PROFESSOR: So here it is. Suppose you have a serious expansion in lambda. So this is the state-- when lambda is equal to 0, this should be the state. But when lambda is different from 0, that will not be the state. We'll have lambda correction.

So this is the first-order correction to this state. So that's why I put the 1. And you should think of it first order-- oh! --because it comes with a lambda.

This is the second-order correction to the state, because it comes with a lambda squared. And the same thing here. So the superscript is telling you what order in lambda you are working-- to what accuracy.

So what is the most urgent thing to find the first order of corrections? If you find them, and we still have time [LAUGH] for the second order, you go more and more. OK, let's continue. Let's solve some of this.

So our next task is to solve this problem. And here we go. Let's solve that. So what am I going to do? I'm going to just write this equation slightly differently.

I'll write it as h of lambda, which is-- OK, I'll write it differently. h0 plus lambda delta h-- that's h of lambda-- minus En of lambda on the state n of lambda is equal to 0. That's your Schrodinger equation, the time-independent Schrodinger equation, we're trying to solve. And now I'm going to just write it out, so that you can see what we get.

So it's going to take a little bit of writing. Let me collect the terms that have no lambda. It's h0. This has a lambda, but En begins with En0 0 that has no lambda. So, from this parentheses, this is a term without a lambda. It came from here, En0-- it's here.

Now let's look at the terms with a lambda. So I want to see how I'm writing. I want to write it with a minus sign. So, with a lambda, we have minus-- from here, we have a term En1 minus delta h.

That is all the terms with a lambda. So I should put the lambda, as well, here. Probably I want to put it in front. Minus lambda [INAUDIBLE]. En1, from there, and the lambda delta h, with a double minus sign.

Then it goes simple, now. I've taken into account these two terms. All the rest come from here.

So you have a minus lambda squared En2, and, at some point, a minus lambda to the k Enk. And then it goes on. And then we write it like a big bracket, here. That's the parentheses.

And now the state. You have n0 lambda n1 plus lambda squared n2 plus lambda to the k, the k-th correction to the state. And it goes on forever. And it is here. And all that is equal to 0. [LAUGH] Looks daunting, but it's not.

What should we do? Well, here is, again, lambda helpful for you. Lambda is a parameter.

The left-hand side is a polynomial on lambda. It should vanish for all values of lambda, because the Schrodinger equation should hold for all values of lambda. When a polynomial vanishes for all values of lambda, the argument of the polynomial, all the coefficients must vanish, of the polynomial. Therefore, we must look at what is 0-th order in lambda, here, and see what we get.

Well, 0-th order in lambda, we get this equation, h0 minus En0 [? on ?] n0 equals 0. That's 0-th order in lambda, and that's an equation that is not new. [LAUGH] You knew it! That's a statement that n0 was an eigenstate of the original Hamiltonian.

So it's good. You know, the 0-th order things had to work, because we said, to 0-th order you have the known Hamiltonian. Let's look at the term with order lambda.

Lambda can get from this term in the Hamiltonian acting on n1. That's order lambda, so let's write it here. h0 minus En0 on n1. And the other term comes from a lambda in the first factor and no lambda in the second. So it's this term, this acting on that state. Look-- there's a lambda, there's a minus sign, so you can put it on the right-hand side. And we get En1 minus delta h acting on n0.

Let's be a little daring and try to get the lambda to the k. So h0 minus En0. And I want to see what are the terms that have lambda to the k, power k.

Well, H0 minus En0 acting on this one has lambda to the k. So you have n to the k, here, nk-- not "to the k." And then, to get lambda to the k, I could have a lambda here, and the term that is before this, lambda to the k minus 1, nk minus 1. And it goes with a minus sign to the right-hand side.

So you would have En1 minus delta h on n k minus 1. And then you'll have En2 on Enk minus 2. And it will go all the way until you'll have Enk acting on n0, the original state.

So let me box this, and, uh-- those are the equations that we get. And we have to solve them. And we can solve them. That's the nice thing about this.

Well, this one, we argued, it's simple enough. We don't have to do much about it. Then we have to solve for n1. Oh, but the second equation actually has two unknowns. We don't know the state n1, the first correction, and we don't know the energy correction.

But that's kind of the useful thing that is Schrodinger equation. You don't know the energies, [LAUGH] and you don't know the eigenstate. So you couldn't expect this.

It's kind of interesting. If you have solved for n1 and En1 and n2 and En2, up to some point, the next state involves nk, the energy of the state nk, and all the things that you already know-- the lower energies, and the lower states. So you can solve this recursively, one equation at a time. Depending how much work you want to do, you go more and more equations. We'll typically go the first and the second and sometimes make some remarks about these things.

There's one important simplifying assumption we can make that helps us a lot. I can claim you can choose n1 and all the higher ones, n2, to be orthogonal to n0. Think a little about this. What does that tell us?

It says, oh, this vector should have no component along n0. And these vectors should have no component along n0. The intuitive reason why this is the case that you can choose that and simplifies your life is that, if it had some component along n0, you could just sort of move it here, and now you would have n0 plus a function of lambda times n0, and you can divide this by this function and rescale the state back, to have an n0 here.

The normalization of this state-- originally, we have them normalized, but it would make our life extremely more complicated if we tried to do this perturbation series and keep the normalization. The states are not going to be normalized, but you know that's not the problem. If they are not normalized but are normalizable, you can always work with them.

So we won't normalize them. But the idea is that any piece that is proportional to and n0, you could reabsorb it. Now, that's vague. If you didn't understand that argument, I commend you, because it's a vague argument.

So let me do a more precise argument. Suppose, for example, you're solving this equation, and you solve n0. And suppose you've solved now for n1. And you got your n1.

You're done, you solve the second equation, you're perfectly happy, but somebody says, you know, it has some component along n0. What can you do? OK, you say, look-- if n1 solves this equation, n1 plus any number c times n0 still is a solution, I claim. Why?

Because n1, the state n1 that you're trying to find, only appears on the left-hand side. And n0 is killed by this combination in the first equation. So, if you have a solution n1, you can replace it by this one. And you can choose c to cancel whatever n0 you had in here.

So you can always produce a state that is orthogonal to it. And it's easier to work with that. And this goes on forever. Suppose you've solved now n1 that has no piece along n0, n2, n3, n4-- all those-- and you go up to here, and nk has a piece along n0. You can still add the constant to nk times n0 and make it work. So you can always do that. They're orthogonal to n0.

So let me say one more thing that is very amazing. Let's look a little a first look at the equation lambda 1 of order lambda 1. Or-- I'm sorry, these parentheses are not good. This is lambda to the 1. This is lambda to the k. The parentheses is bad. Lambda to the 0. Lambda 1.

So this is our equation. We'll have h0 minus En0 n1 equal En1 minus delta h n0. I'm going to do one thing. I'm going to push a bra n0 on the left. Should I do it on that same equation? Let's save a little time.

That equation, we already had it here. So let's put an n0 here. bra n0 here. OK.

So here is the challenge. We've put a lot of notation on the blackboard. And maybe by now all the symbols are floating in your head and not making much sense. I want you to figure out what is the value of this left-hand side.