

**PROFESSOR:** So atom-light interactions. So we will focus just on the electric field,  $E$  field. Magnetic field effects are suppressed by the velocity of the electrons divided by  $c$ . And that you know is the fine-structure constant. So magnetic field effects are suppressed.

Ignore magnetic  $v$  over  $c$  corrections. With  $v$  over  $c$ , again, of the order of the fine-structure constant. We will also think of typically optical frequencies.  $\lambda$  in the optical range. So from about 4,000 to 8,000 angstroms. And that's much, much bigger than a  $\lambda_0$ , which is about 0.5 angstrom.

So that's good for our approximation. That the wave is relatively constant, being the wavelength so large it's constant over the extent of the atom. So we will think of the electric field of the atom,  $E$  at the atom. Our electric field, a bit of notation, will depend on time and it will be a real function of time times a unit vector to begin with.

This will get more interesting because we're going to be dealing with thermal radiation. So eventually, this vector,  $\hat{n}$ , will be pointing and we will average it over all directions because thermal radiation comes with all polarizations and in all directions. So there will be a little bit of averaging necessary for that. That will happen next time as we wrap up this discussion.

So this picture is of an atom sitting here. And in particular, its electron, which is the particle that reacts the most in the electric field. And there is a unit vector,  $\hat{n}$ . And there's  $E$  of  $t$  here.

So the electric field,  $E$  of  $t$ , is  $2 E_0 \cos(\omega t) \hat{n}$  not in our conventions cosine  $\omega t$  times the vector,  $\hat{n}$ . So what is the vector on the scalar potential  $\phi$ ,  $t$ . It's minus  $r$  times  $E$  of  $t$ . This is the formula that gives you  $E$  as minus the gradient of  $\phi$ . This formula is not true in the presence of magnetic fields in general. There is a time derivative of the vector potential. But again, we're ignoring magnetic effects. So this is good enough for us.

If you take the gradient of this formula, the only  $r$  dependence is here. There's no  $r$  dependence in this electric field. And in particular, we consider the wavelengths to be very big. And this is good enough.

So what is the perturbing Hamiltonian due to the coupling of the electric field to the charged particle? And we'll say atom, and we'll put an electron or something, and we'll say the charge is  $q$ . Eventually it will be minus  $E$  for the electron. But let's keep it at  $q$ .

Delta H is q times phi of r and t. So this is minus q times r times E of t vector. Or minus q times r times n times E of t.

So so far, simple things. We're just considering the electric field and how it adds on a charged particle. This is, of course, the simplest situation. We will be considering in this course soon, in fact, starting next lecture, the general interaction of charged particles with electromagnetic fields. But for the time being and in this approximation, this is enough.

And we will define now a dipole operator. It's something that you should keep in mind. It's the usual thing when you define dipoles is you sum or integrate over charges times position vectors. So this is a dipole operator. And I emphasized the operator because of the r. When you have matrix elements, transitions between states, everything will have to deal with those matrix elements of r. And that's why we'll have a dipole term there.

So delta H at this moment has become what? Minus the dipole times the electric field. So let's do this. Minus the dipole. I'll do it here. Minus the dipole dotted with electric field. Vector.

Or minus d dot n with 2 times the magnitude of the electric field. Or minus v dot n times 2 E not cosine of omega t. Factors of 2 keep us busy always. And we have to get them right. Remember when we did our definition of perturbed Hamiltonian we said that delta H was going to be equal to 2H prime cosine of omega t. And our transition, amplitudes, and everything were written in terms of H prime. So this was our definition.

So at this moment, we can isolate H prime here. So H prime is everything except for the 2 and the cosine of omega t. So H prime, for our problem of atoms interacting with electromagnetic fields, is a dipole interaction. And it's given by this nice simple formula. That's our kind of important end result.

So if we have this Hamiltonian, we have calculated the probability for transitions, for example, from b to a, as a function of time. When we have a harmonic perturbation coupled to a two level system, we have a probability. We don't have yet a rate. We just have a probability. And this is equal to the other one, to the reverse one. It's 4H<sub>ab</sub> prime squared over h squared sine squared of omega<sub>ba</sub> minus omega over 2 t over omega<sub>ba</sub> minus omega squared.

That was the formula we had. In our case now, H prime ab is that. So we'll get the following. 4 E<sub>0</sub> squared. The E<sub>0</sub> factor here goes out. And then we will have the matrix elements of the dipole operator, d.n ab. So it will all be matter of the dipole operator. H squared. And these

same factors. Sine squared  $\omega_b a \text{ minus } \omega$  over  $2T$  over  $\omega_b a \text{ minus } \omega$  squared.

So this is the transition probability. We don't have a rate. But the rate will come when we integrate over all the photons that contribute to this process. And we'll get an exact analog of Fermi golden rule. So that will be next time.