PROFESSOR: We have found in this solution with some energy like this, that there's a decaying exponential over this side. And the question is often asked, well what happens if you tried to measure the particle in the forbidden region? Must be a problem. If you find the particle in the forbidden region, it has energy E that is less than v0, so you you have found the particle with negative kinetic energy. How does it look? How can it happen? What's going on? Can you really find the particle in the forbidden region? And then how does this negative kinetic energy look like?

The answer is that it's kind of funny what happens here. You can make two statements. It would be contradictory, contradictory if you could make-- could say the following things. One, that the particle is in the forbidden region, forbidden region. And two, that the particle has energy less than v0. Because then it would mean negative kinetic energy. So if you can say these two things, it seems contradictory.

So quantum mechanics evades this problem. Now, this is not discussed as far as I can see, except in some lecture notes of Gordon [? Boehme. ?] And because the argument is not 100% precise, but they think the spirit of the argument is clear. So I want to share it with you.

So here is the catch. This particle, remember it's governed by e to the minus kappa x is the forbidden region. So the length scale here where you can find it, the particle. The length scale is, this forbidden region stretches to about x of the order 1 over kappa. If you are going to find it, it is in the region of a distance 1 over kappa. At 10 1 over kappa you're not going to find it. The exponential is too small.

But remember, what was kappa? Kappa squared was 2m v0 minus E over h squared. That's what it was. Now if you want to see and declare that you have this particle, you would have to be able to measure position with some precision, with a precision a little smaller than this. Otherwise if you measure with precision 10 times that, well maybe it's to the left, maybe it's somewhere else. So you need to measure position with delta x a little smaller than 1 over kappa, otherwise you cannot really tell it's inside the forbidden region.

But now the problem is that if you do a position measurement, and you localize the wave function, there is some momentum uncertainty. The particle that you're looking at, as opposed to the particle to the left, has no momentum. It's a different kind of wave function. There's no momentum really associated or well-defined momentum to it.

So because you make a position, you're localizing x, whatever wave function you have. You're going to have some uncertainty, and some momentum that is going to be kind of bigger than h bar over delta x. So a momentum that is bigger than, or a little bigger, than h bar kappa. If delta x is less than that inequality, it goes in the same direction. So there's going to be an uncertainty P.

And therefore, this particle has now some kinetic energy due to this uncertain momentum. So uncertainty in the kinetic energy is how much? It's P squared over 2m, where P is this uncertain momentum. So this is equal to h bar kappa squared over 2m, which is equal to v0 minus E.

So actually, if you think about it, here is v0. This difference is v0 minus E. And you were going to say, oh, I found the particle, it has negative kinetic energy. But no. The uncertainty principle says, you found it localized? OK. Your kinetic energy, I'm sorry, no. There's an uncertainty. How much? v0 minus E.

So whatever you wanted to prove, it has been disproved. You can't do it. The total energy, total energy is now E plus the uncertainty in the energy, which is E plus v0 minus E. And it's therefore greater than or equal to v0. And no real contradiction.

So the uncertainty principle sort of conspires to prevent you from finding a particle with negative kinetic energy. And if you do detect a particle in the forbidden region, it will have total energy 0, or total kinetic energy 0. It will be a normal particle. Nothing strange about it.