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Do normal wave analysis to demonstrate that indeed these things should not quite happen. So for that, so ordinary waves and Galilean transformations. So when you have a wave, as you've probably have seen many times before, the key object in the wave is something called the phase of the wave. Phase, the phase. And it's controlled by this quantity $kx - \omega t$. k being the wave number, ω being the angular frequency and we spoke about. And the wave may be sine of that phase or cosine of that phase or a linear combination of sines and cosines, or E to this wave, any of those things could be your wave.

ZWIEBACH:

And whenever you have such a wave, what we say is that the phase of this wave is a Galilean invariant. Invariant. What it means is that two people looking at this wave, and they look at the point on this wave, both people will agree on the value of the phase, because basically, the reality of the wave is based on the phase, and if you have, for example, cosine of this phase, the place where this cosine is 0 is some of the phase, and if the cosine is 0, the wave is 0, and everybody should agree that the wave is 0 at that point. So if you have a place where the wave has a maximum or a place where the wave is 0, this is an ordinary wave, everybody would agree that at that place you have a maximum and in that place you have a 0.

So observers should agree on the value of this phase. It's going to be an invariant. And we can rewrite this phase in a perhaps more familiar way by factoring the k , and then you have $x - vt$ minus ωt , and this is 2π over λ , $x - vt$, and this quantity is called the velocity of the wave, and we'll write it this way. And I'll write in one last way-- $2\pi x$ over λ minus $2\pi V$ over λ t . And this quantity is ω and this quantity is k .

So this is our phase. And we've said that it's a Galilean invariant, so I will say that S should see-- the observer S' should see the same phase-- phase-- as S . So ϕ' , the phase that S' sees, must be equal to ϕ when referring to the same point. When referring to the same point at the same time.

Let's write this. So ϕ' should be equal to ϕ . And ϕ , we've written there. 2π over λ $x - vt$. And this is so far so good, but we want to write it in terms of quantities that S' measures. So this x should be replaced by 2π over λ $x' + vt'$ minus Vt' like this. And I could even do more if I wish. I could put t' here, because the t and t' are the same.

So ϕ prime, by the condition that these phases agree, it's given by this, which is by the relation between the coordinates and times of the two frames, just this quantity. So we can rewrite this as $2\pi \frac{x'}{\lambda} - 2\pi \frac{t'}{T} - \frac{v}{c} \frac{2\pi x'}{\lambda}$. I think I got the algebra right. $2\pi \frac{x'}{\lambda}$, the sine-- yes, I grouped those two terms and rewrote in that way.

So that is the phase. And therefore, we look at this phase and see, oh, whenever we have a wave, we can read the wave number by looking at the factor multiplying x , and we can read the frequency by looking at the factor multiplying t . So you can do the same thing in this case and read, therefore, that ω' , this whole quantity is this, ω' . And this is k' , because they can respond to the frame as prime. So ω' is equal to this $2\pi \frac{v}{\lambda}$, which is ω , times $1 - \frac{v}{c}$. And k' is equal to k or, what I wanted to show, that λ' for a normal wave is equal to λ for ordinary wave moving in the medium.

So at this moment, one wonders, so what happened? What have we learned? Is that this wave function is not like a sound wave. It's not like a water wave. We're doing everything non-relativistic. But still, we're seeing that you're not expected to have agreement. That is, if somebody looks at one wave function and you look at the same wave function, these two people will not agree on the value of the wave function necessarily.

So the things that we conclude-- so the conclusions are that waves are surprising. So size are not directly measurable-- measurable-- because if you had a quantity for which you could measure, like a sound wave or a water wave, and you could measure aspects to it, they should agree between different observables. So this is going to be something that is not directly measurable-- not all of ψ can be measured. Some of ψ can be measured, and you're already heard the hints of that. Because we said any number that you multiply, you cannot measure, and in the phase that you multiply, you cannot measure. So complex numbers can't be measured, you measure real numbers. So at the end of the day, these are not directly measurable, per se.

The second thing is that they're not Galilean invariant, and that sets the stage to that problem 6. You see, the fact that this phase that controls these waves is Galilean invariant led you to the equality of the wavelengths, but these wavelengths don't do that. The de Broglie wavelengths don't transform as they would do for a Galilean invariant wave. Therefore, this thing is not Galilean invariant, and what does that mean? That if you have two people and you

ask, what is the value of the wave function here at 103, the two observers might give you a different complex number for the wave function. They will just not agree.

Not all is lost, because you will find how their measurements can be compared. That will be the task of the problem. How-- if you have a wave function, how does your friend, that is moving with some velocity, measure the wave function? What does this other person measure?

So the end result, if you have a point here at some time t , the wave function ψ of x and t is not going to be the same as the wave function measured by the prime observer at x' and t' , so this point is the point x and t or x' and t' . These are two different labels for the same point. You're talking about the wave function at the same point at the same time. You still don't agree. These two people will not agree. If they agreed, this wave function would have a simpler transformation law with a wavelength that this can serve.

So by simply discussing the Galilean properties of this wave, we're led to know that the de Broglie waves are not like normal matter waves that propagate in a medium or simple.