PROFESSOR: OK. So what I'm going to try to do now is set up again this equation and do the analog of what we're doing there and try to determine this function fk in some nice way. All right. So let's think of this equation.

I want to do it in pieces. So psi of r is going to be equal to some formula. And then it has to be equal to this right hand side. Let's write the e to the ikz the way we've done it before. It's up there. So I'll write it here. Square root of 4 pi over k square, sum of l equals 0 to infinity, square root of 2l plus 1, i to the l yl 0 1 over 2 i e to the ikr minus l pi over 2 over r minus e the minus ikr minus l pi over 2.

Wow. It's tiring this r. Plus f of k of theta e to the ikr over r. OK. So what do we have here? We've written the right hand side of this equation. I copied it. I have not done anything except taking r much greater than a.

Because otherwise in the plane wave into the ikz, I could not have expanded the Bessel functions unless I took r greater than a. But that's good, because we now have our waves. OK. We have our waves there. Now, look at this right hand side. Where is the incoming wave in this right hand side? The incoming wave is here.

That's the only term that is incoming, because this is an outgoing wave, and this is an outgoing wave. So if I want to write the left hand side, the incoming wave of the left hand side has to be equal to this wave. And of course, the outgoing wave of the left hand side will also have to be equal to whatever is outgoing here, but the incoming must be this.

So I'm going to write this left hand side and already use this and put 4 pi over k square sum of l equals 0 to infinity 2l plus 1 i to the l y l 0 1 over 2 pi, big parentheses, and one outgoing wave and one incoming wave minus ikr minus l pi over 2 over r.

And here, I don't know what to put, but I've put already there on the left hand side of this equation for psi of r for the full solution, a wave that matches the right hand side, because it has the same incoming wave. And now, I'm going to use some physical intuition to guess what we'll have to put on this part. This is the step that requires a little imagination, not too much, because we already did something similar here.

So what's happening here and here is intuition I think you should keep after weeks of this course when it's all forgotten, there's some intuition that you should keep. And it's about this scattering happening for each partial wave independent. Yes.

AUDIENCE: 2i.

PROFESSOR: 2i, yes. No, 2 pi there. Yes. Thank you. So here it is. This is a solution, and we've got the intuition already. I will justify this later, of course, very precisely. But I think that this one you need to have a little bit of an intuition of what you should do. And first, we said each l works separately to create a solution of the Schrodinger equation. That's superposition, and it's [INAUDIBLE]. Each l is working separately. Each l is like a scattering problem.

Each l has a wave that comes in and a wave that comes out, because these things, j and n, have waves, and they have an in and out. So these have some in wave and some out wave. And if each wave works separately, it has an in wave and then out wave, in a scattering problem, these waves must have the same amplitude, because otherwise they wouldn't have the same probability current, and probability would get stuck.

So this must be an outgoing wave having this same amplitude as this wave. And by the argument we have here, it just differs by a phase. So we'll put here e to the ikr minus l pi over 2 plus 2i delta l over r. So that this wave, spherical wave, that it's outgoing, it has the same amplitude as this one, and cannot be the same. The only difference can be a phase shift, and that's the phase shift.

So your picture is scattering in three dimensions. Looks like, OK, you threw in a plane wave and out came a spherical wave out. The other picture that is more consistent with the way you solve it is that you have an infinite set of partial waves for different l's, each one scattering, the l equals 0, the equal 1, the l equal 2, all of them scattering.

So this corresponds to an [? ansatz ?] in terms of phase shifts, and now you can say you've parameterized your ignorance in a physical way. You've discovered that all that characterizes the scattering is, as it was in one dimension, a phase shift. In one dimension, there was a single phase shift, because you didn't have all these general solutions that you had in three dimensions. Your energy eigenstates were momentum eigenstates there were non-degenerate really. There was just a couple of momentum eigenstates, wave in and waves out. Here is infinitely degenerate. There's spherical stuff. So there's a phase shift for each value of l.

So we've parameterized the physics of the scattering problem in terms of phase shift, and now, it's interesting to try to figure out what is this quantity after all in terms of the phase shifts. It's already here. We just have to solve it. So from this equation, I now can say that this term cancels with this term, and now, I can solve for this term f of theta e to the ikr by collecting these other two terms together.

And therefore, fk of theta e to the ikr over r is equal, and it's exactly the same thing I did here, cancel here, pass, and the mathematics is going to be completely analogous, except that they have to carry all that sum there. No big deal. So what is it? Square root of 4 pi over k, sum from l equals 0 to infinity, 2l plus 1, i to the l y l0. 1 over 2i.

OK, 1 over 2i. OK, I'll do it this way. e to the 2i delta l minus 1 e to the ikr e to the minus il pi over 2. I think I got everything there. I put the first term on the right hand side, I moved it to the left hand side. The coefficient was all the same. This was the coefficient they both had the same coefficient. Then I just have to subtract these two exponentials over r. So I did forget the r. So I just subtract the two exponentials. Both exponentials have the e to the ikr minus l pi over 2. And the difference is that the first exponential on the top has the extra e to the 2i delta l minus 1, and the other one doesn't.

So what do we get here? This part is e to the i delta sine delta, and this part is e to the minus i pi over 2 to the power l, which is minus i to the l, and i to the l times minus i to the l is happily just 1. i times minus i is 1, and 1 to the l is 1. So this term and this term cancel.

So finally, and I can cancel happily they r dependence is all the same. I can cancel this r dependence, this r dependence. And finally, we've got fk of theta equals square root of 4 pi over k sum from l equals 0 to infinity square root 2l plus 1 y L0 of theta e to the i delta l sine delta l.

So that said, that's our formula for fk in terms of the phase shift. So what have we achieved? We want f of k, because that gives us the cross section. What we have figured out is that the calculation of fk really requires knowing the phase shift. And the phase shifts are defined by that formula over there, where we have estimated how one wave is connected to the other one, the incoming and the outgoing for a given fixed f, for a given partial wave, how they're offset by this phase shift.