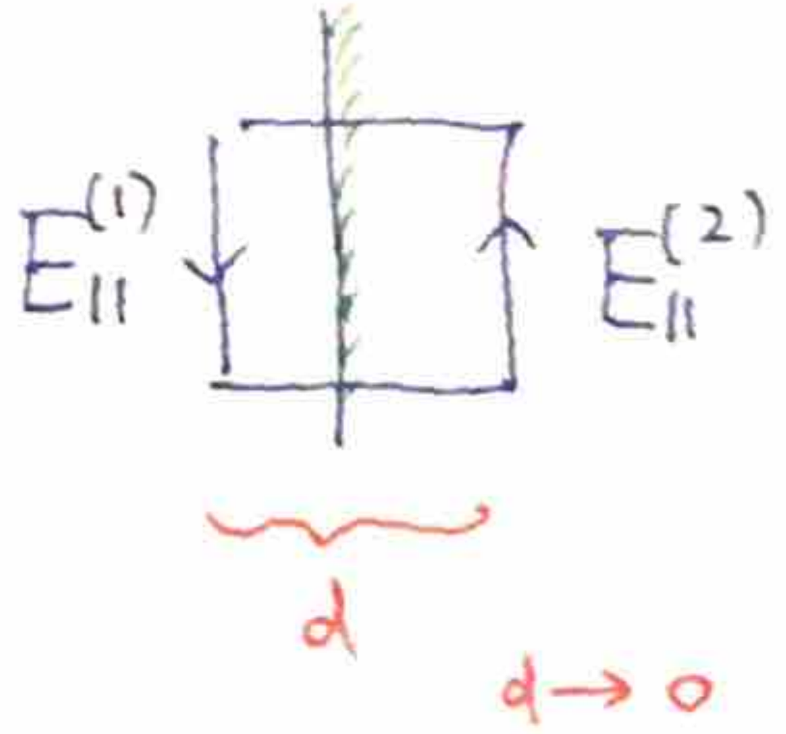
$\oint D\cdot da = 0$




∥ direction

(iii) $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\oint E\, dl = -\frac{d}{dt}\int B\cdot da$




$\Rightarrow E_\parallel^{(1)} = E_\parallel^{(2)}$

If we assume that the polarization of the incident wave is parallel to the plane $(x, z)$:

Goal: express $\vec{E}_{0R}$, $\vec{E}_{0T}$ in terms of $\vec{E}_{0I}$

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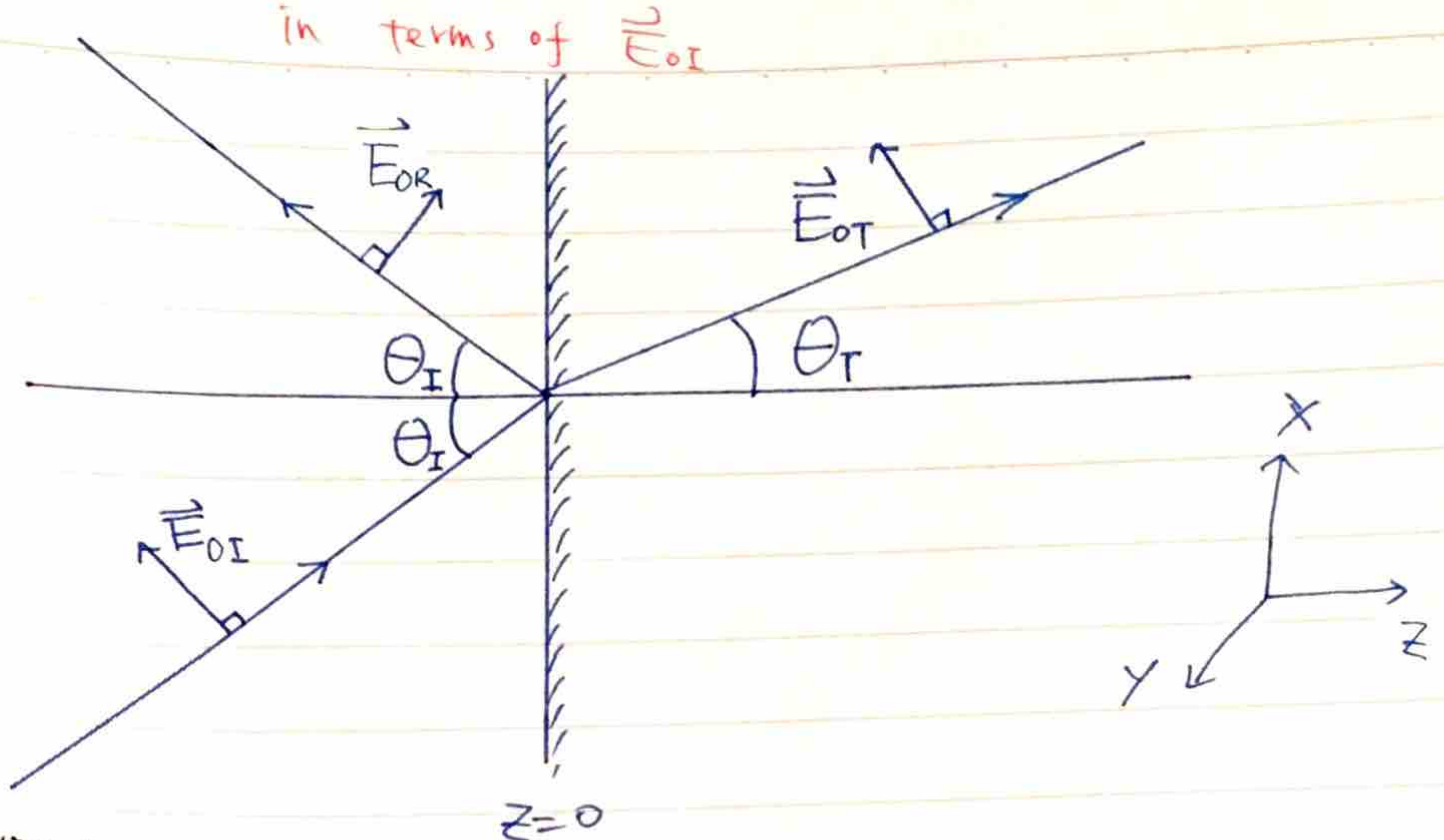



$\perp$ direction:

$$\perp \text{(i)} \quad \varepsilon_1\left(-E_{0I}\sin\theta_I + E_{0R}\sin\theta_I\right) = -\varepsilon_2 E_{0T}\sin\theta_T$$

$\parallel$ direction:

$$\parallel \text{(iii)} \quad E_{0I}\cos\theta_I + E_{0R}\cos\theta_I = E_{0T}\cos\theta_T$$

$$\text{(i)} \Rightarrow \left(E_{0I} - E_{0R}\right) = \frac{\varepsilon_2}{\varepsilon_1}\frac{\sin\theta_T}{\sin\theta_I} E_{0T}$$

$$= \frac{\varepsilon_2}{\varepsilon_1}\frac{n_1}{n_2} E_{0T} = \beta E_{0T}$$

$$\text{(iii)} \Rightarrow \left(E_{0I} + E_{0R}\right) = \frac{\cos\theta_T}{\cos\theta_I} E_{0T} = \alpha E_{0T}$$

$$\Rightarrow E_{0R} = \frac{\alpha-\beta}{\alpha+\beta} E_{0I} \Rightarrow R = \frac{\alpha-\beta}{\alpha+\beta}$$

$$E_{0T} = \frac{2}{\alpha+\beta} E_{0I} \Rightarrow T = \frac{2}{\alpha+\beta}$$

Break?

What do we learn from this ?

(1) Normal incidence :

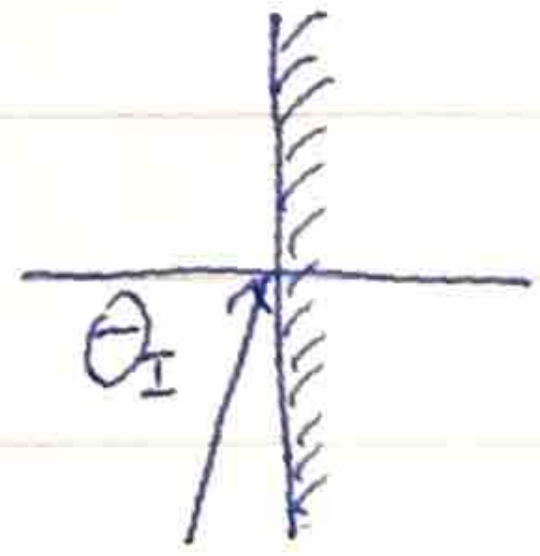
$$\alpha = \frac{\cos\theta_T}{\cos\theta_I} = 1$$

if $\mu_1 \sim \mu_2 \sim \mu_0$

$$\Rightarrow \beta = \frac{\varepsilon_2}{\varepsilon_1}\frac{n_1}{n_2} = \frac{n_2}{n_1}$$

$$\Rightarrow R = \frac{n_1 - n_2}{n_1 + n_2} \qquad T = \frac{2n_1}{n_1 + n_2}$$

(2) Grazing incidence :




$$\theta_I \approx 90^\circ$$

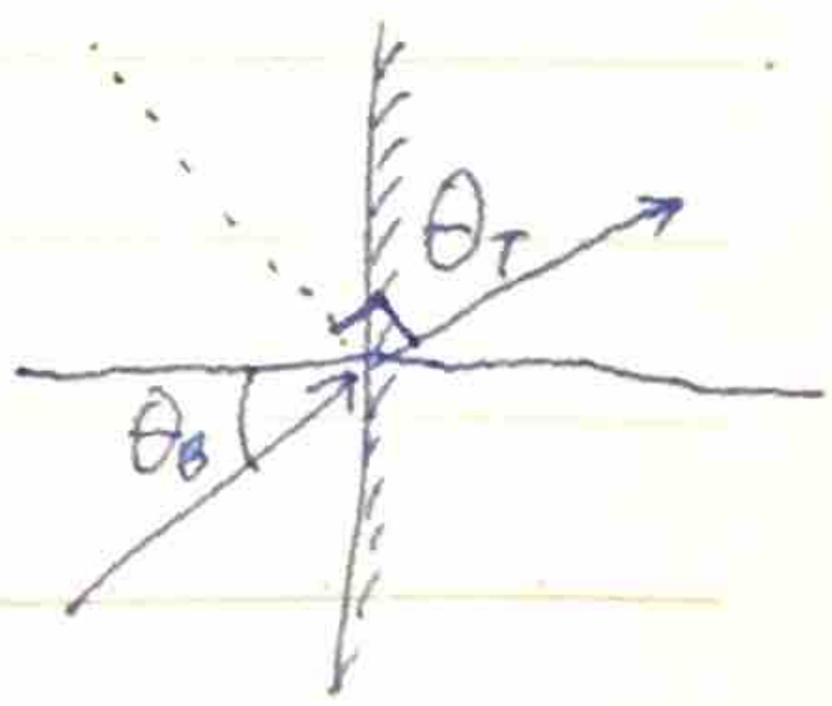
$$\Rightarrow \alpha \to \infty$$

$$\Rightarrow R \sim 1 \, , \, T \sim 0$$

(3) A very special angle : $\theta_B$

Brewster's Angle !

When $\alpha = \beta \Rightarrow R = 0, \; T = 1$ !



$$\alpha=\beta \Rightarrow \frac{\cos\theta_T}{\cos\theta_B} = \frac{n_2}{n_1} = \frac{\sin\theta_B}{\sin\theta_T}$$

$$\sin\theta_T\cos\theta_T = \sin\theta_B\cos\theta_B$$

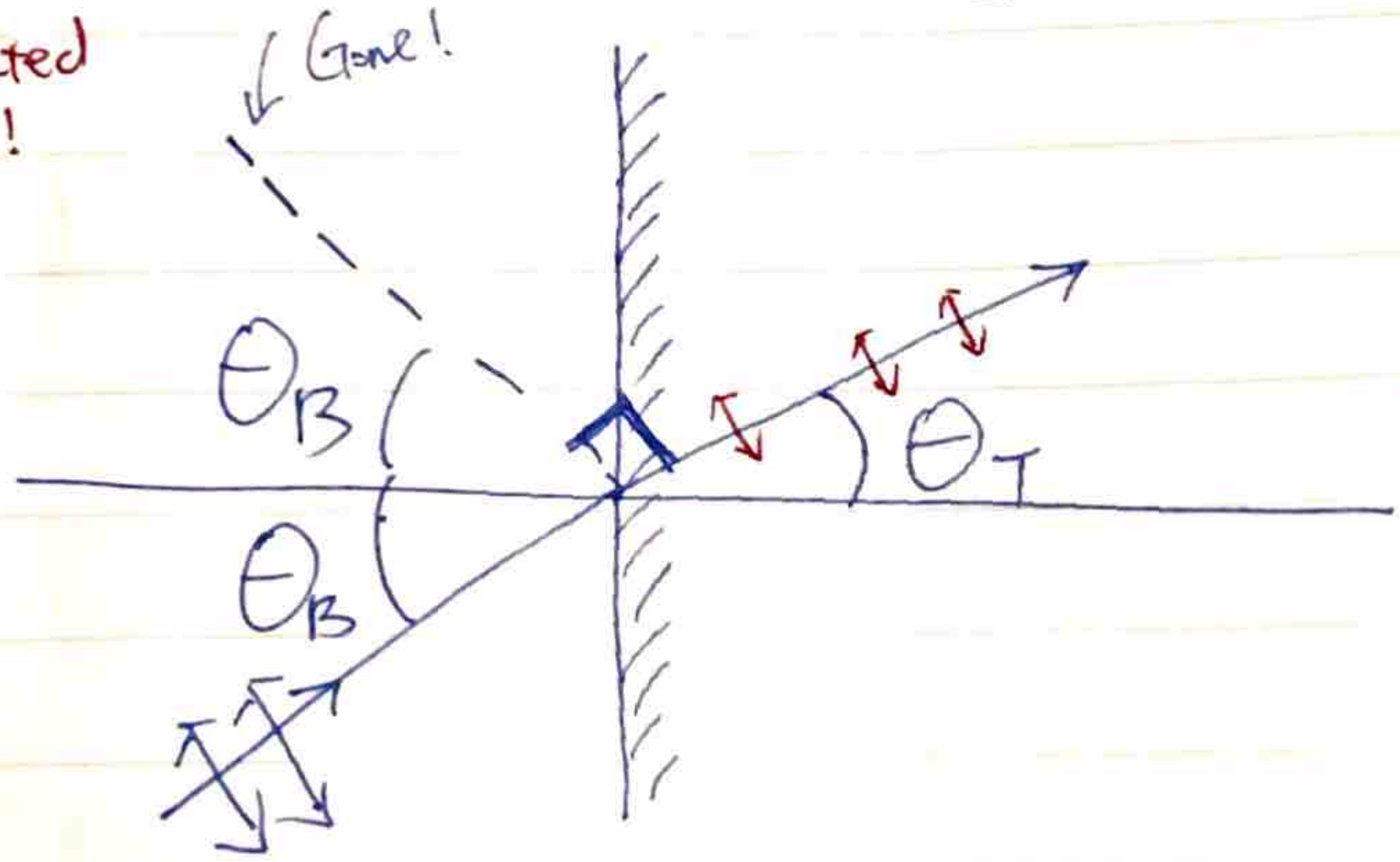
$$\Rightarrow \sin 2\theta_T = \sin 2\theta_B$$

$$\Rightarrow 2\theta_B = 2\theta_T \quad \times \qquad \because n_1\sin\theta_B = n_2\sin\theta_T$$

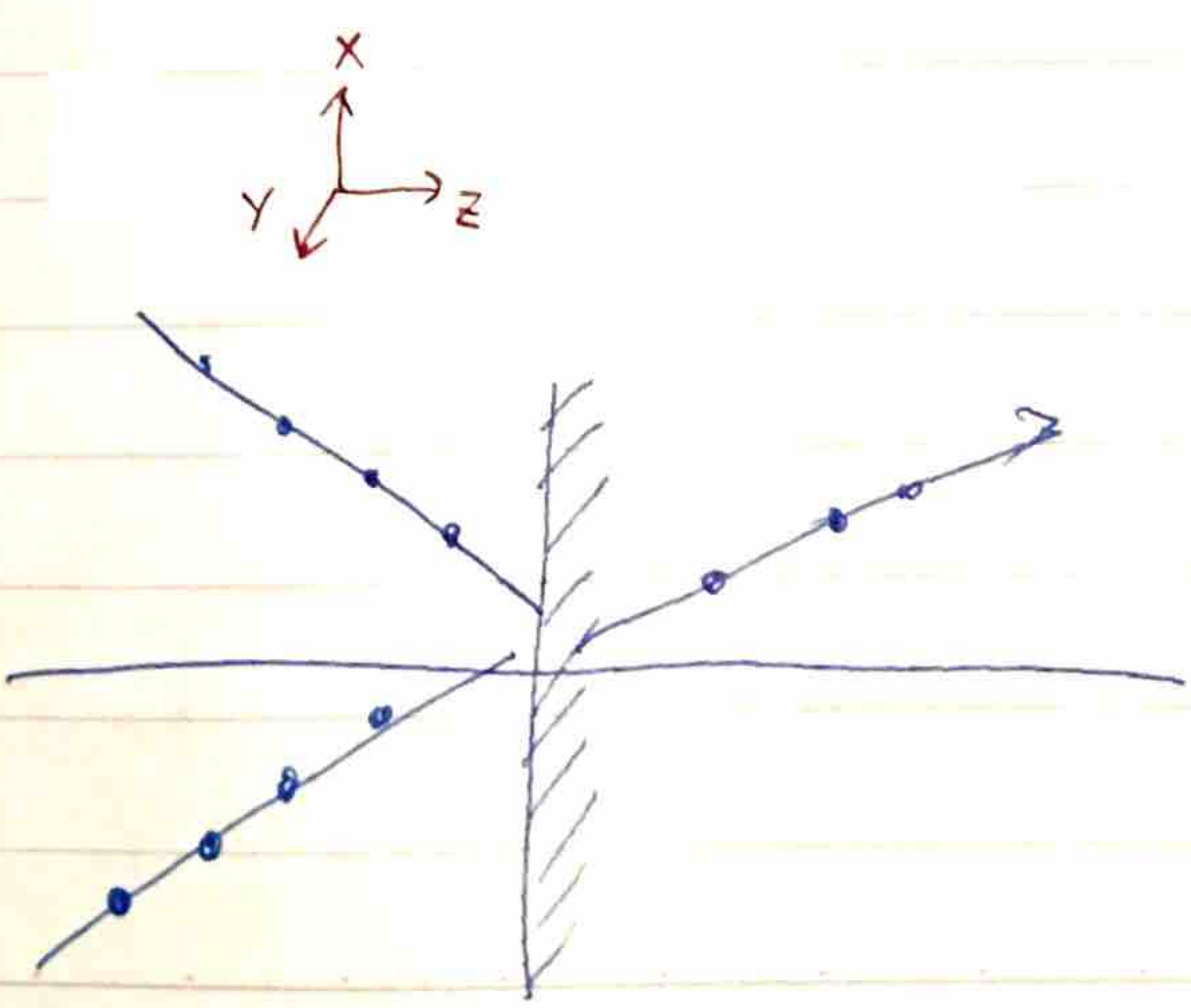
$$\text{or} \quad 2\theta_B = \pi - 2\theta_T$$

(a) $\Rightarrow \theta_B + \theta_T = \frac{\pi}{2}$ !

No reflected Wave!!!




(b)




If polarization is in y direction

→ No Brewster's angle