PROFESSOR: OK, let me get going. Last time we were talking about multi-particle states and tensor products. And for that, we explained that if we have a system, a quantum mechanical system of one particle described by a vector space V, and the quantum mechanical system of another particle described with a vector space W, the quantum mechanics of the total system composed by the two particles is defined on a new vector space called the space V tensor W. And that was a construction that showed that in particular it was not true to say that, oh if you want to know the system of particle 1 and 2, you just tell me what state particle 1 is and what state particle 2 is, and that's the end of the story. No, the story is really more sophisticated than that.

So the typical elements on this space were of the form aij vi cross wj. And it's a sum over i and j numbers times these vectors. So you pick a vector in the first vector space, a vector in the second vector space, you put them in here and take linear combinations of them. So that's the general state in the system.

Now we said a few things about this. One thing I didn't say too much about was the issue of the vector 0 in this tensor space. And well, vector 0 is some element of any vector space is an important element. And we could get a little confused about how it looks.

And here's for example, the vector 0 in v cross w. An example of the vector 0 is the vector 0 tensor wi. If you put in the first input, the vector 0, that's it. That is also the vector 0 in here.

Vi tensor the vector 0 in w. Here is 0 in w. Here is 0 in v. This is also 0. It's maybe a little surprising. Now how do we see that? Well we had a property. For example, this one. av tensor w is equal to av tensor w, where a is a number.

So pick a equals 0. Well 0 times any vector is the 0 vector. 0 cross w. But 0 times any vector is also the vector 0. So this is the 0 in v cross w. So 0 cross w is the vector 0. Once you put 0 in one of the two inputs, you're there. You're at 0. You don't have more.

So that's just a comment on the vector 0. Now we did a few things. And one thing I didn't do last time was to define an inner product on the new vector space. So let's define a way to get numbers from one vector in the tensor space and another vector in the tensor space.

So inner product. And again, here you're supposed to define it to your best understanding and the hope that, once you make the right definitions, it has all that axiomatic properties it should have. So let me take the following thing. The inner product with this thing aij vi omega j with bpq vp wq.

So I will define this by assuming the linearity in the inputs on the right inputs and the anti-linearity here on the left input. So this would be the sum over inj here. So I'll put sum over inj aij star sum over pq bpq and then vi wj comma vp wq.

So by declaring that this is the case, I'm saying that the inner product in the tensor space has the-- I'm demanding it has the properties that we expect. If you have a vector plus another vector here, well you get this times the first plus this times the second. So you can take the sums out and arrange it this way.

But I still haven't got a number, and the inner product is supposed to be a number. So how do we get a number at this stage? I have this thing, and nobody has told me what this is supposed to be. At this stage, the only thing you can say is, well, you know I suspect that, if I had an inner product in V and I had an inner product in w, I must have an inner product here, and somehow I should use that.

So they still define to be ij pq aij bpq. And then what you do is use the inner product in v to get a number from these two vectors. This is going the v inner product. And use the inner product on w to get a number from the two w vectors. And that's it.

The end of the definition.

Now here maybe this is the sort of most interesting step, where this part was set equal to this. And consistent with what I was telling you about 0, suppose any of this vi was 0. If this vi was 0, we would have 0 with vp. That would be 0, so this whole number is 0.

So the way this can happen is one of the vectors must be 0 here. And well, you have the 0 vector here, and the zero vector inner product with anything is 0. So it's, again, consistent to think that, once you put one of these entries to 0, you've got the 0 vector.

So where are we going today? Well, we have now the inner product, and I want to go back to a state we had last time. What we're going to do today is define what we called an entangled state. Then we will consider basis of entangled states, and we will be able to discuss this sort of nice example of teleportation, quantum teleportation.

So that's where we're going today. I wanted to remind you of a calculation we were doing last time. We had established that there was a state in the tensor product of 2 spin 1/2 particles. And the state was alpha plus tensor minus minus minus tensor plus.

Now you can sometimes-- this is an example of a superposition of vectors of the from in the v cross w. So here is a vector of that form. There is a vector of this form. Sometimes we put here 1 and 2. And sometimes it will be useful to put those labels. Because if you don't put the labels, you better make sure that you're always talking that the first ket, is the one for the first vector space, and the second ket is the one for the second vector space.

There's nothing really known commutative here. So if somebody would write for you 1 minus 2, or they would write minus 2 1, both of you would be talking about the same state. But if you don't put the labels, you know you're not something about the same state, because you assume always the first one goes to the first Hilbert space.

The second one goes with the second vector space.

So we considered an entangled state of two spin 1/2 particles. I'm not using-- it's not fair to use the word entangled yet, but we'll be able to say this very soon. So the one thing we can do now given the inner product is try to normalize this state.

So how do we normalize this state? Well, we must take the inner product of this state with itself. So psi psi. So then what do we do? Well, given these rules, we're supposed to take all this vector here, all that vector there, 1 alpha-- the alpha that is on the left goes out as an alpha star. The alpha that this on the right goes out as an alpha. And we have plus minus minus minus plus inner product with plus minus minus minus plus.

Now this is easier than what it seems from what I'm writing. You will be able to do these things, I think. Or you can already maybe do them by inspection. Basically at this stage, you have to do each one with each one here. And let's see what we get. Well, what is the inner product of this with this? This works, because the inner product of plus with plus is 1 and minus with minus is 1.

This on the other hand, doesn't give any contribution, because the first one is a plus has 0 inner product with a minus. A minus has 0 with a plus. That doesn't matter. It's an overkill. So this one couples with this, and this one couples with that.

Another way people would do this is to say oh don't worry just take the bra here. So it's plus minus. Here is one. I'll put the labels too. Minus the bra of the minus is the minus like that. 1 plus 2.

And now you do this with this ket, the plus minus 1 2 minus the minus plus 1 2. And bras and kets, you know that this one goes with this one. Plus plus, minus minus, this one goes with this one. And here I put the labels, because when I form the bra, it's not obvious which one you would put first, but it doesn't really matter.

So back here, we have norm of alpha squared. And this with this is 1. And minus is one, this is another one. So this is 2 alpha squared. So if I want it to be normalized, I take alpha 1 over square root of 2. And this is the well normalized state. So this is the unit normalized state.

So we have this state. This state is something you've played with over last week. Is that state that we started very fine in lecture that had 0 z component of angular momentum, 0 x component of angular momentum, and 0 y component of angular momentum. Total angular momentum as we defined it. And this has a state with absolutely no angular momentum.

And what you verified in the homework was that that state, in fact, is rotational invariant. You apply a rotation operator to that state by rotating in both spaces, and out comes the same state. The state is not changed. So it's a very interesting state that will be important for us later.

All right, so having taken care of inner products and normalizations, let's talk a little about entangled states. So entangled states. So these are precisely those states in which you cannot say, or describe them by saying particle one is doing this, particle two is doing that.

You've learned that v cross w includes a state superpositions alpha ij vi cross omega j. The question is, if somebody hands you a state like this, maybe you could do some algebra or some trickery. And is it equal, you ask, to some sort of vector u star tensor v star times some vector w star. Is it equal? Is there vectors v star and omega star belonging to v and belonging to w in such a way that this thing, the sum, can be written as a product of something and that.

If you would have that, then you would be able to say look, yes, this is an interesting state, but actually it's all simple here. Particle one is to state v star. Particle two is in state w star. If this has happened, if so, this state of the two particles is not entangled.

So if you can really factor it, it's not entangled. If there are no such vectors v star and w star, then it is entangled. So you can say, well, it's a complicated factorization problem. And indeed, it might take a little work to figure out if a state is entangled or not.

It's not a basis dependence problem. It's not like it's entangled in one basis or not. Here is a state, and you find any two things that tensor this way give you the state.

So the simplest example to illustrate this is two dimensional vector spaces, v and w. Two dimensional complex. So v will have a basis e1 and e2. w will have a basis f1 and f2. And the most general state you could write is a state, general state, is a number a11 e1 f1 plus a2 e1 f2 plus a21 e2 f1 plus a22 e2 f2. That's it.

There's two basis states in v, two basis state in w. v cross w is dimension for product of dimensions for basis states, the products of the e's with the f's. So that's it. That's the general vector.

The question is if this is it equal to something like a1 e1 plus a2 e2. Some general vector, you write the most general vector in v, and you write the most general vector b1 f1 plus b2 f2 in w. And you ask is it equal to a product, tensor product, of some vector in v with some vector in w. So the question is really are their numbers a1, a2, b1, and b2 so that this whole thing gets factorized.

So that's happily not a complicated problem. We could see if those number exist, if a1, a2, b1, b2 exist, then the state is not entangled. You've managed to factor it out.

So let's see. Well, we know the distributive laws apply. So actually e1 f1 can only arise from this product. So to have a solution you must have that a11 is equal to a1 b1. a12 can only appear from the product of e1 with f2. So a12 must be equal to a1 b2. a21 must be equal to a2 b2. And a22 must be equal to a2 b2. And we must try to solve for these quantities.

Actually, there is a consistency condition. You see these quantities repeat here in a funny way. If this holds from this, a11 a22 minus a12 a21 is equal to what? a11 a22 would be a1 b1 a2 b2. And a12 a21 also have the same things. a1 b2 a2 b1. Well, both terms have both a's and both b's, so this system only has a solution if this product is 0.

So if you give me four numbers, if you hope to factorize it, you must have the determinant of this matrix-- if you collapse it into a matrix, a11 a12 a21 a22, if you encode the information about this state in a matrix, it's necessary that the determinant of the matrix a be equal 0. So the determinant of a is equal to 0 is certainly necessary for the factorization to take place. But a very small argument that will be in the notes, or you can try to complete it, shows that the determinant equal to 0, in fact, guarantees that then you can solve this system. There's a solution.

And this is not complicated. So determinant equals 0 is actually the same as not entangled. We've done not entangled. So there's a solution implies determinant a equals 0, but determinant of a equals 0 also implies not entangled. You do that by solving this.

Let's not spend time doing that. The basic way to do it is to assume-- consider, say, a11 equals 0 and solve it. Then a11 different from 0, and then you can show that you can choose these quantities. So it can be factored.

And you have that, if these numbers are such that the determinant is 0, then the state is entangled. And it's very easy to have a determinant of this non-zero. For example, you could have these two 0 and these two non-zero. That will be entangled because the determinant is non-zero. You can have this two that will be entangled. There are many ways of getting entangled states.

So in fact, there's enough ways to get entangled states that we can construct a basis. We had a basis here of e1 f1 e2 f2. This thing. This four vector basis. We can construct a basis that is all the states, all the basis vectors are entangled states. That's what we're going to do next. But maybe it's about time for questions, things that have become a little unclear as I went along. Yes?

AUDIENCE: So what exactly does an entangled state mean? What are the [INAUDIBLE] to give me an entangled state.

PROFESSOR: Well, the main thing that it happens is that there will be interesting correlations when you have an entangled state. If you have an entangled state and you find a state that is not entangled, you can say particle one is doing this and particle two is doing that. And particle two is doing this independent of what particle one is doing.

But when a state is entangled, whatever is happening with particle one is correlated with what is happening in particle two in a strange way. So if particle one is doing something, then particle two is doing another thing. But if particle one is doing another thing, then particle two is doing something. And these particles can be very far apart, and that's when it gets really interesting.

So we're going to do a lot of things with entangled states. Today we're doing this teleportation using entangled state, and you will see how subtle it is. Next time we do EPR, these Einstein Podolsky Rosen arguments and the Bell inequalities that answered that with entangled states. There's a couple of problems in the homework set also developing entangled states in different directions. And I think by the time we're done, you'll feel very comfortable with this.

So a basis of entangled states. Here are those. We're going to use spins. So we're going to use v is the state space of spin 1/2. And we're going to consider a v tensor v where this refers to the first particle and this this to the second particle.

So let's take one state, phi 0, defined to be 1 over square root of 2, and I don't put indices. And probably at some stage, you also tend to drop the tensor product. I don't know if it's early enough to drop it. Probably we could drop it. We'll put plus plus minus minus.

Of course, people eventually drop even the other ket and put it plus plus. So those are the evolutions of notation. As you get to more and more calculations, you write less, but hopefully, it's still clear. But I will not do this one. I will still keep that because many times, I will want to keep labels. Otherwise, it's a little more cumbersome.

So this state is normalized. Phi0 phi0 is equal to 1. It's the state we built. Oh, in fact, I want it with a plus. Sorry. It's similar to the state we had there.

And by now, you say, look, yes, it's normalized. Let's take the dual. Plus plus with plus plus will give me 1. The minus minus with minus minus will give me 1. This is 2. 1 over square root of 2 squared, 1. It should become sort of easy by inspection that this is normalized.

And this is entangled state because in the matrix representation, it's a 1 here and a 1 there. You have the 1 1 product and the 2 2 product. So 1 1, the determinant is non-zero. There's no way, we've proven, you can find how to factor this. There's no alpha. There's no way to write this as an alpha plus, plus beta minus, times a gamma plus, plus delta minus. Just impossible. We've proven it. It's entangled.

So this is an entangled state, but the state space is four dimensional. So if it's four dimensional, we need three more basis states. So here they are. I'm going to write a formula for them.

Phi i for i equals 1, 2, and 3 will be defined to be the following thing. You will act with the operator 1 tensor sigma i on phi 0. So three ways of doing. Let's do 1, for example, phi 1. What is it?

Well, you would have 1 times sigma 1 acting on the state phi 0, which is 1 over square root of 2 plus, plus, plus minus, minus. Well, the 1 acts on the first ket, the sigma acts on the second ket.

So what do we get here? 1 over square root of 2-- let me go a little slow-- plus sigma 1 plus, plus, minus sigma 1 minus. And this is phi 1 equals sigma 1 plus is the minus state, and sigma 1 minus is the plus state. 1 over square root of 2. Those are things that you may just remember sigma 1 is this matrix. So you get 1 over square root of 2 plus, minus, plus, minus, plus. So that's phi 1.

And phi 1 is orthogonal to phi 0. You can see that because plus minus cannot have an overlap with plus plus, nor with minus minus. Here minus plus, no. In order to get something, you would have to have the same label here and the same label here so that something matches.

Well, we can do the other ones as well. I will not bother you too much writing them out. So what do they look like? Well, you have phi 2 would be 1 tensor sigma 2 on phi 0. And that would give you-- I will just copy it-- an i because sigma 2 has i's there. So i over square root of 2 plus, minus, minus, minus, plus.

Finally, phi 3 is 1 tensor sigma 3 phi 0. And it's 1 over square root of 2 plus, plus, minus, minus, minus. We got the states here. Let's just check they're orthonormal.

Well, here's one thing. If you take phi 0 with 1 tensor sigma i phi 0, which is phi 0 with phi i. Well, this is 0. You could say, well, how do you know? How do you prove it easily?

Well, I think the best way is just inspection, so let's look at that. Phi 1, we said, is orthogonal to phi 0 because it has plus minus and minus plus, and that can never do anything with that. Phi 2 also has plus minuses and minus pluses, so we can never have anything to do with phi 0.

The only one that has a chance to have an inner product with phi 0 is phi 2 because it has a plus plus and a minus minus. On the other hand, when you flip them, this term with a plus plus of phi 0 will give you 1, but here's a difference of sign. So this with the second term of phi 00 will give you a minus, and therefore, it will be 0. So these things are all 0 by inspection.

You don't really have to do a calculation there. The one that takes a little more work is to try to understand what is the inner product of phi i with phi j. Now, you could say, OK, I'm going to do them by inspection. After all, there's just six things to check.

But let's just do it a little more intelligently. Let's try to calculate this by saying, well, this is phi 0. Since the Pauli matrices are Hermitian, this phi i is also 1 tensor sigma i. They're Hermitian, so acting on the left, they're doing the right thing.

Given our definition, here is a definition as well. So you take the bra and that's what it is. It would have been dagger here but it's not necessary. And then you have the phi j, which is 1 tensor sigma j. And that's phi 0 here.

That sounds like the kind of thing that we can make progress using our Pauli identities. Indeed, first thing is that the product of operators, they multiply just in that order in the tensor product. So phi 0, you have 1 times 1, which is 1 tensor sigma i sigma j phi 0.

And this is equal to phi 0 1 tensor. Now, the product of two Pauli matrices gives you an identity plus a Pauli matrix. You may or may not remember this formula, but it's 1 times delta ij plus i epsilon ijk sigma k phi 0.

Now, what do we get? Look, the second term has a sigma k on phi 0, so it's some number with a psi k here, while the first term is very simple. What do we get from the first term? From the first term, we get-- well, 1 tensor 1 between any two things is nothing because the 1 acting on things and the 1 acting on another thing is 0. So the unit operator in the tensor product is 1 tensor 1. That's nothing whatsoever.

So what do you get here? Delta ij times phi 0 phi 0 plus i epsilon ijk phi 0 phi k. But that is 0. We already showed that any phi i with phi 0 is 0. And this is 1.

So what have we learned? That this whole thing is delta ij. And therefore, the basis is orthonormal. So we've got a basis of orthonormal states in the tensor product of two spin 1/2 particles.

And the nice thing about this basis is that all of these basis states are entangled states. They're entangled because they fill different parts of the matrix. Here you have 1 and 1 and minus 1 here. This would be plus minus, would be an i here and a minus i there. The determinants are non-zero for all of them, and therefore, they can't be factored, and therefore, they're entangled.

So the last thing I want to do with this is to record a formula for you, which is a formula of the basis states in the conventional way, written as superposition of entangled states. So for example, you say, what is plus plus?

Well, plus plus, looking there, how would you solve it? You would solve it from phi 0 and phi 3. You would take the sum so that the minus minus states cancel. Phi 0 and phi 3, and therefore, this state must be 1 over square root of 2, phi 0 plus phi 3. A useful relation.

Then we have plus minus. Then we have minus plus. And finally, minus minus. Well, minus minus would be done by 1 over square root of 2 phi 0 minus phi 3.

The other ones, well, they just leave complex numbers. Phi 1 has this plus minus, and this has a plus minus in phi 2. The only problem is it has an i, so you must take this state minus i times this state will produce this state twice and will cancel this term. That's what you want.

So phi 1, this should be 1 over square root of 2 phi 1 minus i phi 2. And this one should be phi 1 plus i phi 2. And if this was a little quick, it's just algebra, one more line. You do it with patience in private.

So here it is. It's the normal product, simple product basis expressed as a superposition of entangled states. This is called the bell basis, this phi 1 up to phi 4, the bell basis.

And now, I have to say a couple more things and we're on our way to begin the teleportation thing. Are there questions? Any questions about bell basis or the basis we've introduced? Any confusion? Errors on the blackboard?

So we have a basis, and I want to make two remarks before we get started with the teleportation. It's one remark about measurement and one remark about evolution of states. Two facts.

The first fact has to do with measurement in orthonormal basis. If you have an orthonormal basis, the postulate of measurement of quantum mechanics can be stated as saying that you can do an experiment in which you find the probability of your state being along any of these basis states of the orthonormal basis. So you can do an experiment to detect in which of the basis states the state is.

Now, the state, of course, is in a superposition of basis states, but it will collapse into one of them with some probability. So the Stern-Gerlach experiment was an example in which you pick two basis states, orthogonal, and there was a device that allowed you to collapse the state into one or the other. So this is a little more general, not just for two state systems. If there would be a particle with three states, well, orthonormal states, then there is in principle an operator in quantum mechanics that allows it to measure which of these basis states you go into.

So let me state this as saying, given an orthonormal basis, e1 up to en, we can measure a state, psi, and we get that the probability to be in ei is, as you know, ei overlapped with a state squared.

And if you measure that this probability, the state will collapse into one of these states. So after the measurement, the state goes into some ek. There are different probabilities to be in each one of those basis states, but the particle will choose one.

Now, the other thing I want to mention is that a fact that has seemed always a gift, the Pauli matrices are not only Hermitian, but they square to one, and therefore they're also unitary. So the Pauli matrices are unitary. So actually, they can be realized as time evolution.

So you have a state and you want to multiply it by sigma 1. You say, OK, well, that's a very mathematical thing. Not so mathematical because it's a unitary operator, so it could respond to some time evolution. So we claim there is a Hamiltonian that you can construct that will evolve the state and multiply it by sigma 1.

So all these Pauli matrices, sigma 1, sigma 2, and sigma 3 are unitary as operators. They can be realized by time evolution with a suitable Hamiltonian. So if you're talking spin states, some magnetic field that lifts for some few picoseconds according to the dipole, and that's it. It will implement sigma one.

Just in fact, you can check, for example, that e to the i pi over 2 minus 1 plus sigma i. This is i this, and this is Hermitian. Well, this is 1 and sigma i. 1 and sigma i commute, so this is equal to e to the minus i pi over 2 times e to the i pi sigma 1 over 2.

The first factor is a minus i, and the second factor is 1 times cosine of pi over 2 plus i sigma 1 sine of pi over 2. So this is minus i times-- this is 0-- times i sigma 1. So this is sigma 1.

So we've written sigma 1 as the exponential of i times the Hermitian operator. And therefore, you could say that this must be equal to some time times some Hamiltonian over h bar. And you decide, you put the magnetic field in the x, y, z direction. You realize it. So sigmas can be realized by a machine.

We're all done with our preliminary remarks, and it's now time to do the teleportation stuff. Quantum teleportation. So we all know this teleportation is the stuff of science fiction and movies and kind of stuff like that, and it's pretty much something that was, classically, essentially impossible. You have an object, you sort of dematerialize it and create it somewhere else. No basis for doing that.

The interesting thing is that quantum mechanically, you seem to be able to do much better, and that's the idea that we want to explain now. So this is also not something that has been known for a long time. The big discovery that this could be done is from 1993. So it's just 20 years ago people realized finally that you could do something like that.

In that way, quantum mechanics is, in a sense, having a renaissance because there's all kinds of marvelous experiments-- teleportation, entanglement, ideas that you could build one day a quantum computer. It's all stimulating thinking better about quantum mechanics more precisely, and the experiments are just amazing.

This thing was done by the following people. We should mention them. Bennett at IBM, Brassard, Crepeau-- can't pronounce that-- Jozsa, all these people in Montreal. Peres, at Technion, and Wootters at Williams College. 1993.

So big collaboration all over the world. So what is the question that we want to discuss? In this game, always there's two people involved, and the canonical names are Alice and Bob. Everybody calls Alice and Bob. It's been lots of years that people talk about Alice and Bob. They use it also for black hole experiments. Depending on your taste, Alice stays out and Bob is sucked into the black hole, or Bob stays out, Alice goes down. But it's Alice and Bob all the time.

So this time, the way we're going to do it, Alice has a quantum state. It has been handed to her, and it's a state of a spin 1/2 particle. Spin 1/2 is nice because you have discrete labels.

So she has this state. It's alpha plus beta minus. And she has it carefully there in a box, just hoping that the state doesn't get entangled with anything and disappear, or doesn't get measured. And her goal is to send this state to Bob, who's far away.

So Alice is sitting here and has this state, and Bob is sitting somewhere here and has no state. And she wants to send this state. This is the state to be teleported.

Now, there's a couple of things you could try to do before even trying to teleport this. Why teleport it? Why don't you create a copy of this state and just put it in FedEx and send it to Bob, and he gets it?

The problem is that there's something in quantum mechanics, something called no cloning, that you can't create a copy of a state, actually, with a quantum mechanical process. It's really a funny thing. You've got a qubit-- this is called a qubit-- a quantum bit. Bit is something that can be 0 or 1. Quantum, it can be two things. So instead of calling it a spin state, sometimes people call it a qubit. For us, it's a spin state. It has two numbers.

And there's no cloning. We will not discuss it here. It's a nice topic for a recitation. It's a simple matter. You can't make a copy. So given that you can't make a copy, let's avoid that idea, save ourselves $15 of FedEx and just try to do something else.

So the one thing Alice could do is that she could say, all right. Well here is alpha and beta. Let me measure the state. Find alpha and beta. And then I'll of send that information to Bob. OK. But she has one copy of the state. How is she going to measure alpha and beta with one copy of the state. She puts it through a Stern-Gerlach experiment, and the particle comes out the plus side. Now what?

The probability that it went to the plus. You've got some information about the alpha squared. Not even because you just did the experiment once and your cubit is gone.

So Alice actually can't figure out alpha and beta. So if she's handed the qubit, she better not measure it. Because if she measures it, she destroys the state, goes into a plus or a minus, and it's all over. The state is gone before she could do anything. So that doesn't work either.

Now there's the third option. Maybe Alice cannot talk to Bob, and Alice created that state with some Hamiltonian. And she knows because she created it what alpha and beta is. So she could in principle tell Bob, OK. Here is alpha and here is beta. Create it again. That would be a fine strategy, but actually there's even plausibly a problem with that. Because maybe she knows this state, but alpha is a number. It is 0.53782106, never ends. Doesn't repeat. And she has to send that infinite string of information to Bob, which is not a good idea either. She's not going to manage to send the right state.

So these are the things we speculate about because it's a natural thing to one wonder. So what we're going to try to do is somehow produce an experiment in which she'll take this state, get it in, and somehow Bob is going to create that state on his other side. That's the teleportation thing that we'll try to do.

So let's do a little diagram of how we're going to do this. So here is going to be the state that is going to be teleported. We'll call it the state C. So I'll write it as psi alpha plus in the state space C sub particle plus beta minus in this state space C. And C is the state she is going to try to teleport.

But now they're not going to be able to do it unless they use something different. They try something different. And the whole idea is going to be to use an entangled state. So basically what we're going to do is we're going to put the source here, entangled state source. And we're going to produce and an entangled state of two particles. And one particle is going to be given to A, to Alice. And one particle is going to be given to Bob. So particle B for Bob is going to be given to Bob. And particle A is going to be given to Alice. And this is an entangled pair.

So there it is. Now what's going to happen? What are we going to do? Entanglement really correlates what goes here with what goes in there. Now entanglement happens instantaneously, and we can discuss this. You have no way of sending information through entanglement in general. There's no such thing as learning something about A when B doesn't measure, learning anything nontrivial about A. So the entangled state is there, and that's what we're going to try to use in order to do the teleporting.

Now morally speaking, suppose I wanted to teleport myself from one place in this room to another. What I would have to do is create an enormous reservoir of entangled states. So here's my generator, and I create billions of entangled pairs. And I put them all here, all the ones here and all the corresponding pairs over there. And then I sort of-- somebody takes me and these billions of entangled pairs, one side of the pair, and does a measurement in which every atom or every quantum state in my body is measured with some entangled state. They've done the measurement, and boom. I reappear on the other side. That's what's going to happen.

So we're going to do this. We're going to have this state, and now we're going to a measurement between this state and this state. Alice is going to do a measurement. That's going to force this particle to actually pretty much become the state you wanted to teleport. So that's the goal.

So let me say a couple more things. Alice will have to send some information actually. Because she is going to have to do a measurement, and she has a console with four lights, zero, one, two, and three. Four lights. And when she will do her measurement, one of the lights will blink. And she will have to tell Bob which one blinked. So she will have to send the number and information of two bits. Because with two bits, you can represent any of four numbers, binary code.

So she will send information of which clicked. And then Bob will have a machine with four entries here. And according to the information that he gets, he will make the state to go through one of those machines, the zero, the one, the two, or the three. So he will push B into one of them out, we claim, will come this teleported state.

So that's the set up. You have to get a feel for the set up. So are there questions on what we're doing?

AUDIENCE: So after teleportation would have some kind of copy [INAUDIBLE]?

PROFESSOR: No. After the replication, this state will be destroyed beyond repair as you will see. So there will not be a copy created by this procedure. You destroy. It's really what teleportation was supposed to be. Not to create another copy of you there, but to take you there. Destroy you here and recreate you there. So no other copy. Other questions? Yes.

AUDIENCE: Does this also work if C is an entangled state?

PROFESSOR: If what?

AUDIENCE: If C say itself contains different parts which are entangled with each other? PROFESSOR: Well, it's a more complicated thing. I'm pretty sure it would work. Maybe you would need more than one entangled pair here. You would need a source that is more complicated. More questions.

AUDIENCE: What do you mean about pushes the state into one of the [INAUDIBLE]?

PROFESSOR: What do I mean by pushes it through one of them? Well you know, Hamiltonians. You get your state. You can put them in a magnetic field. Let them evolve a little bit. Those are machines. So any of these machines are some unitary time evolution. It does something to the state.

AUDIENCE: But one [INAUDIBLE]

PROFESSOR: Sorry.

AUDIENCE: Are there Hamiltonians that are based off of what Alice measures?

PROFESSOR: Yes. So they will be correlated as you will see. So if Alice measures that the light zero beeps, the instruction for Bob is to send the state through the zero Hamiltonian, and one, two, and three Hamiltonian. More questions? It's good to really have a good feeling of this or what we're trying to do and why it's nontrivial.

Yes.

AUDIENCE: This might be a little too intuitive, but in a state which-- Can a Hamiltonian which Bob needs to send B through in order to yield the same state that Alice had, can that also be transmitted quantumly through qubits? Or would you just get like an infinite line of qubits needing to--

PROFESSOR: No no. You know, this is a devise that they can build by themselves. As you will see once we do the calculation, Alice will construct a device that has these four lights and she knows what they mean. And Bob will construct a device that has these things, and they can use it to transport any state. So these machines are independent of the state you want to teleport. You teleported this, you want to teleport another state with alpha prime and beta prime? Sure. Use exactly the same machines, give me another entangled pair, and do it.

AUDIENCE: Well, I think what I meant is that the information between the two machines, does that have to be transmitted classically, or is there some way to transmit--

PROFESSOR: There's no real information. The machines were built, say, in the same laboratory of IBM. And then they're built, and we will tell you how to build each of these machines. And then just put aside, taken away by these two people, and then we'll do it. There's no mystery of sending information about it. That probably will become clear with the computation, which I better start doing soon. Yes.

AUDIENCE: The difference--

PROFESSOR: Louder.

AUDIENCE: Just a question about the first part on the left side of the board. So, when we first do a measurement, does that mean it's something that's like a microscopic quantity, like an energy or something? Or does it just refer to any?

PROFESSOR: When we refer to measurements and quantum mechanics, we talk-- Let me give you just a little bit of intuition here. We typically talk about measuring permission operators, because they have [INAUDIBLE] values. So we don't have to say what they are-- energy, momentum. It's a permission operator you measure. And projector operators into basic states of permission operators. So you could imagine that's one way of thinking about these measurements.

OK. So let's do this. All right. The state to be teleported is this one, and the A B pair is an entangled state. So it will be one of the bell states, psi zero AB 1 over square root of 2 plus A plus b plus minus A plus minus B. So this is the state they share. Of course, Alic only has a handle on particle A, and Bob only has a handle on particle B. Nevertheless the state is entangled even though this could be 200 kilometers apart.

So the total state-- well, we've been tensoring two things. Well, tensoring three is three particles. So I don't think you will be too unhappy to just tensor the whole thing. So psi zero AB tensor alpha plus C plus beta minus C.

So here comes the interesting point. Alice has available the state A. The particle A is not the state A because A is in a funny thing. It's entangled. But it has a particle A available, and it has a particle C available. So Alice is going to do a measurement, and it's going to be a sneaky measurement. It's going to use a bases. Since she has two particles, she can choose a basis of two particle states. Any orthonormal basis will do well by the idea that we can measure with any orthonormal basis. So what she's going to try to do is use the bell basis for A and C.

So let's try to think of what that means. That requires a small calculation here. So this is equal to 1 over square root of 2 plus-- so I anticipate that this will become clear in a second, what that measurement means-- plus minus A minus b Tensor alpha plus plus beta minus C. So I just wrote what this is. OK.

Some algebra. This is the total state, psi total. Let's multiply these things out, and I will keep the labels all the time because I don't want there to be any confusion about what's happening. So what do we get first? Alpha multiplying plus of A. I should write in plus of B, but the order doesn't really matter if I keep the labels. So I'll put plus of C times plus of B.

Then keep multiplying. So we have plus beta, from this with that. So I'll have plus of A minus of C and plus of B. Maybe it's easier to read if I use another line. So I now must multiply the second state times this. So I get plus alpha minus of A with plus of C and minus of B. So this is this times that, minus of A plus of C minus of B plus beta minus of A minus of C minus of B.

OK. So there here my state. But now I have written it in a way that I have here A and C A and C A and C and A and C. So I could decide to measure in this basis. This is an orthonormal basis for A and C. But it's not a very smart basis because it's not entangled. So let's go to the entangled base. So let's rewrite this state, this total state. Nothing has been done yet to the state. We're just mathematically rewriting it, nothing else. We have this, this, this, and that. And I want you now to use these formulas to do this. So I'll do this on this blackboard. We'll have to erase those important names.

So what do we get? Well a little of algebra. Let's do it. A with C plus plus would be that. So I'll write it with one over square root of 2 becomes one half. A with C would be psi zero AC plus psi three AC multiplying alpha plus on B. So I took care of the first term. The alpha is there. The B is there. And AC is there, in which, you know, you can put any labels you want to here. AB, this is the AB state. The entangled AB state. We used AC.

Second term plus one half. Now we have plus A minus C. So it's the second line in there. So it would be psi one AC minus I psi 2 AC beta plus B. Next line, I'll just copy it, one half. Well not. Alpha minus B and here you'll have the minus plus which is the same thing, psi 1 AC plus I psi 2 AC.

And the last term is plus one half psi 0 AC minus psi 3 AC. And we get beta minus B.

OK, almost was there. Let's rewrite this as-- let's collect the psi zeroes, psi 0 and psi 0. You see we're do nothing yet. We're just mathematically rewriting the states in a different basis, the total states. So it is equal to one half psi 0 AC. and look what you get here, very curiously. You get alpha plus B plus beta minus B. Very curious, that was precisely the state we wanted to teleport. Alpha plus plus beta minus.

All right. Let's see what else happens. Here we get plus one half psi-- which other one do I want to copy? Psi 1 AC.

You see this is the state we wanted to teleport. It's here. And it sort of has appeared in the B space. Psi 1 AC, well this time I have this term and this term. So actually it seems a little different. Now we get beta plus B plus alpha minus B.

Then we go to the next. One half of psi 2 AC. So psi 2 is here. So you get I alpha minus B minus I beta plus B. OK. Finally linear combinations.

And finally psi 3. What is psi 3? Well two terms also for psi 3. This one and this one. So you get alpha plus B minus beta minus B. Kind of the end of math by now. You've proven a funny identity actually in doing this. And maybe this blackboard should-- to make sure you understand.

This is the calculation of total state. And here we go. So let me show you one thing. This is actually the state we wanted. So this will be called psi in the B basis, in the B space. The state that you wanted to teleport that was psi in the C basis, now it's psi in the B basis.

Those ones look a little funny, but this one actually looks like this thing, looks like sigma 3 times psi. Because if you have sigma 3 on this state, it gives you a plus 1 here and a minus [INAUDIBLE] value. So that's sigma 3 psi.

This actually has flipped the plus and the minus. So that actually is sigma 1 psi. And this state is actually sigma 2 psi.

OK everything is in place now. We've just done math, but now comes the physics. Alice is going to measure in the bell space of A and C.

So these are the four bases states. So she's going to measure in one of these bases states. And as see measures, she falls and the wave function of her collapses into one of them. So when she gets the zero basis state, this light blanks. If doing the measurement on AC, because she has both particles A and C, she gets this basis state-- recall the postulate of measurement-- light one blinks. If she gets the third like 2 and the fourth here.

Suppose the state light zero shines. Well the state collapsed into this. She is now sitting with psi 0 AC that has no memory whatsoever of the original state C, but B is sitting with this state, the state we wanted to teleport. So if light zero shines, she tells Bob, let it go to machine zero where there's no magnetic field, nothing. So actually the same state goes out.

If she gets psi 1 as the measured state, again no memory in this state about alpha and beta. But Bob gets sigma 1 psi 1. So he puts it into the first Hamiltonian for a picosecond, produces a sigma 1. This Hamiltonian, this box I takes a state into sigma I state. It's a unitary operation. So puts a sigma 1 and gets psi. If light two shines, goes to the machine two, which produces a sigma 2, and so he gets the state. Light four shines, the third Hamiltonian, he gets the state. Any of the four options, he gets the precise state. The state has been teleported. You needed to send only the information of which light shone, and the state is on the other side of the ocean.

All right. That's it for today.