PROFESSOR: This is very important. This is the beginning of the uncertainty principle, the matrix formulation of quantum mechanics, and all those things. I want to just tabulate the information of matrices. We have an analog, so we have operators. And we think of them as matrices.

Then in addition to operators, we have wave functions. And we think of them as vectors. The operators act on the wave functions or functions, and matrices act on vectors. We have eigenstate sometimes and eigenvectors.

So matrices do the same thing. They don't necessarily commute. There are very many examples of that. I might as well give you a little example that is famous in the theory of spin, spin 1/2. There is the Pauli matrices. Sigma 1 is equal to 1, 1, 0, 0. Sigma 2 is 0 minus i, i 0, and sigma 3 is 1 minus 1, 0, 0.

And a preview of things to come-- the spin operator is actually h bar over 2 sigma. And you have to think of sigma as having three components. That's where it is. Spins will be like that. We won't have to deal with spins this semester. But there it is, that spin 1/2. Somehow these matrices encode spin 1/2.

And you can do simple things, like sigma 1 times sigma 2. 0, 1, 1, 0 times 0 minus i, i, 0. Let's see if I can get this right. i, 0, 0 minus i. And you can do sigma 2 sigma 1 0 minus i, i 0, 0, 1, 1, 0 equals minus i, 0, 0, i, i. So I can go ahead here.

And therefore, sigma 1 commutator with sigma 2 is equal to sigma 1, sigma 2 minus sigma 2, sigma 1. And you can see that they're actually the same up to a sign, so you get twice. So you get 2 times i 0, 0 minus i. And this is 2i times 1 minus 1, 0, 0. And that happens to be the sigma 3 matrix. So sigma 1 and sigma 2 is equal to 2i sigma 3.

These matrices talk to each other. And you would say, OK, these matrices commute to give you this matrix. This thing commutes to give you a number so that surely it's a lot easier. You couldn't be more wrong.

This is complicated, extraordinarily complicated to understand what this means. This is very easy. This is 2 by 2 matrices that you check. In fact, you can write matrices for x and p. This correspondence is not just an analogy. It's a concrete fact. You will learn-- not too much in this course, but in 805-- how to write matrices for any operator. They're called matrix representations.

And therefore, you could ask how does the matrix for x look. How does the matrix for p look? And the problem is these matrices have to be infinite dimensional. It's impossible to find two matrices whose commutator gives you a number.

Something you can prove in math is actually not difficult. You will all prove it through thinking a little bit. There's no two matrices that commute to give you a number. On the other hand, very easy to have matrices that commute to give you another matrix.

So this is very strange and profound and interesting, and this is much simpler. Spin 1/2 is much simpler. That's why people do quantum computations. They're working with matrices and simple stuff, and they go very far. This is very difficult. x and p is really complicated.

But that's OK. The purpose of this course is getting familiar with those things. So I want to now generalize this a little bit more to just give you the complete Schrodinger equation in three dimensions. So how do we work in three dimensions, three-dimensional physics?

There's two ways of teaching 804-- it's to just do everything in one dimension, and then one day, 2/3 of the way through the course-- well, we live in three dimensions, and we're going to add these things. But I don't want to do that. I want to, from the beginning, show you the three-dimensional thing and have you play with three-dimensional things and with one-dimensional things so that you don't get focused on just one dimension.

The emphasis will be in one dimension for a while, but I don't want you to get too focused on that. So what did we have with this thing? Well, we had p equal h bar over i d dx. But in three dimensions, that should be the momentum along the x direction.

We wrote waves like that with momentum along the x direction. And py should be h bar over i d dy, and pz should be h bar over i d dz-- momentum in the x, y, and z direction. And this corresponds to the idea that if you have a wave, a de Broglie wave in three dimensions, you would write this-- e to the i kx minus omega t, i omega t.

And the momentum would be equal to h bar k vector, because that's how the plane wave works. That's what de Broglie really said. He didn't say it in one dimension. Now, it may be easier to write this as p1 equal h bar over i d dx1, p2 h bar over i d dx2, and p3 h bar over i d dx3 so that you can say that all these three things are Pi equals h bar over i d dxi-- and maybe I should put pk, because the i and the i could get you confused-- with k running from 1 to 3.

So that's the momentum. They're three momenta, they're three coordinates. In vector notation, the momentum operator will be h bar over i times the gradient. You know that the gradient is a vector operator because d dx, d dy, d dz. So there you go. The x component of the momentum operators, h bar over i d dx, or d dx1, d dx2, d dx3. So this is the momentum operator.

And if you act on this wave with the momentum operator, you take the gradient, you get this-- so p hat vector. Now here's a problem. Where do you put the arrow? Before or after the hat? I don't know. It just doesn't look very nice either way. The type of notes I think we'll use for vectors is bold symbols so there will be no proliferation of vectors there.

So anyway, if you have this thing being the gradient acting on this wave function, e to the i kx minus i omega t, that would be h over i, the gradient, acting on a to the i kx vector minus i omega t. And the gradient acting on this-- this is a vector-- actually gives you a vector. So you can do component by component, but this gives you i k vector times the same wave function. So you get hk, which is the vector momentum times the wave function.

So the momentum operator has become the gradient. This is all nice. So what about the Schrodinger equation and the rest of these things? Well, it's not too complicated. We'll say one more thing.

So the energy operator, or the Hamiltonian, will be equal to p vector hat squared over 2m plus a potential that depends on all the coordinates x and t, the three coordinates. Even the potential is radial, like the hydrogen atom, is much simpler. There are conservation laws. Angular momentum works nice. All kinds of beautiful things happen. If not, you just leave it as x and p.

And now what is p hat squared? Well, p vector hat squared would be h bar over-- well, I'll write this-- p vector hat dotted with p vector hat. And this is h over i gradient dotted with h over i gradient, which is minus h squared Laplacian.

So your Schrodinger equation will be ih bar d psi dt is equal to the whole Hamiltonian, which will be h squared over 2m. Now Laplacian plus v of x and t multiplied by psi of x vector and t. And this is the full three-dimensional Schrodinger equation.

So it's not a new invention. If you invented the one-dimensional one, you could have invented the three-dimensional one as well. The only issue was recognizing that the second dx squared now turns into the full Laplacian, which is a very sensible thing to happen.

Now, the commutation relations that we had here before-- we had x with p is equal to ih bar. Now, px and x failed to commute, because d dx and x, they interact. But px will commute with y. y doesn't care about x derivative.

So the p's failed to commute. They give you a number with a corresponding coordinate. So you have the i-th component of the x operator and the j-th component of the p operator-- these are the components-- give you ih bar delta ij, where delta ij is a symbol that gives you 1 if i is equal to j and gives you 0 if i is different from j.

So here you go. X and px is 1 and 1. Delta 1, 1 is 1. So you get ih bar. But if you have x with py or p2, you would have delta 1, 2, and that's 0, because the two indices are not the same. So this is a neat way of writing nine equations. Because in principle, I should give you the commutator of x with px and py and pc, y with px, py, pc, and z with px, py, pc. You're seeing that, in fact, x just talks to px, y talks to py, z talks to pz.

So that's it for the Schrodinger equation. Our goal is going to be to understand this equation. So our next step is to try to figure out the interpretation of this psi. We've done very nicely by following these things.

We had a de Broglie wave. We found an equation. Which invented a free Schrodinger equation. We invented an interacting Schrodinger equation. But we still don't know what the wave function means.