PROFESSOR: OK, so we're good to consider, therefore, the two cases. So let's consider the first case, that this can be pretty important, this case one, when omega fi plus omega is nearly 0. In this case, what's happening? You have Ef minus Ei plus h bar omega is nearly 0, or Ef roughly equal to Ei minus h bar omega.

So what has happened if you have a number-- so the question is, when this happens, which means that your omega-- that is, your perturbation-- is tailored to produce this, then Ef is Ei minus h omega. So you can think of the energy scale here. And Ef is lower than Ei. And the difference is h bar omega.

So what is this process? This process is called stimulated emission. And why is that called stimulated emission? Because you're going from a state of energy Ei to a state of energy Ef that has lower energy. In that process, you're releasing energy h bar omega to the perturbation.

So it's almost-- you would say it's stimulated because the perturbation did it. Still, it almost seems like something the system would do by itself. It has higher energy. It can go to lower energy and emit something with frequency with energy h omega. But it does that only because there is a perturbation. If there was no perturbation, this would not happen.

That's why, say if you consider a hydrogen atom or a system, it's not that the electron that is in a higher state just jumps by itself down. It jumps by itself out because it can couple to other degrees of freedom. And in particular, it can couple to an electromagnetic field and send out the photon. So here, you have stimulated emission of energy h bar omega. So stimulated emission-- emission.

And then the other term corresponds to the case when omega fi minus omega is equal to 0, in which case Ef is equal Ei plus h bar omega. This time, Ef is higher than Ei h bar omega. And this process is called absorption-- absorption.

You absorb energy h bar omega from the perturbation. The perturbation is capable of giving the system energy h bar omega to enable a transition between Ei and Ef. So this harmonic perturbation has two tricks up its sleeve. It can push you up by giving you energy, or it can stimulate you to go to a lower state and give energy to the system represented by the perturbation. So two good things it can do, and both cases are pretty important.

So we're going to develop one of these cases. The other one has exactly the same formulas. And we need the Fermi golden rule for this situation. So that's what I'll do right now.

To fix our notation, let's just do the case of absorption. But both are going to be taken care simultaneously. So absorption-- so what is C?

So when you're doing absorption, you're saying, OK. I'm basically having this process in which Ef is related to Ei in this way. This term is becoming 0. And this term is negligible. So you can completely ignore one term when you're doing absorption. And you can ignore the other term when you're doing a stimulated emission.

You would say, oh, but it's not exact. What about if I keep it? Well, there are many things that we don't do exact. This term is much bigger than the other-- in principle, infinitely bigger-- because we're going to be integrated over a narrow set of states. So it would make no sense to keep those other things. Those other terms would be much smaller, probably, at even the next order of perturbation theory. So we keep, therefore, the second term. And so what do we have?

Cf of t naught 1 minus H fi prime over h bar, e to the i omega fi minus omega t over 2. You don't see that term, of course. But I'm going to take out half of this phase so that I get a sine function out here. So I took out half of the phase. And then I get 2i sine omega fi minus omega t over 2. OK, that's this thing.

So the probability to go from initial to final state at time t naught would be the C f1 of t naught squared. And that's 4H fi prime, because of the two here. It's 4H fi prime squared over h squared sine squared of omega fi minus omega over 2 times time over omega fi minus omega squared. And this is all for Ef roughly equal to Ei plus h bar omega.

So I didn't do much. I really didn't do all that much so far. I just took the second term, rewrote it with a sine, and calculated its norm squared. That's the probability to go from initial to final states.

So the last step is integrating over a final state. So you have this sum over final states of this probability to go to the final state. And we'll write it as the integral over the set of states rho of e final dE final probability final to initial of t naught.

And this is the same calculation we were doing before. And for this calculation, we just

substitute what P fi is. We assume that as we integrate, we're going to have a narrow range so that this function, rho of Ef and h prime fi can go out of the integral. So what are we going to have?

I'll write it here. 4-- from the matrix element, from this thing-- H fi prime squared over h squared. That goes out.

Then the rho of Ef goes out. So I'll write it rho of Ef. And rather than leaving it like Ef, like that, because it seems like a variable, still, of integration-- if I take it out, I should be explicit what this Ef is. And in this case, Ef-- we're doing absorption. So this is equal to Ei plus h bar omega.

That's this central contribution. And that's where you're taking rho out. So this is pretty important. This rho is evaluated at the final energy, which in this process is h omega in addition to Ei. And then the rest of the integral dE f-- you still have dE f-- and the sine function. So sine squared of omega fi minus omega t over-- t naught, I'm sorry. I'm missing t naught everywhere. But I did copy it. t naught and t naught's up there. Over 2 over omega fi minus omega squared.

And this is the story of the lobes. This is the function that has the lobes as a function of Ef. As a function of Ef, this quantity varies and starts developing the lobes. And the lobes happen for values of Ef that are separated by some h bar divided by t naught.

So this is the exact same integral we analyzed for the constant transitions. And it is the same integral that we argued that could be done as a sine squared x over x squared. So I will not do it again. This integral gives h bar linear in t naught over 2 pi.

When you study this, you will have the notes and you can review it. If you've taken notes, you will see that is exactly the same integral we had last time, except that we didn't have the omega, which really doesn't change things. It just shifts the zero. So the peak of this contribution is when omega fi is equal to omega. Before when we did it was when omega fi was equal to 0. That's why, in the constant transitions you conserved energy.

So with this result in there, the whole answer here is 2 pi over h bar-- h bar only because one h bar cancels, indeed-- rho at Ef equal Ei plus h omega H fi prime squared times t naught. That's all that is left, which is great because that is our Fermi's golden rule. Remember, that's a transition probability. And the rate is obtained by dividing this by the time that has elapsed to find the probability per unit time.

That's the rate. It's probability for transition per unit, times 2 pi over h bar rho at Ef equal Ei plus h bar omega times the matrix element f H prime initial squared. That's Fermi's golden rule for harmonic transitions, and in particular, for absorption.

But stimulated emission, the calculation is completely analogous. So the end result for stimulated emission is just minus H omega. The final energy now is this one. So the top sine-- plus is for absorption, minus for stimulated emission.

And if you want a reminder here, it was that delta H was 2H prime cosine omega t. So this reminds you that this H prime you have here is the matrix element of the perturbation with this convention. This omega you have here is the frequency of the perturbing Hamiltonian. And that's what has happened.

So this is Fermi's golden rule. It's over. We've done it, done it basically for two cases-- the constant perturbation and the harmonic perturbation. And there's a lot of physics here that we will be exploring starting now, but continuing with atoms and radiation in general, atomic transitions.

W is the rate. So this is called the transition rate per unit time-- transition rate per unit time. So the probability of transition or probability of transition per unit time-- probability of transition per unit time-- that's probably more understandable than the word rate. So this P fi gives you the probability of transition after a time t naught. Happily, it's proportional to t naught, so you divide by the time that has elapsed, and it's a probability of transition per unit time. So once you compute this number, you get the probability of transition per unit time. So if this W is something and you have a billion atoms, you multiply that probability by the number of atoms you have, and you know how many have decayed already.