**BARTON ZWIEBACH:** --that has served, also, our first example of solving the Schrodinger equation. Last time, I showed you a particle in a circle. And we wrote the wave function. And we said, OK, let's see what is the momentum of it. But now, let's solve, completely, this problem.

So we have the particle in the circle. Which means particle moving here. And this is the coordinate x. And x goes from 0 to L. And we think of this point and that point, identify. We actually write this as, x is the same as x plus L. This is a strange way of saying things, but it's actually very practical. Here is 2L, 3L.

We say that any point is the same as the point at which you add L. So the circle is the whole, infinite line with this identification, because every point here, for example, is the same as this point. And this point is the same as that point. So at the end of everything, it's equivalent to this piece, where L is equivalent to 0.

It's almost like if I was walking here in this room, I begin here. I go there. And when I reach those control panels, somehow, it looks like a door. And I walk in. And there's another classroom there with lots of people sitting. And it continues, and goes on forever.

And then I would conclude that I live in a circle, because I have just begun here and returned to the same point that is there. And it just continues. So here it is. You are all sitting here. But you are all sitting there. And you are all sitting there, and just live on a circle.

So this implies that in order to solve wave functions in a circle, we'll have to put that psi of x plus L is equal to psi of x, which are the same points. And we'll have 0 potential. V of x equals 0. It will make life simple. So the Hamiltonian is just minus h squared over 2m d second dx squared.

We want to find the energy eigenstate. So we want to find minus h squared over 2m d second psi dx squared is equal to E psi. We want to find those solutions.

Now it's simple, or relatively simple to show that all the energies that you can find are either zero or positive. It's impossible to find solutions of this equation with a negative energies.

And we do it as follows. We multiply by dx and psi star and integrate from 0 to L. So we do that on this equation. And what will we get? Minus h squared over 2m integral psi star of x d dx of d dx psi of x is equal to E times the integral psi star psi x dx.

And we will assume, of course, that you have things that are well normalized. So if this is well normalized, this is 1. So this is the energy is equal to this quantity. And look at this quantity. This is minus h squared over 2m.

I could integrate by parts. If I do this quickly, I would say, just integrate by parts over here. And if we integrate by parts, d dx of psi of x, we will get a minus sign. We'll cancel this minus sign, and will be over.

But let's do it a little bit more slowly. You can put dx, this is equal to d dx of psi star d psi dx minus d psi star dx d psi dx. I will do it like this, with a nice big bracket.

Look what I wrote. I rewrote the psi star d second of psi as d dx of this quantity, which gives me this term when the derivative acts on the second factor. But then I used an extra term, where the derivative acts on the first factor that is not present in the above line. Therefore, it must be subtracted out. So this bracket has replaced this thing.

Now d dx of something, if you integrate over x from 0 to L, the derivative of something, this will be minus h bar squared over 2m psi star d psi dx integrated at L and at 0. And then minus cancels. So you get plus h squared over 2m integral from 0 to L dx d psi dx squared equal E.

And therefore, this quantity is 0. The point L is the same point as the point 0. This is not the point at infinity. I cannot say that the wave function goes to 0 at L, or goes to 0, because you're going to infinity. No, they have a better argument in this case.

Whatever it is, the wave function, the derivative, everything, is periodic with L. So whatever values it has at L equal 0 it has-- at x equals 0, it has at x equals L. So this is 0. And this equation shows that E is the integral of a positive quantity. So it's showing that E is greater than 0, as claimed.

So E is greater than 0. So let's just try a couple of solutions, and solve. We'll comment on them more in time. But let's get the solutions, because, after all, that's what we're supposed to do.

The differential equation is d second psi dx squared is equal to minus 2mE over h squared psi. And here comes the thing. We always like to define quantities, numbers. If this is a number, and E is positive, this I can call minus k squared psi. Where k is a real number. Because k real, the square is positive. And we've shown that the energy is positive.

And in fact, this is nice notation. Because if you were setting k squared equal to 2mE over h squared, you're saying that E is equal to h squared k squared over 2m. So, in fact, the momentum is equal to hk. Which is very nice notation.

So this number, k, actually has the meaning that we usually associate, that hk is the momentum. And now you just have to solve this. d second psi dx squared is equal to minus k squared psi. Well, those are solved by sines or cosines of kx. So you could choose sine of kx, cosine of kx, e to the ikx. And this is, kind of better, or easier, because you don't have to deal with two types of different functions. And when you take k and minus k, you have to use this, too. So let's try this. And these are your solutions, indeed. psi is equal to e to the ikx. So we leave for next time to analyze the [INAUDIBLE] details. What values of k are necessary for periodicity and how we normalize this wave function.