MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Physics Department

8.044 Statistical Physics I — Spring Term 2003

Practice Exam #2

Problem 1 (25 points) Elastic Rod

For a certain elastic rod of length L, tension $\mathcal{F}$, and temperature T the following facts are known.

i) The isothermal Young's modulus $\equiv L(\partial\mathcal{F}/\partial L)_T = L(a + bT)$.

ii) The expansion coefficient $\equiv (1/L)(\partial L/\partial T)_{\mathcal{F}} = \dfrac{-b}{(a + bT)}\left(\dfrac{L - L_0}{L}\right)$.

iii) The tension vanishes when $L = L_0$.

In the above expressions, a, b, and L_0 are constants. Find the tension as a function of temperature and length, $\mathcal{F}(T, L)$.

Problem 2 (35 points) Adiabatic Demagnetization

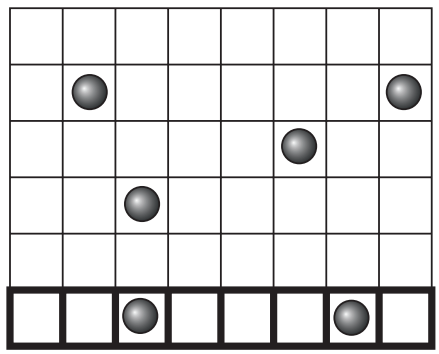
N magnetic ions in a solid behave as Curie Law paramagnet with an equation of state

$$M = \frac{aH}{T}$$

where a is a constant proportional to the number of ions in the solid. Experiments show that the internal energy of the system is independent of magnetization if the temperature is held constant, and that the heat capacity at constant magnetization, C_M, is a constant independent of M or T.

The system is initially magnetized to M_0 at a temperature of T_0. The magnetization is subsequently lowered in an adiabatic, quasistatic process all the way to zero ($M = 0$). Find the final temperature of the system.

Problem 3 (40 points) Adsorption



If a gas is confined in a container a fraction of the atoms will inevitably be found on the wall, a process known as physical adsorption. We will study this process by neglecting the kinetic energy of the atoms and using a discrete model for the locations of the atoms in the bulk and on the surface.

Let M be the number of possible spatial cells the atoms may occupy in the bulk and N be the number of spatial cells on the surface. The gas consists of N atoms (just enough to completely fill the surface states). Let n be the number of atoms actually on the surface: $n \leq N$. An atom has an energy $-\epsilon$ while it is on the surface and 0 while it is in the bulk; thus $E = -\epsilon n$. M, N, and ϵ are all constants; n is a variable. M, N and n are all very large.

a) Using the microcanonical ensemble find the entropy as a function of n.

b) Derive an expression relating n to the temperature of the system. You do <u>not</u> have to solve the expression to find an explicit relation $n = n(T)$.

c) Find n when $T = 0$.

d) Find the limit of n/N as $T \to \infty$.

PARTIAL DERIVATIVE RELATIONSHIPS

Let x, y, z be quantities satisfying a functional relation $f(x, y, z) = 0$. Let w be a function of any two of x, y, z. Then

$$\left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w = \left(\frac{\partial x}{\partial z}\right)_w$$

$$\left(\frac{\partial x}{\partial y}\right)_z = \frac{1}{\left(\frac{\partial y}{\partial x}\right)_z}$$

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$$

COMBINATORIAL FACTS

There are $K!$ different orderings of K objects. The number of ways of choosing L objects from a set of K objects is

$$\frac{K!}{(K-L)!}$$

if the order in which they are chosen matters, and

$$\frac{K!}{L!(K-L)!}$$

if order does not matter.

STERLING'S APPROXIMATION

When $K \gg 1$

$$\ln K! \approx K \ln K - K \quad \text{or} \quad K! \approx (K/e)^K$$

DERIVATIVE OF A LOG

$$\frac{d}{dx} \ln u(x) = \frac{1}{u(x)} \frac{du(x)}{dx}$$

VOLUME OF AN α DIMENSIONAL SPHERE OF RADIUS R

$$\frac{\pi^{\alpha/2}}{(\alpha/2)!} R^\alpha$$

LIMITS

$$\lim_{n\to\infty} \frac{\ln n}{n} = 0$$

$$\lim_{n\to\infty} \sqrt[n]{n} = 1$$

$$\lim_{n\to\infty} x^{1/n} = 1 \quad (x > 0)$$

$$\lim_{n\to\infty} x^n = 0 \quad (|x| < 1)$$

$$\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x \quad (\text{any } x)$$

$$\lim_{n\to\infty} \frac{x^n}{n!} = 0 \quad (\text{any } x)$$

WORK IN SIMPLE SYSTEMS

System	Intensive quantity	Extensive quantity	Work
Hydrostatic system	P	V	$-PdV$
Wire	$\mathcal{F}$	L	$\mathcal{F}dL$
Surface	$\mathcal{S}$	A	$\mathcal{S}dA$
Reversible cell	E	Z	EdZ
Dielectric material	$\mathcal{E}$	$\mathcal{P}$	$\mathcal{E}d\mathcal{P}$
Magnetic material	H	M	HdM