PROFESSOR: So case A-- weak Zeeman effect. So what are our states here? We discovered that and we know those are the coupled basis states. And the states of H0 tilde eigenstates-- they are approximate eigenstates-- are the states n, l, j, mj. And the energies were energies dependent on n and j.

So the 1S 1/2, 2S 1/2, 2P 1/2, and 2P 3/2. Roughly, to remind you of what happened, the original states were shifted, and we used the j quantum number in here. And those are our multiplets. Those are our multiplets. And we have a lot of degeneracy as usual.

So this is degeneracy here and degeneracy here because a multiplet P 3/2 is j equal 3/2. And that's four states. Here you have two states, and here you have two states as well. So quite a bit of states that are degenerate.

So in principle, when we do the Zeeman splitting, we may have to consider the full matrix n, l, j, mj, delta H Zeeman, nl prime j m prime j. So what are our degeneracies? Our degeneracies are when you have a given value of j.

So a degenerate subspace can have different l's-- for example, here-- but the same j, and therefore different mj's. Or within a given j multiplet, it might have different mj's. So this is the scope of the degeneracy. And in principle, we may have to diagonalize a matrix like that by looking at the degenerate spaces. If you're doing the level two, you would have to discuss these four states here. You would have to discuss this other four states.

Happily, we don't have to do that much because, as usual, delta H Zeeman is proportional to l z plus 2Sz. And this commutes with l squared with delta H Zeeman. l squared commutes with any l operator. It certainly commutes with any S operator. They don't even talk to each other. And therefore, l squared commutes with l Zeeman, which means that when l is different from l prime, this matrix element has to vanish.

This is our remark from perturbation theory long, long ago. You have another operator for which the states have different eigenvalues, commutes with your perturbation. The matrix element of the perturbation must vanish between those states. So we don't have eigenstates like that. And when l is equal to l prime already-- so we focus on l equals to l prime-- we only need to worry within multiplets.

So you have n, l, j, mj, delta H Zeeman now, and l, j, mj prime. It's an issue of mj prime now. But Zeeman thing commutes with Jz. Jz commutes with delta H Zeeman. Jz is Lz plus Sz, and z components in angular momentum-- two identical components always commute, of course. So Jz commutes with delta H Zeeman. So this thing will vanish unless m is equal to m prime.

And that's great because you're back to nondegenerate perturbation theory. The whole matrix, this Zeeman thing could have turned out to be complicated matrices. No. It's perfectly diagonal in this basis. There's nothing to worry about here, except that it's still not easy to compute, as we will see.

So what do we need to compute? We'll have the first order corrections due to Zeeman on the n, l, j, mj basis is equal-- well, the Zeeman Hamiltonian had an e over 2mc. So let's put it there. e. Let's put the B close to the e. 2mc. And now we have to do n l j mj Lz plus 2Sz n l j mj.

Perfectly diagonal. And that's nondegenerate perturbation theory. It's going to give us all the energies we want. All the splittings we want. So basically what's going to happen, as you can see here, is that the things down mix. Everything is there. Honestly, these two levels are going to split. These two levels are going to split. These four levels are going to split. Everything is going to split here.

The remarkable thing of this formula, and it's going to keep us busy for about 10, 15 more minutes, is that this thing, this matrix element, is proportional to mj. So the states split proportional to the m quantum number. The state with the m equal 3/2 will split three times as much as the state with m equal 1/2. And that's not obvious here.

It's a remarkable result. It's part of what's called the Wigner-Eckart theorem, something that you study in graduate quantum mechanics. But we're going to see a bit of it, the beginning of it, in this computation. And it's a fairly remarkable result. So the remarkable result here-- remarkable-- is that E [INAUDIBLE] are proportional to mj. And that defines a linear splitting because it's linearly proportional to the magnitude of the magnetic field and divides the states nicely.

So we want to understand this matrix element. And there's a little thing we can do. Notice that Lz plus 2Sz is equal to Jz plus Sz. You can take one of the Sz's and complete Jz, and you're left with that. So the matrix element n l j m Lz plus 2Sz n l j mj is equal-- if you have a Jz, that gives you just something proportional to Hmj plus n l j mj Sz n l j mj.

So OK. A little bit of the mystery maybe seems to you at least consistent here. I said this matrix element turns out to be proportional to mj. And certainly, this piece, having to do with the J component here, is proportional to mj. The mystery that remains is why this matrix element would be proportional to mj.